Proceedings of the 2nd International Conference on Applied Physics and Mathematical Modeling
DOI: 10.54254/2753-8818/95/2024.21350

Application of Differential Equations in Malaria Transmission

Keyu Mao

Xi'an Jiaotong-Liverpool University, School of Science, Suzhou, 215000, China

15287117847@163.com

Abstract. This paper focuses on the development of a mathematical model suitable for calculating equilibrium points in the transmission process of malaria. The research takes into account human-to-human transmission: maternal and infant transmission, in addition to most of the transmission modes considered: human-to-mosquito transmission. In the process of maternal and infant transmission, there are two routes of infection that affect the probability of infection and transmission: one is sexual transmission; the other is transfusion transmission. The mathematical model is based on the SEIR model, and the data come from the report of the World Health Organization and statistics from other literature. The model includes susceptible S(t), exposed E(t), infectious I(t), and recovered R(t) classes for humans and mosquitoes. By solving the differential equations, the obtained results are simulated and analyzed. This study found that disease-free equilibrium and optimal treatment points during malaria transmission when considering sexual transmission and blood transfusion transmission. The results in this study may offer some references for further studying the malaria transmission and treatment.

Keywords: Differential equations, Malaria transmission, maternal and infant transmission, mosquito, sexual and transfuse impact.

1. Intrudoction

In recent years, malaria has been spreading in large numbers in South America and Africa. There were an estimated 249 million cases of malaria in 2022, and the estimated number of malaria deaths stood at 608 000. In 2022, the African Region was home to 94% and 95% of malaria cases and deaths, respectively [1]. Moreover, infants and young children continue to suffer the greatest mortality; in 2022, an estimated 4 out of 5 malaria-related deaths in the African Region were among children under 5 years of age [2]. In studying the transmission of malaria, most researchers have focused on the transmission between humans and mosquitoes. In 2009, Ma, Z. [3] wrote a basic malaria transmission model, but the paper only took into consideration three classes; the exposed class wasn't considered. Additionally, J. Labadin [4] in 2009 researched that in person-to-person transmission, the effective temporary immunity in the recovering population is taken into account and the effective duration is calculated. Moreover, S. Olaniyi [5] states that in the process of human-mosquito transmission, malaria parasites will bite and parasitize mosquitoes, so susceptible mosquitoes will produce antibodies related to malaria, which will make susceptible mosquitoes that produce antibodies less susceptible to infection when they bite infected people. In the process of malaria transmission, the infection of mother and child is also more serious. During childbirth, pregnant women will need a blood transfusion due to heavy bleeding. In Africa, malaria infection due to blood transfusion accounts for many diseases [6], and in areas with low malaria transmission rates, the disease severity of children with malaria is often low. It is characterized

© 2025 The Authors. This is an open access article distributed under the terms of the Creative Commons Attribution License 4.0 (https://creativecommons.org/licenses/by/4.0/).

DOI: 10.54254/2753-8818/95/2024.21350

by low parasitemia and high hemoglobin levels [7], but this article does not discuss the influence of hemoglobin in this article. In the field of malaria transmission models, there are no papers discussing the malaria disease-free equilibrium and stable point affected by the mother-infant pathway. Therefore, this paper mainly focuses on considering the parameter changes under mother-to-child transmission in the process of malaria transmission by applying difference equations, which will be more in line with the actual situation and have a more accurate malaria disease-free equilibrium and stable point. This can better prevent the spread of malaria.

This paper presents the malaria transmission model in section Two, taking into consideration the sexual and transfuse in maternal and infant transmission which affects the parameter variations in person-to-person transmission. After this study adjusts the model parameters using clinical data and conducted simulations to produce trend graphs of malaria transmission and find the point of disease-free equilibrium in section Three. Finally, conclusions are remarking in section Ⅳ.

2. Models Foundation

2.1. Development of a sexual transmission model

The development of sexual and transfusion affected mental and infant transmission is also one of the main transmission routes apart from mosquito transmission in malaria. In general situations, researchers normally consider the transfer between humans and mosquitoes as the crucial mode. However, mental and infant transfer affect the changes in parameter values of variables in human-to-human transmission. Thereby, this study initially developed a model for sexual transmission.

The total human population size at time t, denoted by Nh(t), is divided into susceptible humans Sh(t), exposed humans Eh(t), infectious humans Ih(t), and recovered humans Rh(t). therefore:

$$N_h(t) = S_h(t) + E_h(t) + I_h(t) + R_h(t) \tag{1}$$

Assuming that the probability of infection per contact is β0 , and the number N of individuals contacting with infectives in unit time is defined as the contact rate U(N), which depends on the total human population. Then β0U(N) is called adequate contact rate [3]. Except for the susceptible class, other compartments wouldn't be infected when they take into contact with infectives, so the infection rate is:

$$I_r = \frac{\beta_0 U(N) S_h(t) I_h(t)}{N_h(t)} \tag{2}$$

When Uh(N) = k, which means that the contact rate is constant. Hence the infection rate is:

$$I_r^1 = \beta U(N) S_h t I_h t N_h t, \text{where} \beta = \beta_0 k \tag{3}$$

In normal, the saturate contact rate U(N) satisfied the following:

$$U(0) = 0, U^{'}(N) \geq 0, \left(\frac{U(N)}{N}\right)^{'} \leq 0, \lim_{N \to \infty} U(N) = U_0 \tag{4}$$

However, within this paper, the situation in which there is no population doesn't exist. Thereby, U(0) = 0 is not accounted for here [3].

2.2. Transfusion transmitted model

Transfusion transmitted is an alternative infection method, that has an extremely dangerous impact on South America. According to the statistics, about half of all malaria cases were from Mexico (67.3%), 40.3% from the United States of America, and 6.6% from Brazil. Moreover, most of those who get malaria through blood transfusions are women during childbirth. Gyneco-obstetrical conditions (67.3%), surgical procedures (20.6%) and complications from neoplasia (6.1%) are normal conditions for transfusion. Packed red blood cells (RBCs) (50.7%) and whole blood (43.3%) were common

DOI: 10.54254/2753-8818/95/2024.21350

productions associated with transfusion [6]. Incubation of 2-3 weeks is most normal, accounting for 28.6%, followed by 2-5 weeks at 61.9%.

2.3. Model construction

This study considers virus-carrying mosquitoes that transmit diseases to humans to be female, as male mosquitoes do not feed on blood which means that male mosquitoes do not bite humans. When the virus-carrying female mosquito bites a susceptible individual, it secretes saliva to prevent blood clotting. Meanwhile, the virus contained in the mosquito's saliva is injected into the human bloodstream, causing the susceptible individual to move into the exposed class (Eh). Moreover, in the process of malaria transmission, we consider two modes of human-to-human transmission: sexual and blood transmission. Infected individuals (Ih) can transmit the virus to susceptible individuals (Sh) through these two routes, which will also cause susceptible individuals to move into the exposed population (Eh). However, the parasites in the people, during the exposed time, have not developed into gametocytes, which are required for malaria transmission. As a result, the exposed human cannot transmit the virus to susceptible mosquitoes. When the exposed individuals pass through the incubation period, they move into the infected population (Ih). At this stage, gametocytes develop in infected individuals, making them susceptible to mosquitoes. After a period of treatment, the infected individuals recover and acquire temporary immunity. However, since this does not guarantee they will not be reinfected, they become part of the susceptible class again (Sh). Naturally, the population densities within these four human compartments will be influenced by mortality rates and emigration rates, which cause the number of people to decrease.

For the process sequence for mosquitoes, there are three classes: susceptible Sm, exposed Em and infectious class Im.

$$N_m(t) = S_m(t) + E_m(t) + I_m(t) \tag{5}$$

When susceptible mosquitoes bite infective and recovering humans who have developed gametocytes in the bloodstream. However, at this stage, they do not produce gametocytes which leads to susceptible mosquitoes moving into the exposed class. When exposed mosquitoes produce coordinating gametocytes, which is a symbol that mosquitoes are infectious, then the exposed move into the inflective class. Since mosquitoes with disease cannot recover, there is no vector of recovery class [5]. What's more, the number of mosquitoes will also be affected by mortality rates and emigration rates. The complete compartmental model is shown in Figure 1, and the variables for models are described in Table 1.

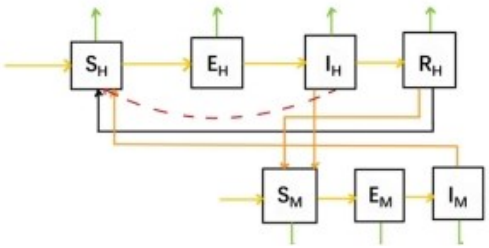


Figure 1. The complete compartmental model between human and mosquitoes

$$\frac{dS_H}{dt} = b_1 N + M + \gamma R_H + LI_H - \left(D_1 + D_2 N\right) S_H - (\beta_1 + BY) S_H \frac{I_H}{N} \tag{6}$$

$$\frac{dE_H}{dt} = (\beta_1 + BY) S_H \frac{I_H}{N} - \left(D_1 + D_2 N\right) E_H - PE_H \tag{7}$$

DOI: 10.54254/2753-8818/95/2024.21350

$$\frac{dI_H}{dt} = PE_H - \left(D_1 + D_2N\right)I_H - \frac{1}{\frac{1}{q}+\frac{1}{r}}I_H \tag{8}$$

$$\frac{dR_H}{dt} = \frac{1}{\frac{1}{q}+\frac{1}{r}}I_H - \left(D_1 + D_2N\right)R_H - \gamma R_H + LI_H \tag{9}$$

$$\frac{dS_M}{dt} = b_2N_0 - \left(\alpha_1 + \alpha_2\right)S_M - \left(\beta_H S_M \frac{I_H}{N} + \beta_H^1 S_M \frac{I_H}{N}\right) \tag{10}$$

$$\frac{dE_M}{dt} = \beta_H S_M \frac{I_H}{N} + \beta_H^1 S_M \frac{I_H}{N} - \left(\alpha_1 + \alpha_2\right)S_M - \mu E_M \tag{11}$$

$$\frac{dI_M}{dt} = \mu E_M - \left(\alpha_1 + \alpha_2\right)S_M \tag{12}$$

$$\frac{dN_0}{dt} = b_2N_0 - \left(\alpha_1 + \alpha_2N_0\right)N_0 \tag{13}$$

$$\frac{dN}{dt} = bN + M - dI_H - \left(D_1 + D_2N\right)N \tag{14}$$

Table 1. Description of variables for models

Variable	Description	Variable	Description
SH	the number of susceptible people	b1	Birth rates that depend on population density
EH	the number of exposed people	M	Immigration rate that does not depend on population density
IH	the number of infectious people	D1	Emigration rates and mortality rates independent of human population density
RH	the number of recovery people	D2	Emigration rates and mortality rates dependent of human population density
SM	the number of susceptible mosquito	α1	Emigration rates and mortality rates independent of mosquito population density
EM	the number of exposed mosquito	α2	Emigration rates and mortality rates dependent of mosquito population density
IM	the number of infectious mosquito	β1	The probability of infection per contact which is related with sexual and blood transmission
N	the total number of people in the area	βH	The probability that a mosquito will infect an infected person per time
N0	the total number of mosquito in the area	βH1	The probability that a mosquito will infect an infected person per time
		P	People move from the exposed class to the infected class in unit time
		γ	Recovery humans establish effective temporary immunity to restore susceptible populations per time
		L	The immune failure of the infected population directly restores to the susceptible population

DOI: 10.54254/2753-8818/95/2024.21350

Table 1. (continued).

	μ	Mosquitoes move from the exposed class to the infected class in unit time
	d	The rate of disease-induced death in human
	B	The probability of being infected by a mosquito bite
	Y	The average number of bites a mosquito makes

3. Results Analysis

3.1. Model results

Before solving nonlinear system equations, these equations use the data from R. Matthew Chico in 2016 [8] to calculate the probability of contracting malaria through sexual transmission toughly accounting for 52.2%. Additionally, as mentioned above, the probability of gynecological and obstetrical diseases caused by blood transfusion is 67.3%. But it is not clear that the data of gynecological and obstetrical diseases is all caused by malaria or that a part of the data may be resulted by malaria. Therefore, there is an assumption that the probability caused by blood transfusion is 60.0%. The system of equations (6)-(12) are nonlinear coupled ordinary differential equations that need to be worked out numerically. Thus, this study uses numerous data from various clinical medical data and resources.

b1 =2.417x10-3[9], M =1.217x10-3[9], γ =8.33x102[4], L =8.333x10-2[4], D1=4.808x10-4[10], D2=1.000x10-5[11], b2=4.227[12], α 1=3.623[11], α 2=6.722x10-7[13], βH=4.7x10-1[13], β1H=2.35x10-1, μ=3.04375[12], q =8.33x102[4], r =5.558x10-2[14], d=1x10-7[15], B=1.2x10-2[16].

This study constructs the initial numerical value: SH =1800, EH = 0, IH =400, RH =0, SM = 900, EM =0 and IM =100. As far as we know, the stability of difference functions is: $\frac{dS_H}{dt} = \frac{dE_H}{dt} = \frac{dI_H}{dt} = \frac{dR_H}{dt} = \frac{dS_M}{dt} = \frac{dE_M}{dt} = \frac{dI_M}{dt} = 0$. Moreover, the final of this paper finds out the stability of malaria. Thus, this study predicts where the number of different people would go in the next 50 days according to the results of model in Figure 2. As can be seen, the number of susceptible class presents an decreasing trend, even though increasing in ten days. And the infectious people decline dramatically, which may be caused by temporary immunity and other precautionary measures. It is worth noting that the reason why the number of recovered people presents the trend toward 0 is the change in the Y-axis is so small for a 500 interval that it looks like it hasn't changed at all. The same is true for the dramatic changes in infected class from 0-10 days.

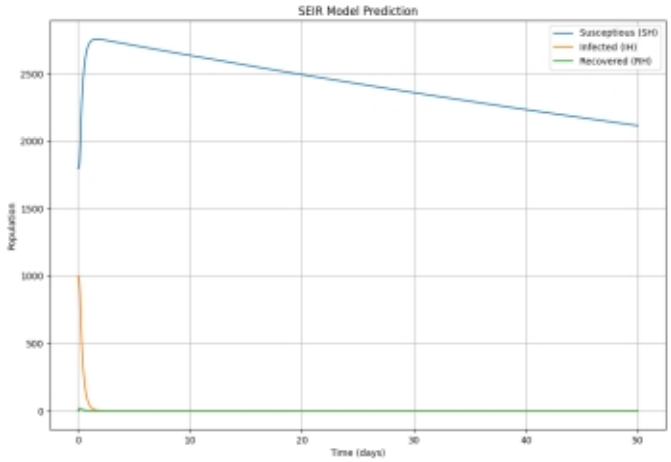


Figure 2. The prediction of model for different classes

DOI: 10.54254/2753-8818/95/2024.21350

3.2. Results discussion

Before the results, the assumption of results should present that the infectious class rises in a period of time and falls down with the recovered class increasing. However, the practical result is the opposite of the hypothesis. The main reason may be caused by the value of parameters. Because the creation of essay takes consideration of sexual transmission into mother-to-fetus transmission, but there may exist other ways or mechanism to influence the procession between sexual and mother-to-fetus transmission, which may cause that the value of parameter for sexual transmission is different from practical situations. And the other reason may be the time. In the process of debugging the results, we actually found that when the transfer time changes, the image waviness changes dramatically. All in all, the results may not be relevant with real life for inaccurate the value of parameter and time, but it fits the procession of immunity.

4. Conclusion

In recent years, the spreading malaria has caused a large number of deaths in South America and Africa, among which, infants and young children has suffered the greatest mortality. Many researches make studies about the malaria transmission, but in the field of malaria transmission models, there are no papers discussing the malaria disease-free equilibrium and stable point affected by the mother-infant pathway. Therefore, this paper mainly constructs the difference model suitable for calculating equilibrium points in the transmission process of malaria, which takes account for sexual and transfer transmission, and analyzes it. Sexual and transfer are not common to consider in the malaria model since these transmissions directly affect the maternal and infant transmission rather than malaria. However, these transmissions are crucial parts of malaria in real life. Additionally, the parameters mentioned in the previously studied literature are not suitable for situations where sexual transmission and blood transfusion transmission are considered. Thus, this model adjusts some parameters to adapt factual situations, which aims to prevent malaria more accurately at the stable point of malaria under numerical analysis. However, this paper only proves the trend of malaria, which considers sexual and transfer transmission under the condition of immunity. This model has never been able to achieve the desired trend, which may be caused by the clinical medical data or the value of parameters. Therefore, the model may need to be practiced in real conditions to predict more practical malaria conditions.

References

[1] Malaria,“Key facts”. https://www.who.int/health-topics/malaria#tab=tab_.
[2] Malaria,”Database”. https://www.who.int/campaigns/world-malaria-day/2024.
[3] Zhien M. (2009) Some Recent Results on Epidemic Dynamics obtained by Our group World Scientific / Higher Education Press publishing The Shaanxi.
[4] J. Labadin, C. Kon M. L. and S. F. S. Juan (2009) Deterministic Malaria Transmission Model with Acquired Immunity. Proceedings of the World Congress on Engineering and Computer Science 2009, Vol II 978-988-18210-2-7.
[5] S. Olaniyi, O.S. Obabiyi. (2013) MATHEMATICAL MODEL FOR MALARIA TRANSMISSION DYNAMICS IN HUMAN AND MOSQUITO POPULATIONS WITH NONLINEAR FORCES OF INFECTION. International Journal of Pure and Applied Mathematics.Volume 88, No. 1, 2013, pp.125-156.
[6] Regina M. Alho, Kim Vinícius Amaral Machado, et al. (2017) Alternative transmission routes in the malaria elimination era: an overview of transfusion-transmitted malaria in the Americas. Malaria Journal. DOI 10.1186/s12936-017-1726-y.
[7] Henrietta E. Mensah Brown, et al. Awandare (2017) Assessing the impact of differences in malaria transmission intensity on clinical and hematological indices in children with malaria. Malaria Journal. DOI 10.1186/s12936-017-1745-8.
[8] R. Matthew Chico, Enesia Banda Chaponda, Cono Ariti and Daniel (2017) Chandramohan1Sulfadoxine-Pyrimethamine Exhibits Dose-Response Protection Against

DOI: 10.54254/2753-8818/95/2024.21350

Adverse Birth Outcomes Related to Malaria and Sexually Transmitted and Reproductive Tract Infections. Clinical Infectious Diseases. 2017;64(8):1043–51.

[9] Cheong H. Tan, Indra Vythilingam, Asmad Matusop, Seng T. Chan, Balbir Singh. "Bionomics of Anopheles latens in Kapit, Sarawak, Malaysian Borneo in relation to the transmission of zoonotic simian malaria parasite Plasmodium knowlesi", Malaria Journal, 2008.

[10] WHO website, "Frequently Asked Questions, Malaria", Global Malaria Programme, Available: http://apps.who.int/malaria/faq.html.

[11] Central Intelligence Agency, "World Factbook 1993", Libraries of the University of Missouri- St. Louis http://www.umsl.edu/services/govdocs/wofact93/wf940147.txt

[12] Francis E. G. Cox, Modern Parasitology. A Textbook of Parasitology, Blackwell Publishing, 1993.

[13] Nakul Chitnis, J. M. Cushing, J. M. Hyman Bifurcation analysis of a mathematical model for malaria transmission, SIAM J. APPL. MATH, Volume 67, Society for Industrial and Applied Mathematics, 2006.

[14] Bloland, P.B, Williams, H.A. Roundtable on the Demography of Forced Migration, and Joseph L. Mailman School of Public Health, Malaria Control during mass population movements and natural disasters, National Academies Press, Washington, DC, 2002zz.

[15] Malaysia Ministry of Health, Annual Report 1993.

[16] J. Nedelman. Inoculation and recovery rates in the malaria model of Dietz, Molineaux and Thomas, Mathematical Biosciences, 1984, pp. 209-233.