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A Review on the Application of Partial Differential Equations in Atmospheric Research

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Abstract: In the last decade, the large amount of chemicals produced by industrial development and human activities has triggered climate problems such as the heat island effect and the greenhouse effect, making climate change a major challenge of global concern. For this reason, it is important to study the atmospheric temperature distribution, especially in application areas such as climate change prediction, weather forecasting and environmental protection. In this paper, we review the theoretical basis and recent progress in modeling and analyzing atmospheric temperature distribution using partial differential equations, discuss the main influencing factors such as solar radiation, surface albedo, convective processes, etc., and illustrate the advantages of the method in impact assessment of complex eventswith examples, which provide a scientific basis for mitigating the climate problems. Therefore, this article will provide a theoretical overview and the latest progress in the simulation and analysis of atmospheric temperature distribution using partial differential equations.

Keywords: Partial differential equations, Atmospheric temperature distribution, Climate change modeling.

1. Introduction

In the past decade, the large amount of chemicals produced by industrial development and human activities, such as carbon dioxide and methane, has caused numerous climate problems, especially the warming caused by the heat island effect and greenhouse effect. . Therefore, climate change has become one of the serious problems facing all mankind. From this perspective, the study of atmospheric temperature distribution is urgent, especially in the application research of climate change, weather forecasting, and environmental protection. In the above research, using partial differential equations for modeling is one of the mainstream research methods. Due to the fact that atmospheric changes in real life are not only influenced by one factor, partial differential equations can be used to construct relationships between multiple independent and dependent variables, making them suitable for studying the impact of complex events on the atmosphere. Li, YK, and Chao, JP used linear theory to arrange local thermal cycles into sixth order temperature partial differential equations, and solved them using Fourier transform and inverse Fourier transform. They found that for deep PBL situations where the Ekman height is equivalent to the vertical motion scale, both exponential decay and Ekman spiral structures play a role in forcing atmospheric circulation[1]. KIERAN and JAMES MICHAEL (1983) used conservation of mass partial differential equations and time integrated them using the Gear method to obtain the most important physical properties of the

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aerosol layer and estimate the impact of disturbances such as volcanic eruptions on the layer [2]. Zaurbekov, N and Aidosov, A et al. (2018) conducted a modeling based on turbulent diffusion partial differential equations and used the model to predict the concentration levels of toxic pollutants two meters above ground level, as well as the vertical concentration distribution of landscapes determined based on meteorological data. By reading a large amount of literature, we can find that the partial differential equation modeling method is mainly applied to the impact of complex events (usually causing changes in multiple influencing factors, such as volcanic eruptions) on the atmosphere and thermal cycle, as well as to predict the changes in this impact over time or space [3]. Pan, JN, and He, KY (2024) proposed a logarithmic Gaussian Gombe Bayesian hierarchy that allows for the sharing of stochastic partial differential equations and first-order autoregressive models, and found similar spatiotemporal interactions between PM 2.5 and ozone, indicating that the spatiotemporal variations of these pollutants exhibit relatively similar consistency in California[4]. From this, we can see that the partial differential equation modeling method can predict the impact of some human activities (such as the increase in global farmers) on atmospheric temperature and other aspects, which has a great role in solving or alleviating some climate problems. Therefore, this article will provide a theoretical overview and the latest progress in the simulation and analysis of atmospheric temperature distribution using partial differential equations.

2. Physical Model of Atmospheric Temperature Distribution

In the second part, some basic knowledge about atmospheric physics models, such as the influencing factors of atmospheric temperature distribution, will be introduced to facilitate the use of four commonly used partial differential equations in atmospheric physics research in specific studies in the third part.

2.1. Influencing factors of atmospheric temperature distribution

Numerous factors directly influence atmospheric temperature distribution; however, primary factors in complex event analysis include solar radiation, ground albedo, convective processes, heat conduction, and humidity. For example, when analyzing the temperature changes over time in the affected area caused by volcanic eruptions, the impact of volcanic gases on solar radiation and ground reflection is discussed and studied. Through some scientific explanations and relevant theorems in atmospheric physics, it can be concluded that the five common factors mentioned above have a specific impact on the distribution of atmospheric temperature. Firstly, there are three factors with clear intrinsic relationships: solar radiation, ground reflection, and humidity. Among external factors (such as geothermal energy, which is a factor that the Earth itself possesses and cannot undergo significant changes through human activities or natural disasters, and belongs to its own factors rather than external factors), solar radiation (i.e. heat flux), ground reflection, and humidity are the most important factors affecting the surface temperature of the Earth. The sun releases a type of electromagnetic wave (i.e. solar radiation) that carries a large amount of energy and transfers it to the surface through thermal convection. Due to the absence of particles such as dust and small water droplets in the atmosphere other than the troposphere, electromagnetic waves can pass through with minimal energy loss. But when entering the troposphere, particles and small water droplets in the air will reflect some light, causing electromagnetic waves to lose some energy and affect the temperature of the atmosphere. As is well known, humidity refers to the amount of small water droplets per unit volume of air, while surface reflection refers to the number of reflection phenomena caused by substances in the air in the troposphere. From the above explanation, it can be concluded that the energy efficiency during electromagnetic wave transmission (i.e. solar radiation) is inversely correlated with the surface reflection and humidity caused by particles in the air passing through the

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transmission process. Secondly, the relationship between heat conduction and convection and atmospheric temperature is relatively direct, that is, the higher the efficiency of heat conduction and radiation, the higher the temperature. Although there are many factors that affect the efficiency of these two heat conduction methods, such as temperature difference, thermal conductivity, and the thickness and cross-sectional area of the heat-conducting object, most of these factors are positively correlated with heat conduction efficiency, so we will not discuss them in detail here.

2.2. Introduction to Atmospheric Structure

The atmosphere is conventionally segmented into five layers based on physical properties: the troposphere, stratosphere, mesosphere, thermosphere, and exosphere. The trend of temperature change varies among these five different levels. In the troposphere, the temperature decreases with increasing altitude, and for every 100 meters increase in altitude, the temperature decreases by approximately 0.65 ℃. This is mainly because the warming of the atmosphere in the troposphere mainly comes from the absorption of longwave radiation emitted by the Earth, and the farther away from the surface, the less heating it receives from the surface, resulting in lower temperatures. In the stratosphere, the temperature rises with altitude, while the top temperature hovers around 270K. This is because the gas at the top of the stratosphere absorbs ultraviolet radiation from the sun and is heated, resulting in a temperature change trend opposite to that in the troposphere. This temperature change trend also makes there almost no thermal convection in the stratosphere. In the middle layer, due to the lack of ozone absorbing ultraviolet radiation, most of the solar shortwave radiation that can be absorbed by nitrogen and oxygen is also absorbed by the more outer layers. Therefore, the temperature of the middle layer decreases significantly with increasing altitude. In the thermosphere, the phenomenon of temperature increasing with altitude reappears in the atmosphere. This is because the air in the thermosphere can absorb solar ultraviolet radiation with wavelengths less than 0.175 microns, causing the temperature within 80 kilometers to rise to 970 ℃. In the exosphere, the temperature is extremely high and increases with altitude. The reason why the exosphere has high temperature is basically the same as that of the thermosphere, where atmospheric molecules are heated by absorbing shortwave radiation from the sun. Since the focus of this article is not on explaining the reasons for gas changes in each layer, we will not go into too much detail here, which underscores the necessity for multi-layered modeling approaches in atmospheric studies.

2.3. The Relationship between Physical Phenomena and Partial Differential Equations

The literature review reveals that partial differential equations (PDEs) effectively capture intrinsic relationships among multiple physical quantities, notably in energy conservation, heat conduction, and convective diffusion. In this section, we will take the heat conduction equation as an example to explain how physical phenomena are expressed in the form of partial differential equations. The following figure shows the heat conduction equation in a three-dimensional homogeneous medium.

$$\frac{\partial \mathrm{y}}{\partial \mathrm{x}} = \mathrm{k}\left(\frac{\partial^2}{\partial x^2}u + \frac{\partial^2}{\partial y^2}u + \frac{\partial^2}{\partial z^2}u\right) = k\left(u_{xx} + u_{yy} + u_{zz}\right) \quad (1)$$

In this equation, u=u (t, x, y, z) represents temperature, which is a function of the time variable t and the spatial variable (x, y, z); ∂ U/∂ t is the rate of change of temperature over time at a certain point in the space; Uxx, Uyy, and Uzz are the second derivatives of temperature with respect to three spatial coordinate axes; K is the thermal diffusivity, determined by the material's thermal conductivity, density, and heat capacity, providing a quantitative foundation for modeling the spatiotemporal evolution of atmospheric parameters.

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3. Application of Partial Differential Equations in Atmospheric Temperature Simulation

In atmospheric physics research, the following four partial differential equations are mainly applied, namely the heat conduction equation, convection diffusion equation, Navier Stokes equation, and radiative transfer equation, to obtain the impact of complex events on the atmosphere. However, the Navier Stokes equations are equations constructed for the conservation of energy during convection and diffusion of incompressible fluids (substances whose density does not change during flow), so they will not be discussed in detail in this section. In the third part, we will briefly introduce the main contents of the four equations mentioned earlier and analyze their applications in specific research.

3.1. Heat conduction equation

In the previous section (2.3), the basic meaning represented by each symbol in the heat conduction equation has been explained in detail. Simply put, the heat conduction equation describes the rate of temperature change over time within a given three-dimensional region. This equation is widely used in most studies on the impact of specific events on temperature, mainly because the thermal diffusivity k in the equation covers most factors that can affect atmospheric temperature. This value is positively correlated with thermal conductivity and negatively correlated with heat capacity and density. By analyzing the changes in the three factors that affect thermal diffusivity caused by specific events, the impact of specific events on thermal diffusivity can be analyzed, and the impact of specific events on the rate of temperature change over time can also be analyzed. For example, when analyzing the impact of volcanic eruptions on temperature changes over time, volcanic eruptions result in a large amount of volcanic particles and gases (mainly H2O, as well as certain amounts of CO2, H2S, and SO2) in the air. These substances cause an increase in the density, thermal conductivity, and heat capacity of the air, but the degree of increase in heat capacity is much greater at higher temperatures. Therefore, according to the thermal diffusion formula, volcanic eruptions will cause a decrease in thermal diffusion rate, and then according to the thermal conduction equation, the rate of temperature change caused by volcanic eruptions over time can be obtained. By following the same approach as in the previous text, it can be concluded that many specific events have an impact on the rate of temperature change over time. Zhao LiHua, Zhao ShiJia, and He RuDong (2022) established a three-dimensional transient temperature field model of CRTS Ⅰ slab ballast-less track using the heat conduction equation and analyzed the distribution law and influencing factors of temperature field of ballast-less track under extreme low temperature weather conditions in Harbin during winter. The conclusion drawn is that the influencing factors of temperature field distribution of ballast less track include extreme temperature, track slab absorption rate, wind speed, etc.; The temperature changes inside the ballast-less track lag behind the ambient temperature, and the daily temperature changes at the top and edge of the track slab are relatively large; Along the top of the track slab, the temperature field exhibits nonlinear changes, and the amplitude of temperature fluctuations continuously decreases; The higher the absorption rate of the track board, the higher the top temperature and temperature gradient of the board; The higher the wind speed, the lower the temperature at the top of the board, and the smaller the positive temperature gradient and the larger the negative temperature gradient inside the track board[5].

3.2. Convection-diffusion equation

As the name suggests, the convection diffusion equation is an equation used for numerical calculations in studying convection diffusion problems. Through the definition of the convection diffusion equation, three questions will be explained in the following text: firstly, what is the convection diffusion problem; Secondly, what is the specific content of the convection diffusion

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equation; Thirdly, what is the application approach of the convection diffusion equation in research. Firstly, the convection diffusion problem mainly studies a certain physical quantity carried by fluid particles in a fluid. In short, the convection diffusion problem refers to the variation of temperature and concentration of substances dissolved in a fluid during the flow process. River pollution, air pollution, and nuclear waste pollution are all classic convection diffusion problems. Then, the convection diffusion equation is an equation used to study the laws of material motion in a flowing system due to the chemical potential in the medium. The specific equation is shown in the following figure.

$$\frac{\partial C_A}{\partial \tau} + u_x \frac{\partial C_A}{\partial x} + u_y \frac{\partial C_A}{\partial y} + u_z \frac{\partial C_A}{\partial z} = D_{AB}\left(\frac{\partial^2}{\partial x^2} C_A + \frac{\partial^2}{\partial y^2} C_A + \frac{\partial^2}{\partial z^2} C_A\right) \tag{2}$$

In the convection diffusion equation, DAB represents the molecular diffusion coefficient of substance component A in substance component B; RA is the amount of substance component A generated by chemical reactions within a unit time and unit volume space; CA is the concentration of substance component A; τ is time; Ux, uy, and uz are the components of the flow velocity u on the three axes of the three-dimensional coordinate system, respectively. Finally, the application approach of the convection diffusion equation in research is similar to that of the heat conduction equation. In the convection diffusion equation, there also exists a coefficient similar to the thermal diffusivity k: the molecular diffusion coefficient D. The molecular diffusion coefficient D represents the diffusion ability of a specific substance molecule in another substance and has different influencing factors in gases and liquids. In gas, the value of molecular diffusion coefficient is inversely correlated with the total system pressure and the total volume of molecules of two specific substances, and positively correlated with the absolute temperature and the reciprocal sum of the molecular weights of two specific substances. However, in liquids, the molecular diffusion coefficient is inversely correlated with solute solvent viscosity and solute molecule volume, and positively correlated with absolute temperature and solute molecule mass. In most studies, the influence of specific events (mostly toxic gas diffusion or pollutant diffusion in rivers) on the diffusion coefficient of molecules is analyzed after analyzing the state of the diffusion environment of diffusing molecules (liquid or solid), and then the variation law of concentration during the flow process is obtained through the convection diffusion equation. Zaurbekov, N ;Aidosov, A; Zaurbekova,N; Aidosov, G,;Zaurbekova, G,;Zaurbekov, I (2018) established a model for determining pollutant diffusion in emergency situations using convection diffusion equations, including diffusion time, volumetric properties of pollution sources, and wind speed. The model was used to calculate the possible pollution areas around any high pollutant concentration source system under any environmental conditions, predict the pollutant concentration levels two meters above ground, and determine the landscape vertical concentration distribution based on meteorological data[6].

3.3. Radiative transfer equation

He radiative transfer equation refers to the equation in which electromagnetic radiation (usually solar radiation in atmospheric research) is attenuated by the absorption or scattering of a medium during thermal radiation. As mentioned in section 2.1 above, solar radiation and ground reflectance are one of the main factors affecting atmospheric temperature. Many complex events can affect the amount of solar radiation, leading to a decrease in atmospheric temperature, such as volcanic eruptions and an increase in vehicle traffic resulting in an increase in carbon dioxide emissions. The following figure shows the specific content of the radiative transfer equation.

$$I_\lambda(s) = I_{\lambda_{S_1}} e^{-\int_{S_1}^{S_2} K_\lambda(S) d_S} \tag{3}$$

In this equation, K λ is referred to as the attenuation coefficient, which refers to the relative attenuation rate caused by the transmission of electromagnetic waves per unit distance in a given

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medium. In nuclear physics, the attenuation coefficient is directly proportional to the radiation flux density I of the medium and inversely proportional to the thickness Δ x of the medium layer. Among the aforementioned influencing factors, the radiation flux density of the medium refers to the amount of radiation received per unit area of the medium. There are many factors that affect radiation flux density, including temperature, wavelength, spectral reflectance, spectral illuminance of the sun incident on the ground, atmospheric spectral transmittance, photometer field of view angle, and photometer effective receiving area. Among them, temperature is positively correlated with radiation flux density, while atmospheric spectral reflectance (i.e. the ability of the medium surface to reflect solar radiation) is inversely correlated. Due to the possibility of changes in temperature and atmospheric spectral transmittance in practical scientific research, as well as their impact on radiation flux density being much greater than other factors, the influence of other factors on radiation flux density will not be discussed one by one here.

Due to the fact that most events cause changes in solar radiation, the radiative transfer equation is widely used in scientific research. Zhai Xidan, Xing Lixin, Yuan Nannan, Wang Xiaodong, Gao Zhiyong, and Chen Xiao (2014) used the thermal infrared band of Landsat8 satellite data source and applied the radiative transfer equation algorithm to invert surface temperature. The results showed that due to the influence of terrain factors, the thermal anomalies in temperature inversion were distributed in areas receiving more solar radiation [7]. By reading a large amount of literature, it can be seen that the radiative transfer equation plays an important role in studying atmospheric temperature anomalies such as surface temperature anomalies. Establishing a model through the radiative transfer equation can effectively analyze the causes and trends of abnormal phenomena, and can also predict the impact trends and degrees of natural disasters on different regions.

4. Numerical simulation and equation solving methods

In this section, we will first introduce the two mainstream methods for solving the above partial differential equations - finite difference method and finite element method. Then, we will introduce the two common numerical simulation methods.

Before proceeding with the following discussion, it is necessary to introduce the problem of definite solutions in differentiation. A differential equation has two solutions - boundary solution and initial solution. The conditions obtained on the boundary of the spatial domain are called boundary conditions, and problems that do not include time effects and have boundary conditions are called boundary value problems. If a problem involves the time domain, the conditions obtained at the initial moment are called the initial value problem, and the problem that contains the influence of the time domain and has initial value conditions is called the initial value problem. If a problem satisfies all the conditions of both initial and boundary value problems, it is called a mixed initial and boundary value problem.

4.1. Finite Difference

The idea of the finite difference method for solving differential equations is to first analyze the domain given in the problem on a grid, and then replace the derivative in the differential equation with the derivative for each grid point according to an appropriate numerical differentiation formula, thereby transforming the originally discretized problem into a differential problem for solving. Therefore, in summary, the finite difference method has high flexibility and universality, but requires a lot of verification procedures, including the estimation of the error between the solution obtained by the difference scheme and the true solution of the original equation, the proof of the existence and uniqueness of the solution under the difference scheme, and so on. However, the finite difference

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method is easy to form algorithms on computers, so its application in scientific research is relatively wide.

4.2. Finite element method

He finite element method has a more complex process compared to the finite difference method, mainly presented in the following four steps. 1: Discretization of structure. The first step of finite element method is to divide the structure or continuum into many elements. Therefore, when analyzing, it is necessary to model the structure with appropriate finite elements and determine the number, type, size, and arrangement of elements. Step 2: Extract one unit from the region or structure to study. Choose an appropriate interpolation or displacement mode to approximate the displacement field of the unit. Due to the fact that under any given load, the displacement solution of complex structures cannot be accurately known in advance. Therefore, the difference pattern is usually taken in polynomial form. From a computational perspective, polynomials are simple and meet certain convergence requirements. After approximating the element displacement function with polynomials, the problem becomes how to calculate the node displacement. Once the node displacement is determined, the displacement field is also determined. Step 3: Derive the stiffness matrix and load vector of the element. Based on the assumed displacement mode, the stiffness matrix and load vector of the element can be derived using equilibrium conditions or appropriate variational principles. Step 4: Obtain the overall equilibrium equation system from the set of unit equations. A continuum or structure is composed of a combination of many finite element elements. Therefore, when conducting finite element analysis on the entire continuum or structure, combination is necessary. Although the finite element method does not require considering many subsequent arguments like the finite difference method, the selection of network points in the finite element method is more flexible compared to the finite difference method, making it more complex and difficult to implement when using the finite element method for computer programming. Moreover, the finite difference method has more versatility in solving physical problems than the finite element method. Many differential equations in physical problems cannot be solved using the finite element method, but can be solved using the finite difference method. In summary, the finite element method is relatively rare in practical scientific research.

4.3. Spectral method

The core idea of the spectral method for solving differential equations is to approximately expand the image of equations in practical problems into a finite series expansion of a relatively smooth curve, known as the approximate spectral expansion of solutions. Next, combine the expansion equation with the original equation and solve it. If the problem encountered is a non-stationary problem, the system of equations will also be related to time t and need to be solved at a specific time. The essence of spectral methods is an extension of the standard method of separating variables. When using spectral methods for solving problems, Chebyshev polynomials and Legendre polynomials are usually used as the basis functions for approximate expansions. However, for problems with periodic boundary conditions, Fourier series and surface harmonic series are generally used to replace these two polynomials as the basis functions for greater convenience. In practical error estimation research, the accuracy of spectral methods is very convenient because it is only related to the number of terms in the approximate polynomial.

5. Conclusion

Partial differential equations play an important role in simulating atmospheric temperature distribution, accurately describing physical processes such as heat conduction, convection diffusion,

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and radiative heat transfer through mathematical models, helping to understand and predict temperature changes in the atmosphere. In recent years, with the advancement of numerical computing technology, numerical methods such as finite difference method and finite element method have been widely used in solving complex partial differential equation models, greatly improving the accuracy and efficiency of simulations. Despite facing challenges in multi-scale modeling and handling complex boundary conditions, partial differential equation models provide a solid theoretical foundation and application tools for the study of atmospheric temperature. In the future, combining big data and machine learning technologies is expected to further enhance the accuracy and reliability of these models in climate prediction and extreme weather warning.

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