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Comparative Analysis of Human Mortality Models: Insights from Denmark and the Netherlands

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Abstract: This study provides a comparative analysis of human mortality models for Denmark and the Netherlands from 1970 to 2008, using data from the Human Mortality Database (HMD). The analysis explores mortality trends across age groups and genders, highlighting key patterns and differences between the two countries. The Lee-Carter model is applied to analyze mortality dynamics, while the Gompertz model estimates age-specific mortality rates, both proving effective in capturing long-term trends. The study finds consistent differences between male and female mortality, with men experiencing higher mortality rates across all age groups in both countries. Mortality rates have declined significantly over time due to advancements in healthcare, socioeconomic improvements, and better living conditions. To project future mortality trends, the Random Walk with Drift model is used, offering insights into longevity and life expectancy for both populations. The results emphasize the need for ongoing public health improvements and policies to address the challenges posed by aging populations.

Keywords: Mortality Models, Hazard rates, Lee-Carter Model, Gompertz Model.

1. Introduction

When poverty and hunger are no longer pressing issues, people tend to focus more on improving their quality of life and, more fundamentally, extending human longevity. Especially in recent decades, some governments, particularly in developed countries, have already worked on predicting human mortality models, which provide evidence for extending human survival.

Previous research has significantly contributed to the exploration of mortality models and inspired subsequent studies. For example, Lundström and Qvist researched mortality forecasting and trend shifts using the Lee-Carter model for Sweden [1]. The Gompertz model has also been widely applied in detecting plateau periods [2]. Multi-country comparison studies have proven useful for analyzing specific aspects of mortality, considering economic and societal differences [3]. Building on these pioneering works, this study seeks to assess several mortality models by comparing Denmark and the Netherlands as representative examples of developed European countries, aiming to improve health risk management.

The data utilized is sourced from the Human Mortality Database (HMD), which is the world's leading scientific data resource on mortality in developed countries. This is satisfied with the research goal. Original data set for modeling contains Denmark and the Netherlands from 1970 to 2008. Based on the model, period 2008–2058 will be forecasted. It is useful to compare the predictions with the

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actual mortality in that period as validation. The analysis will capture long-term mortality trends, focusing on variations across age groups and genders.

2. Current Mortality Rate

The analysis of the mortality model is based on the hazard rate. The hazard rate, which represents the mortality rate for an individual of a specific age, is accurately approximated by the central death rate:

$$\widehat{\mu_{xt}} = lnD_{xt}/E_{xt}$$

Where D_{xt} denotes the individuals who have died at age x in year t, and E_{xt} signifies the individuals who have been exposed to risk at age x in year t. Figures 1 and 2 below depict the logarithms of central death rates for individuals aged 25 to 100 for the years 1970 and 2008. A differentiation is established between males and females.

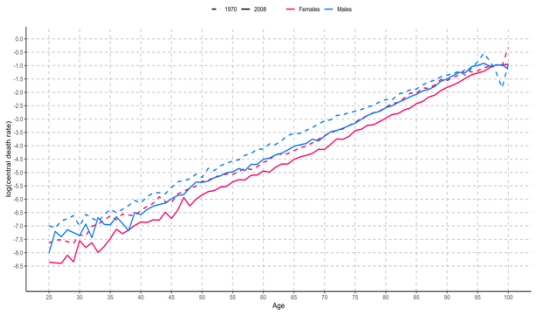

Figure 1: Central death rates for Denmark.

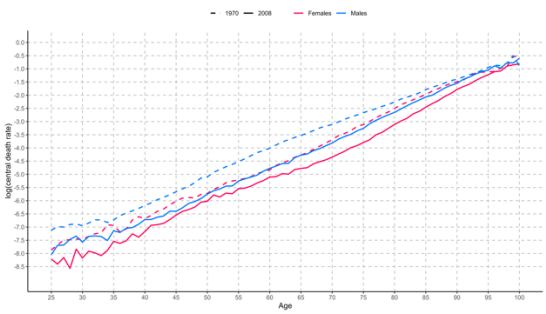

Figure 2: Central death rates for Netherlands.

The Netherlands displays a more uniform graph than Denmark, likely due to its larger population. Denmark shows increased mortality rates among males aged 25 to 50 and those over 95. Judging by the slope, trend, and other key features, there are few differences overall between these two developed European nations, which are geographically close and have high-income economies. However, both countries exhibit variable central death rates across both high and low age groups. The limited observations in these age groups lead to substantial variance in the hazard rate. Conversely, the rates for the middle-aged demographic appear to consistently rise. As age advances, the death rate escalates. Additionally, the mortality rate for men exceeds that of women, largely due to men's higher risk of death resulting from lifestyle choices. Finally, mortality rates have significantly declined between 1970 and 2008, likely due to improved living conditions and advancements in medical science.

The residual life expectancy is calculated to forecast the future lifespan of these individuals. This is accomplished by a recognized approximation [4]:

$$\dot{e}_x = e_x + 0.5$$

Where e_x denotes the curtate life expectancy of individuals aged 0 and 65 in the years 1970 and 2008. This represents the cumulative survival odds up to the age of 100. $\dot{e}_x$ represents the period complete life expectancy, as detailed in Table 1.

Table 1: Life expectancies for the Netherlands and Denmark

Period Complete Remaining Life Expectancy				
	1970		2008	
	Denmark	Netherlands	Denmark	Netherlands
Female, Age 0	96.483	96.173	97.470	97.453
Male, Age 0	95.758	95.346	96.431	96.395
Female, Age 65	31.670	31.333	32.585	32.551
Male, Age 65	31.059	30.653	31.615	31.533

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The life expectancy in Denmark is somewhat higher than that in the Netherlands. The disparity among these countries diminished in 2008. The projected lifespan has risen from 1970 to 2008, with females anticipated to outlive males on average. Generally, elderly individuals possess a diminished life expectancy compared to neonates.

Truncation, the exclusion of data over a threshold, has been considered by incorporating the observations of individuals over age 99 into the mortality and exposure rates of those who died at age 100.

3. Current mortality per age

This section will estimate the mortality rates for females aged 25 to 90 in Denmark in 2008 using the classical Gompertz model [5]. The model presupposes:

$$\mu_{x,2008} = e^{a+bx}$$

The objective is for the ideal parameters a and b to be identified. The model delineates the hazard rate for a single year, implicitly presuming that mortality rates remain constant over time. Three approaches for estimating the current mortality rate are presented:

Method 1: OLS

$$\min_{a,b} \sum_{x=25}^{90} \left[\frac{D_{x,2008}}{E_{x,2008}} - e^{a+bx} \right]^2$$

Method 2: OLS with logarithms

$$\min_{a,b} \sum_{x=25}^{90} \left[ln \frac{D_{x,2008}}{E_{x,2008}} - (a+bx) \right]^2$$

[Attention must be directed to observations that are equivalent to zero. Since taking logarithms requires that the expression within the logarithmic brackets not be zero, this issue is addressed by assigning a value of 1/10 to all zero deaths. This results in a minor error that can be disregarded.]

Method 3: WLS (Weighted Least Squares)

$$\min_{a,b} \sum_{x=25}^{90} D_{x,2008} \left[\frac{D_{x,2008}}{E_{x,2008}} - e^{a+bx} \right]^2$$

The mortality rate increases significantly with age. WLS is utilized here to equilibrate the data and adjust for the mortality of older individuals by weighting during parameter optimization.

Applying these three methodologies on the dataset yields Figure 3 and Figure 4. All the approaches have comparable characteristics. Utilizing the original data, all three methods yield satisfactory fits. Method 1 exhibits superior efficacy with elderly individuals. Method 2 is clearly favored for data subjected to logarithmic transformations.

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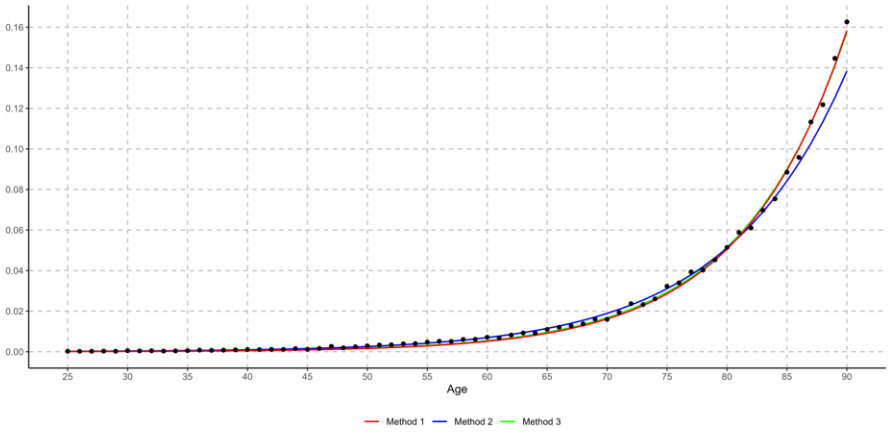

Figure 3: Hazard rate and approximation for females in Denmark in 2008.

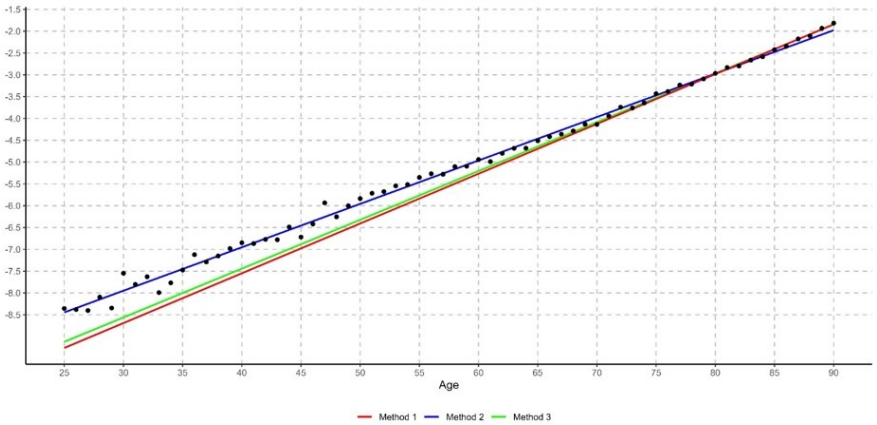

Figure 4: Logarithm of hazard rate and approximation for females in Denmark in 2008.

4. Sensitivities in a dynamic model

The initial statistical model for human mortality was created by Ronald D. Lee and Lawrence R. Carter in 1992 [6]. They delineated a log-bilinear model for the mortality rate $\mu_{x,t}$.

$$ln(D_{xt}/E_{xt}) = \alpha_x + \beta_x k_t + \epsilon_{x,t}$$

Where α_x refers to age-specific constant, β_x is age-specific patterns of mortality change, k_t denotes time trend and $\epsilon_{x,t}$ is error term.

The Lee-Carter model, being an empirical framework, is subject to certain assumptions and limitations. An additive constant to k would alter the mortality rates. This would produce multiple solutions that provide an equivalent fit to the data without constraints, complicating the estimation of parameters in a unique manner.

Based on that, $\sum_t k_t = 0$ and $\sum_x (\beta_x)^2 = 1$ should be introduced. The maximum likelihood estimates of $\alpha_x, \beta_x$and k_x can be achieved from minimizing $\sum_{x\in\mathcal{X}} \sum_{t\in\mathcal{T}} \left(\ln(D_{xt}/E_{xt}) - \widehat{\alpha_x} - \widehat{\beta_x}\widehat{k_t}\right)^2$:

$$F.O.C. = \frac{\partial}{\partial \widehat{a_x}} \sum_X \sum_t \left(\ln\left(\frac{D_{xt}}{E_{xt}}\right) - \widehat{a_x} - \widehat{\beta_x}\widehat{k_t}\right)^2$$

$$= 2\sum\sum \left(\ln\left(\frac{D_{xt}}{E_{xt}}\right) - \widehat{a_x} - \widehat{\beta_x}\widehat{k_t}\right) \cdot (-1)$$

$$\sum\sum \widehat{a_x} = \sum\sum \left(\ln\left(\frac{D_{xt}}{E_{xt}}\right) - \sum \widehat{\beta_x} \sum \hat{k}t\right)$$

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Then $\sum(\hat{\beta}x)^2 = 1$ and $\sum k = 0$.

$$\sum\sum \hat{a}x = \sum\sum \ln(D_{xt}/E_{xt})$$

$$\sum_{x\in X} T\,\hat{a}x = \sum\sum \ln(D_{xt}/E_{xt})$$

$$\hat{a}x = \frac{1}{T}\sum_{t\in T} ln(D_{xt}/E_{xt})$$

In Figure 5, the values for males and females from Denmark and the Netherlands are shown for ages ranging from 0 to 110 years between the years 1970 and 2008.

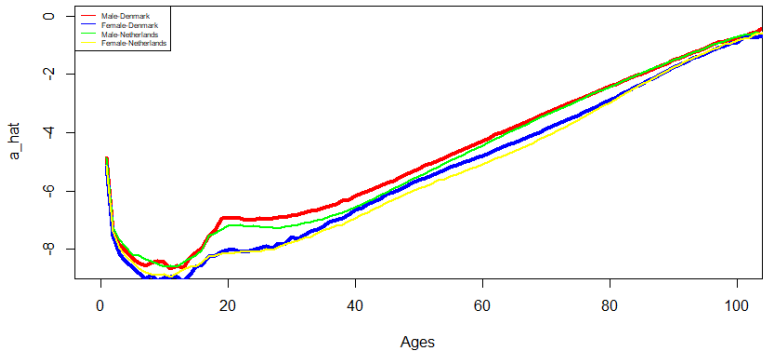

Figure 5: Estimation α.

α begins around 3 and then rapidly declines during the first 14 years of life. The newborn mortality spikes at age 0 could be caused by inadequate neonatal care, while the rest of the graph shows that the risk of dying increases with age. Generally, there is no discernible variation between the four groups in the trend of α. It is also notable that there is a higher α level for men in both countries between the ages of 18 and 30.

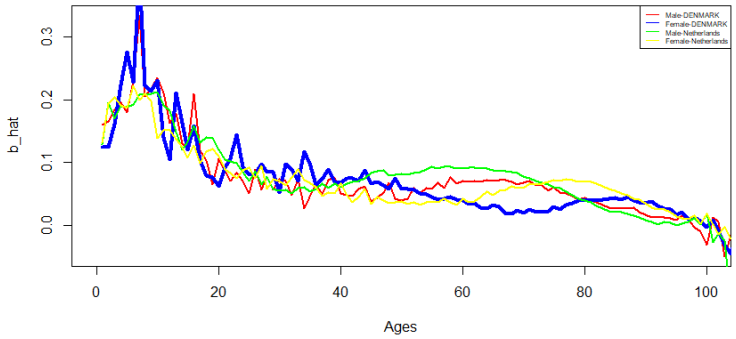

Figure 6: Estimation β.

Figure 6 indicates the value of β from 1970 to 2008. The figures provide insight into how mortality rates alter over time for different ages. An increasing death rate with age is shown by a positive value. The beta for the Danes is much greater than that of the Dutch during adolescence, notably around the age of 10 (0.35 to 0.2). Consequently, all betas steadily decline and converge to 0.12 at age 16. Although the value of beta fluctuates slightly until the age of 45, it shows a declining trend until the age of 100.

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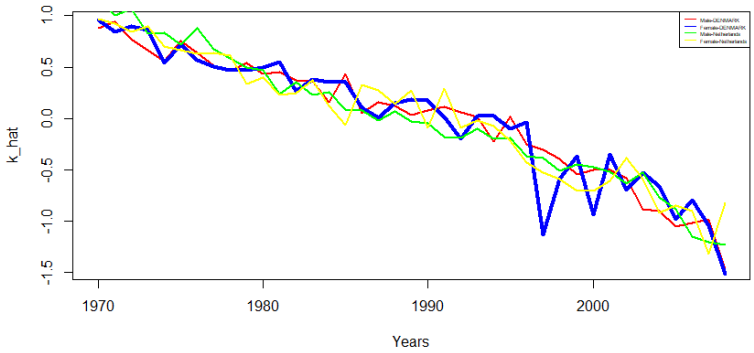

Figure 7: Estimation κ.

K is the time-varying index of mortality improvement in the Lee-Carter model. It shows the yearly rate of mortality change over time. Figure 7 displays the values from 1990 to 2008 for all ages. K begins at 1.0 in 1970 and steadily declines to 0 around the year 1990. Thereafter, K always remains below 0. This may imply that life quality has increased resulting in a general decline in the mortality rate.

5. Simulation of a dynamic mode

Utilizing $\widehat{k_t}$ derived in the preceding section, it can now project the forthcoming years. A Random Walk with Drift model is introduced here, according to standard Gaussian assumptions:

$$\kappa_{t+1} = \kappa_t + \theta + \sigma\delta_t$$

δ_t follows the standard Gaussian assumptions. The time-series $\widehat{k_t}$ serves as the training set for the model, including the years 1970 to 2008. I presume that the current date is January 1, 2008. Subsequently, I will employ the Lee-Carter model for forecasts over the next 50 years. $\widehat{k_{t+1}} - \widehat{k_t}$ is denoted as $\Delta\kappa_t$. Then we can determine the ideal estimates for α and β by linear regression.

Modelling one-year death probabilities by using:

$$q_{xt} = 1 - exp(-\mu_{xt}) = 1 - exp\left(-exp(\widehat{\alpha_x} + \widehat{\beta_x}\kappa_t)\right)$$

The one-year mortality probability of 65-year-old males from Denmark and the Netherlands will be examined from 2008 to 2058. Generating 25 distinct δ_t values yield Figure 8 and Figure 9.

Denmark and the Netherlands are both advanced nations with comprehensive social assistance programs. Their data indicates that the one-year mortality odds for a 65-year-old guy are minimal (ranging from 0 to 0.02) and diminish over time. In general, Denmark's graph exhibits greater fluctuations. The one-year mortality probabilities in the Netherlands exhibit more stability and predictability.

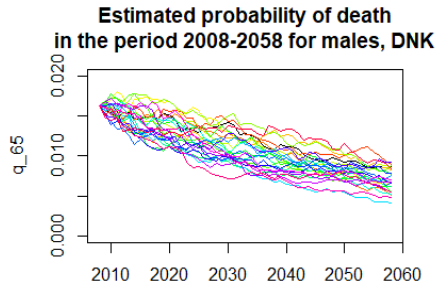

Figure 8: Simulated model for Denmark.

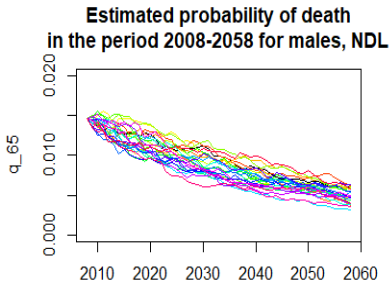

Figure 9: Simulated model for Netherlands.

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6. Conclusion

The Lee-Carter and Gompertz models have proven effective in capturing long-term mortality patterns and projecting future mortality rates. Consistent with prior research, the study found that males tend to have higher mortality rates compared to females, and overall mortality rates have declined significantly due to advancements in healthcare, socioeconomic improvements, and better living conditions. Despite some differences in mortality rates, particularly among specific age groups and genders, both Denmark and the Netherlands exhibit similar trends due to their status as developed European countries. However, Denmark displayed more variability, particularly in younger and older age groups, while the Netherlands showed more uniformity, likely due to its larger population.

Projections using the Random Walk with Drift model suggest a continued increase in life expectancy for both countries. This study emphasis the importance of sustained investment in public health and social policies to address the growing demands of aging populations and ensure that mortality trends continue to improve.

Future work could refine these models by incorporating additional variables such as geographical factors and economic patterns, allowing for a more comprehensive understanding of mortality dynamics.

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