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Enhancing Student Assessment Through Social Network Analysis

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Abstract: This study introduces an innovative application of Social Network Analysis (SNA) in the educational sector, specifically for enhancing student assessments beyond conventional methods. Traditional assessments predominantly rely on exam scores, which may not accurately reflect a student's true abilities due to factors like question difficulty and selection randomness. By incorporating SNA, the research constructs a system linking the student-question model with the Rasch model, facilitating a more comprehensive evaluation of student abilities and question difficulties. Parameters are estimated using maximum likelihood estimation applied to real data, demonstrating the method's effectiveness in accurately mapping out the distribution of student abilities and the relative difficulty of questions. However, the approach encounters challenges, particularly with data sparsity, which can lead to instability in parameter estimation and potentially compromise the accuracy of assessment results. Despite these hurdles, the use of SNA within the Rasch model framework shows considerable promise in refining educational evaluations. It provides a new perspective by analyzing the interactions within the educational network, thus offering a richer, more detailed understanding of educational dynamics.

Keywords: Social network analysis, Machine learning, Maximum likelihood estimation, Rasch model

1. Introduction

In the field of education, it is very important to accurately evaluate students' learning situations and ability levels [1]. It not only helps teachers understand student learning progress and provide a basis for personalized teaching but also enables schools and educational institutions to optimize the allocation of teaching resources and enhance the overall quality of education [2]. Traditional student assessment methods mainly rely on exam scores, which have many problems [3,4]. Factors such as differences in question difficulty and randomness in question selection can seriously interfere with the accuracy of assessment results. For example, a highly capable student may score low on an overly difficult test paper, while a less capable student may achieve a high score on an easy one. This phenomenon highlights the problem that traditional assessment methods are difficult to truly reflect students' abilities, calling for a more scientific and comprehensive approach.

Social Network Analysis (SNA) examines the relationship between entities. It's widely used in sociology, marketing, and education. In recent years, its potential in the education field has gradually attracted attention [5]. SNA can break through the limitations of traditional assessments that only focus on individual performances in isolation. By exploring the interaction relationships between

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students and questions, as well as among students, it can uncover in - depth information, providing new ideas and ways for more accurate student ability assessment [6]. In this paper, the rasch model is constructed through SNA, and the students' responses to the questions are visually seen through the bipartite graph. The likelihood function is constructed through the maximum likelihood estimation to solve the two parameters of personal ability and difficulty of the questions. The final results show the gradual distribution of parameters, which is consistent with the results in the references, proving the effectiveness and rationality of the research, and achieving a more accurate assessment of students' personal abilities.

2. Theoretical Framework

2.1. Student-Question Interaction Model

Now suppose if student i answers question j, then $X_{ij} = 1$;if student i did not answers question j, then $X_{ij} = 0$. Obviously this matrix follows a Bernoulli distribution, i.e. X_{ij}~ Bernoulli (0,1). To clearly show the interaction between students and problems, this study constructed a bipartite graph $\mathbf{G}$ [4]. In this bipartite graph, students and problems are located in two different sets of nodes so it can clearly see whether student i answered question j. Through the Rasch model, a matrix of G, called H, can be developed using the adjacent matrix formed by $\{a_{ij}\}$.

On the basis of graph $\mathbf{G}$, this research innovates and sets up a $\widetilde{\mathbf{G}}$ to indicate whether students answer the questions correctly, which helps to analyze the interaction between students and questions more comprehensively. This graphical representation visually shows the complex network of relationships between students and problems, and provides a basic framework for subsequent analysis.

2.2. Overview of Rasch Model

The Rasch model is a method used to analyzing project response data that is simple and easy to understand [7]. It is broad used in educational assessment and psychometrics [8]. The formula for this model is as follows:

$$\mathrm{P}\ (\text{individual}\ \ i\ \text{correctly answers item}\ j\ )\ = \frac{e^{\alpha_i - \beta_i}}{1+e^{\alpha_i - \beta_i}} \tag{1}$$

In this study, it is mainly used to solve the parameters α_i and β_i which show the ability for student i and difficulty parameter for question j. From the formula, it can be seen that the probability of students answering correctly is only related to individual ability and the difficulty of the question.

Ideally, the model assumes that the difference between students' ability and the difficulty of the question determines the possibility of correct answer. If the ability of students is stronger or the difficulty of the question is lower, the probability of answer the question is higher and vice versa. This hypothesis provides an effective theoretical basis for quantifying students' ability and problem difficulty and makes it more accurate to measure students' real level and the actual difficulty of problems in education evaluation.

2.3. Fundamentals of Maximum Likelihood Estimation

In statistics, maximum likelihood estimation is a method used to estimate parameters [8]. In this study, it is employed to estimate the parameters of the Rasch model. The principle is to construct a likelihood function and maximize it. The larger the value of the likelihood function, the more likely the event is to occur. To facilitate calculation, the MLE estimate is obtained by maximizing the log-likelihood function, $\ln L(w|y)$. This is because the two functions, $\ln L(w|y)$ and $L(w|y)$ are monotonically related to each other so the same MLE estimate is obtained by maximizing either one [9].

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In the context of student assessment, considering students and questions as nodes and their interaction behaviors as edges to form a bipartite graph structure, the parameter values of students' abilities and question difficulties can be obtained by maximizing the likelihood function. Compared with other estimation methods, MLE has the property of being asymptotically optimal in large samples, which can provide more accurate parameter estimates [10]. For example, when analyzing a large amount of students' test data, MLE can make full use of the data information to obtain reliable parameter estimation results, providing strong support for subsequent analysis and decision-making [11].

3. Methodology and Data analysis

3.1. Integrating MLE with the Student-Question Model

In the student-question interaction model, the likelihood function of all random variables is developed. The likelihood function reflects the probability of the occurrence of the observed data at different parameter values [12]. The parameter p is used to measure the response rate of the question, i.e., the probability that question j will be answered by the student.

$$f_X(x;p) = \Pi_{i=1}^{r}\Pi_{j=1}^{t} p^{x_{ij}}(1-p)^{1-x_{ij}} \tag{2}$$

Maximize the likelihood function and get the maximum likelihood estimation of p :

$$\hat{p} = \frac{1}{rt}\sum_{i=1}^{r}\sum_{j=1}^{t} x_{ij} \tag{3}$$

Next, based on the conclusion just made, find the estimation of the student's ability parameter θ and the problem difficulty parameter β. Note that this step is based on $X_{ij} = 1$, that is, students can continue to estimate whether the answer is correct only after they have answered the question [13].

$$\mathrm{P}(a_{ij} = 1 | X_{ij} = 1) = \frac{e^{\alpha_i - \beta_i}}{1+e^{\alpha_i - \beta_i}} \tag{4}$$

Under sparse Rasch model (4), the negative log-likelihood function conditional on $\mathrm{G}(\mathrm{r},\mathrm{t})$ is given by, where

$$\begin{aligned} \mathrm{I}(\theta) = &\sum_{X_{ij}=1, i\in[r], j\in[t]}\{a_{ij} log\mu(\alpha_i - \beta_i) + (1-a_{ij})\log(1-\mu(\alpha_i-\beta_i))\} = \\ &\sum_{X_{ij}=1, i\in[r], j\in[t]}\left\{a_{ij} log\mu(\theta_i - \theta_{j+r}) + (1-a_{ij})\log\left(1-\mu(\theta_i-\theta_{j+r})\right)\right\} \end{aligned} \tag{5}$$

Then maximum the negative log-likelihood function. The MLE of θ is defined by $\hat{\theta} = argmax_{\theta\in R^{r+t}}\max\ l(\theta)$. After the partial derivative is obtained, the following formula is obtained, but it is found that the equation is a nonlinear equation. Numerical methods such as Nelder-Mead method is used to solve it.

$$\sum_{j\in[t],\, X_{ij}=1} a_{ij} = \sum_{j\in[t],\, X_{ij}=1} a_{ij}\, \mu(\hat{\theta}_i - \hat{\theta}_{j+r}), i \in [r] \tag{6}$$

$$\sum_{i\in[r],\, X_{ij}=1} a_{ij} = \sum_{i\in[r],\, X_{ij}=1} a_{ij}\, \mu(\hat{\theta}_i - \hat{\theta}_{j+r}), j \in [t] \tag{7}$$

3.2. Data Collection

The real data is from the public Riiid AlEd data set for the reference data, available at https://www.kaggle.com/c/riiid-test-answer-prediction/data. It has lots of data that can use as students and questions in this study. And this study randomly chose 98 items and then selected individuals who responded to at least 50 items among 98 items in the total of 1283 individuals [13]. The response matrix is treated as from the graph H with a sampling probability of 51.93%. The minimum and

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maximum degrees for individuals in H are 12 and 117 respectively, and for items are 4 and 1792 respectively. These data are selected to ensure that they are representative of students' performance in different learning scenarios. The data collection protocol includes determining the frequency and sample size of data collection [14]. In the process of data collection, it also needs to pay attention to the completeness and accuracy of the data, and the missing values and erroneous data that may occur are strictly screened and processed.

3.3. Data Processing Techniques

The performance of machine learning model depends on the quantity, quality and diversity of data. To improve the reliability of the algorithm, it is necessary to filter and process the original data. The data mentioned in 3.2 is extracted from a large public database, so rational selection is required. Data processing is an important part, and the main purpose is to remove outliers, duplicate values and other noisy data in the data [11]. Outliers can be due to data entry errors or a student's unique circumstances, and if left unaddressed, can have a significant impact on the results of the analysis. Duplicate values increase the redundancy of the data and affect the efficiency of the analysis. Standardization is the unified and standardized transformation of different types of data to make them comparable. Regarding the handling of missing values, model - based imputation methods can be employed [13,14]. For example, by using students' historical grades and the performance information of the same group of students, missing values can be estimated and filled through methods such as regression models to ensure data integrity and provide a reliable data basis for subsequent analysis [15].

3.4. Analysis of result

The figure 1 is the distribution of ability parameters, while the right one presents the distribution of difficulty parameters. The study in references shows that the parameter $\theta\hat{}$ follows asymptotic normal distribution when $r, t \to \infty$, i.e. the standardized value converges in distribution to the standard normal distribution [16]. As shown in Figure 2, in the visualization of the ability parameter (the number of the parameter is large enough), they form an approximate normal distribution curve, which means our work is of certain validity.

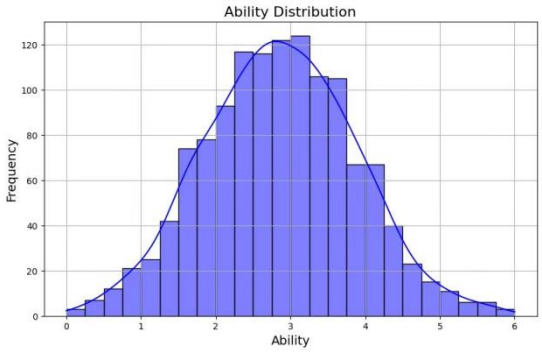

Figure 1: Student ability distribution. (Picture credit : Original)

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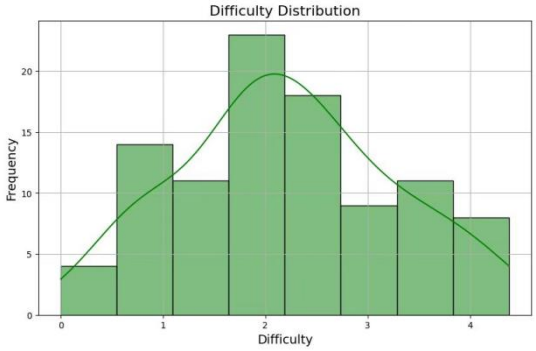

Figure 2: Question difficulty distribution. (Picture credit : Original)

4. Challenges and optimization

4.1. Identifying Challenges

Data sparsity is a significant hurdle in our study because students can not answer all the questions, resulting the data matrix tend to be sparse. This sparsity can cause instability in model parameter estimation and undermine the accuracy of assessment results. For instance, students often select only a subset of questions based on their interests or relevant majors. As a result, the data for some questions is insufficient, making it difficult to accurately estimate their difficulty parameters. Also, student ability and question difficulty parameters are not fixed, but change over time and over learning process, where sparse observations pose challenges to parameter estimation in the Rasch model [17].

4.2. Proposed Solutions

To address the data sparsity issue, innovative methods can be employed. One such approach is to draw on the concept of random pairing, similar to that used in the Random Pairing Maximum Likelihood Estimator (RP - MLE). By randomly pairing user - item responses to create item - item comparisons, the size of the data matrix can be reduced while maintaining statistical independence. This helps in extracting more useful information from sparse data and improves the stability of parameter estimation [18].

Another solution is to use data augmentation techniques. For instance, virtual data similar to real-world student answering data can be generated through Generative Adversarial Networks (GANs) [19]. This increases the data richness and alleviates the data sparsity problem.

In addition, improved estimation methods can be considered. Regularization-based maximum likelihood estimation can be used to constrain parameters. This is achieved by adding a regularization term to the likelihood function, which enables more stable parameter estimates even with sparse data. Applying these methods aims to enhance the reliability of the model when dealing with data sparsity.

5. Conclusion

This study successfully demonstrates the application of Social Network Analysis (SNA) and the Rasch model to enhance the assessment of student abilities beyond traditional methods, which predominantly rely on direct exam scores. By integrating these models, the study provides a more nuanced analysis of student interactions with test questions, effectively mapping out the complexity of student abilities and the difficulties inherent in the examination questions themselves. This approach not only refines the accuracy of student assessments but also introduces a new dimension to understanding educational interactions through network analysis.

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However, the application of this method revealed significant challenges, particularly the sparsity of data which affects the stability and accuracy of parameter estimations. Despite these obstacles, the research highlights the potential of SNA in educational assessments, providing a foundation for future improvements. It suggests that addressing data sparsity through innovative methods such as data augmentation and advanced estimation techniques could further enhance the reliability and efficacy of this model.

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