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Facing the Algorithm Study of Position Control for Five-Axis CNC Machine Tool Processing Parts

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Abstract: With the increasing demand for complex surface parts, the accuracy requirements for orientation control when manufacturing parts using five-axis CNC machine tools are also higher. This paper aims to conduct an algorithm study on the orientation control problem during the machine tool part manufacturing process. After understanding the working principle of a five-axis CNC machine tool, a simple model was established. On the basis that when the panel rotates around the B axis, the Y-axis coordinate of the drilling point and the center of the panel remains unchanged, and when it rotates around the A axis, the X-axis coordinate remains unchanged, the coordinates of the final drilling point are defined according to the data. Furthermore, due to the rotation around the A and B axes during machining, a transfer hole coordinate is set up to facilitate the solution. Through the derived expression and projection on the plane forming angles using geometric relationships, three displacement amounts and two rotation amounts were found. On this basis, constraint conditions were added to ensure that the projection of the optimized drilling line segment with the X-axis forms a known angle. A transfer hole was set up again, and the same algorithm was used to calculate the encoding solution, resulting in the three displacement amounts and two rotation amounts after the initial position change.

Keywords: Five-axis CNC machine tool, orientation control, transfer hole coordinates

1. Introduction

With the rapid development of manufacturing, increasingly complex surface parts are being used in industries such as aerospace and automotive. As a result, traditional three-axis CNC machine tools can no longer meet the current development needs, while five-axis CNC machine tools have become the preferred choice for machining in factories due to their multi-directional control advantages [1]. A five-axis CNC machine tool is a high-tech, high-precision machine tool capable of performing high-precision machining on complex spatial surfaces [2], making it an essential tool for machining critical components such as impellers, blades, and ship propellers. In the high-speed, high-precision linkage machining process of a five-axis CNC machine tool for complex surfaces, there is a high demand for different orientation controls in each movement, where the actual running trajectory may not match the programmed trajectory [3]. This discrepancy is one of the main factors affecting the system error in the final part formation. This paper conducts an algorithm study on the orientation control issues of five-axis CNC machine tools and path optimization [4], using a simple model of machining inclined holes of varying orientations on a rectangular plate as an example.

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2. Control Principles of the Five-Axis CNC Machine Tool

Figure 1 shows an image of a point-contact five-axis CNC machine tool, which has five coordinate axes (three linear movement axes X, Y, Z, and two rotational axes A, B) and can perform coordinated motion under the control of a computer numerical control (CNC) system.



Figure 1: A Simple Five-Axis CNC Machine Tool

When the machining system is in the home position (initial state), the coordinate system is defined as follows: The base of the machining center is taken as the origin O, the horizontal right direction is defined as the positive direction of the *X*, the vertical upward direction is defined as the positive direction of the *Z*, and the *X*, *Y*, and *Z* axes together form a right-handed orthogonal coordinate system. The rotating axis (the *A*) is parallel to the *X* (rotating around a line parallel to the *X*) and lies within the XOZ plane. The rotation angle around the A-axis is referred to as the *A*-angle. The rotating axis (the *B*) is parallel to the *Y*, located above the A-axis, and lies within the YOZ plane. The rotation angle around the B-axis is referred to as the *B*-angle. When the *A* rotates, the *B* also moves accordingly; however, when only the *B* rotates, the *A* remains stationary.

3. Restatement of Issues Studied in This Paper

The paper aims to use the machine tool to machine inclined holes of varying orientations on a rectangular plate. The drill placed above the plate remains in a vertical direction and only performs parallel movements up and down along the *Z*.At the initial moment, the entire machining system is in a home position, with the surface of the plate perpendicular to the *Z*, the center of the plate located on the *Z*, and both the displacement and rotation angles of the machining center set to 0. The CNC center automatically controls the movement and rotation of the machining center based on the instructions (three displacement amounts and two rotation angles) to allow the plate to reach the specified position and orientation, enabling the drill to punch the required hole at the specified angle.

H1: Suppose the system is in the zero position, with the distances from the rotating axes A and B to the upper surface of the plate being 96 mm and 46 mm, respectively. The coordinates of the drill bit are (10, -45, 116) (units: mm, same below). It is assumed that the holes to be processed (simplified as line segments) are all perpendicular to the *X*, and that the angle θ_i between their projection in the YOZ plane and the *Y*, as well as the coordinates of the intersection point with the plate (x_i, y_i, z_i), are all known. Please establish a mathematical model to determine the displacement of the machining center relative to the initial state (X_i, Y_i, Z_i), and the required rotation angles of the A-axis and B-axis (A_i and B_i), as well as the relationship between the parameters of each hole$(x_i, y_i, z_i, \theta_i)$ [5].

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H2: Based on H1, except for the condition that "the holes to be processed (simplified as line segments) are all perpendicular to the *X*," the rest remains the same. It is also assumed that when the system is in the zero position, the angle φ_i between the projection of each hole in the *XOZ* plane and the *X* is known. Please establish a mathematical model to determine the displacement of the machining center relative to the initial state (X_i, Y_i, Z_i), and the required rotation angles of the A-axis and B-axis (A_i and B_i), as well as the relationship between the parameters of each hole $(x_i, y_i, z_i, \theta_i, \varphi_i)$ [6].

4. Model Assumptions

(1) It is assumed that the lengths of the desired machining holes simplified to line segments approach infinity as they approach 0.

(2) It is assumed that when machining the plate, the machine tool rotates around the B-axis first and then around the A-axis.

(3) It is assumed that calculation errors appearing in the machine tool's process are within an acceptable error range.

(4) It is assumed that one rotation of the A-axis and one rotation of the B-axis are sufficient to achieve the required drilling point.

(5) It is assumed that the deviation during drilling is within the error range.

5. Symbol Definitions

Symbol	Meaning	Unit
H_A	Distance from rotation axis A to the surface of the plate	mm
H_B	Distance from rotation axis B to the surface of the plate	mm
x_i	X-axis coordinate in the initial data provided in the attachment	mm
y_i	Y-axis coordinate in the initial data provided in the attachment	mm
z_i	Z-axis coordinate in the initial data provided in the attachment	mm
A_i	Rotation angle around the A-axis	degrees
B_i	Rotation angle around the B-axis	degrees
x_i	Displacement along the X-axis after rotation of the plate from the initial position	mm
y_i	Displacement along the Y-axis after rotation of the plate from the initial position	mm
z_i	Displacement along the Z-axis after rotation of the plate from the initial position	mm
θ_i	Angle formed between the projection of the simplified line segment of the machining hole on the YOZ plane and the Y-axis	degrees
Ψ_i	Angle formed between the projection of the simplified line segment of the machining hole on the YOZ plane and the X-axis	degrees

6. Model Establishment

6.1. Solving Problem H1

6.1.1. Analysis of Problem H1

Based on the working principle of the five-axis CNC machine tool, establish a three-dimensional coordinate system to precisely locate the A-axis, B-axis, and the X, Y, Z coordinate axes. It is assumed that the desired machining holes (simplified as line segments) are perpendicular to the X-axis. A mathematical model is established based on the initial state of the data, such that after the rotation of the plate around the A-axis and B-axis, the projection of the plate on the YOZ plane forms an angle

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θ_i with the Y-axis. The displacement and the values of the rotation angles of the A and B axes are then calculated.

6.1.2. Model Preparation for H1

To solve problem H1 and establish the mathematical model, first prepare the model based on the following data:

The distance from rotation axis A to the surface of the plate is HA = 96 mm, and the distance from rotation axis B to the surface of the plate is HB = 46 mm. Let the coordinates of the initial hole point E be (x_i, y_i, z_i), the coordinates of the initial center (center of the machining plate) F be (0, 0, z_i), the coordinates of the intermediate hole (the transition hole) H be (x_H, y_H, z_k), and the coordinates of the hole end point J be (10, -45, K'). Since the angle formed with the Y-axis in the YOZ plane is θ_i, it follows that K' = 45tan(θ_i). Therefore, the coordinates of point J are (10, -45, 45tan(θ_i)). From the analysis of the problem, it is found that when the B-axis rotates, the Y-axis coordinate remains unchanged, and when the A-axis rotates, the X-axis coordinate remains unchanged. For this problem, we assume that rotation around the B-axis occurs first, followed by rotation around the A-axis. Hence, the Y-axis coordinate of the intermediate hole is the same as that of the initial hole point E, i.e., $y_H = y_i$, and the X-axis coordinate of the intermediate hole is the same as that of the final position point J, i.e., XH = 10. Thus, the coordinates of the intermediate hole H are (10, y_i, z_k).

6.1.3. Establishment of the H1 Model

Define two planes, JBC and BCI, which intersect along the line BC. Point B is the projection of point J onto plane BCI, and point C is the projection of point I onto plane JBC. To simplify the description of the final position of the center, as shown in Figure 2, we extract a portion of the coordinate axes and draw a tetrahedron for coordinate calculation.

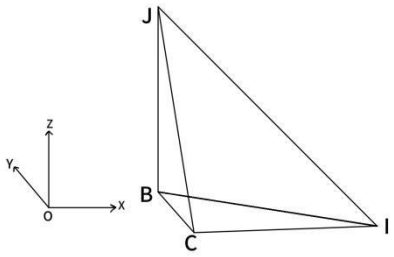

Figure 2: Using a Tetrahedron to Calculate Coordinates

Coordinates of Point B: The coordinates of point B are (10, -45, (45 - y_i)tan(θ_i)),
Coordinates of Point C: The coordinates of point C are (10, -45 - y_i, (45 - y_i)tan(θ_i)),
Coordinates of Point I: The coordinates of point I are (10 - x_i, -45 - y_i, (45 - y_i)tan(θ_i)).

As shown in the figure above, we have the following: CI = xi, BC = 45 + (45 - y_i)tan(θ_i), JB = $y_i$tan(θ_i). Based on the above, we can solve the three displacement equations as follows:

$$\begin{cases} X_i = 10 - x_i \\ Y_i = -90 - (45 - y_i)tan\theta_i \\ Z_i = (45 - y_i)tan\theta_i - z_i \end{cases}$$

After solving these displacement equations, we can proceed with the calculations for the A and B angles. Since we assume the B-axis rotates first, we will start with the calculation for the B angle.

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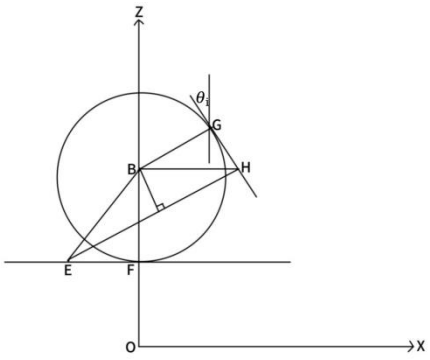

Figure 3: B Angle Calculation Diagram

$$\sqrt{x_i^2 + y_i^2 + (z_i - 152)^2} = \sqrt{10^2 + y_i^2 + (z_H - 152)^2}$$

$$\cos\frac{B_{i1}}{2} = 0.5\sqrt{(10 - x_i)^2 + (z_i - (45 - y_i)\tan\theta_i)^2} \Big/ \sqrt{x_i^2 + y_i^2 + 46^2}$$

Simplifying, we get:

$$B_i = 2\pi - 2Arccos\ (10 - x_i)^2 + (z_i - (45 - y_i)tan\theta_i)^2 \Big/ 2\sqrt{x_i^2 + y_i^2 + 46^2}$$

To calculate the A angle, we set point H at (10, y_i, z_k).

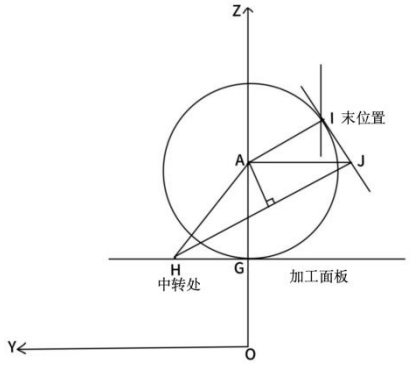

Figure 4: A Angle Calculation Diagram

$$HJ = \sqrt{(y_u + 45)^2 + (z_k - 45\tan\theta_i)^2} \quad r_A = \sqrt{10^2 + y_i^2 + (z_i - z_k + 96)^2}$$

$$A_i = 2\pi - 2Arccos\ (y_i + 45)^2 + (z_k - 45tan\theta_i)^2 \Big/ 2\sqrt{10^2 + y_i^2 + (z_i - z_k + 96)^2}$$

Using C language, the above model preparation, establishment, and constraints can be written into code, which will output the displacement and rotation angles.

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6.2. Solving Problem H2

6.2.1. Analysis of Problem H2

Building upon H1, additional constraints are introduced to the simplified holes, which now form an angle Ψ_i with the X-axis. Based on the given initial data, a mathematical model is established to calculate the displacement, A angle, and B angle.

6.2.2. Model Preparation for H2

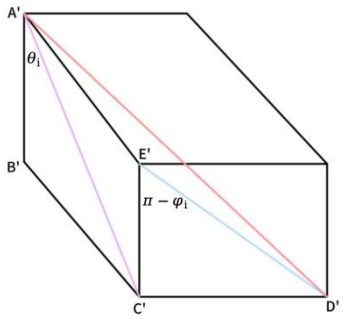
In this problem, based on the first question, a constraint is added where the projection of each hole onto the XOZ plane and its angle with the X-axis, φ_i, is known. As a result, there will be changes to the model preparation. Let the initial position of the hole be (x_i, y_i, z_i), the end position of the hole A' be (10, -45, P'), the coordinates of the initial center be (0, 0, z_i), and the coordinates of the center end position be D'(M', N', P). The projection of the line segment onto the YOZ plane still forms an angle θ_i with the Y-axis. Therefore:

$$P' = 45\tan\theta_i$$

$$P = 45\tan\theta_i - {x^2 + y^2}\Big/{\tan(\pi - \varphi_i)^2 + \tan\theta_i^2 + 1}$$

6.2.3. Establishment of the H2 Model

For ease of finding and expression, the relationship between the two angles θ_i and Ψ_i is abstracted into a rectangular coordinate system, as shown in the figure below:



Let the coordinates of B' be (10, -45, P), C' be (10, N', P), D' be (M', N', P), and E' be (10, N', P'). The relationships are as follows: ∠C'E'D' = π - φ_i, ∠B'A'C' = θ_i.

From this, the following formulas are derived:

$$\frac{C'D'}{E'C'} = \frac{|M' - 10|}{|P' - P|} = tan(\pi - \varphi_i)$$

$$\frac{B'C'}{A'B'} = \frac{|N' + 45|}{|P' - P|} = tan\theta_i$$

Thus, we can express the three displacement equations as:

Let H' be (10, y_i, z_k'). Using geometry, we obtain the following formula:

$$\tan\left(\varphi_i - \frac{\pi}{2}\right) = {10}\Big/{z_k' - (z_i + 46)}$$

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$$R_B = \sqrt{x_i^2 + y_i^2 + 46^2}$$

E'H' = $(10 - x_i)^2 + (z_k' - z_i)^2$

From this, we can determine the B angle Bi':

$$B_i' = 2Arccos \sqrt{(10 - x_i)^2 + (z_k' - z_i)^2} \Big/ 2\sqrt{x_i^2 + y_i^2 + 46^2}$$

Similarly, solving for the A angle A_i':

$$A_i' = 2\pi - 2\text{Arccos}\, (y_i + 45)^2 + (z_k' - 45tan\theta_i)^2 \Big/ \sqrt{10^2 + y_i^2 + (z_i - z_k' + 96)^2}$$

7. Conclusion

Through the research on the positioning control algorithm of five-axis CNC machining, this system comprehensively elaborates the principles of orientation and model description when using a five-axis CNC machine tool for machining complex curved parts. With the development of modern manufacturing, the application of five-axis CNC machines has become increasingly widespread, and the research on orientation control algorithms has become more precise.

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