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Conditions for Maximum Discrepancies of Axes in Elliptic Flow

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Abstract: Event shape engineering is a critical technique in studying quark-gluon plasma, probing the initial state of collision and understanding anisotropic flow, which can be advanced by studying the elliptic flow. Parameters like ellipticity (ϵ_2), triangularity (ϵ_3), event plane angle for elliptic Flow (ψ_2), event Plane Angle for triangular Flow (ψ_3), and impact parameter (b) can influence the pattern of elliptic flow. This study aims to find how these parameters influence the difference between the major and minor axes of elliptic flows and the range of initial conditions that can give the largest 5% difference. A specific condition that can produce the maximum discrepancy is analyzed individually. What's more, the relationship between the major and minor axes' lengths and the number of participants, and their internal relationship are analyzed. These goals were achieved by analyzing the data of real $75000\ \mathrm{Pb}-\mathrm{Pb}$ collision events under various conditions.

Keywords: Initial condition, elliptic flow, quark-gluon plasma, event shape engineering, nucleon distribution

1. Introduction

The elliptic flow produced in the heavy-ion collision is essential for understanding the behavior of the quark-gluon plasma. In non-central heavy-ion collision, the overlap region of the nuclei is not a sphere, this causes the uneven pressure gradient in the early stage of collision, so the hydrodynamics expansion later is anisotropic(uneven), which is the expansion of quark-gluon plasma[1]. The anisotropic expansion leads to anisotropic particle momentum distributions. The area with higher particle densities is in the direction of the major axis and the area with lower densities is in the direction of the minor axis, creating an approximately elliptic pattern. This is the origin of elliptic flow[2]. These distributions can be detected through correlations in the azimuth of the emitted particles relative to the reaction plane. Elliptic flow can be characterized by the second harmonic coefficient of the Fourier expansion of the azimuthal distribution of particles, which is usually noted by v2, a factor that

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is an approximately linear relationship between the ellipticity (ϵ_2) of the overlap zone, so observable elliptic flow is a channel for scientists to study the early stages of nuclei collisions [3]. The properties of quark-gluon plasma can also be further explored by varying impact parameter and it is related the initial conditions[4].

Many previous studies have made efforts in this specialized area[1, 5, 6, 7]. Stoecker (2005) established the role of collective flow, including elliptic flow, in signaling QGP formation by examining how these patterns emerge from the anisotropic expansion of the collision fireball. Teaney et al. (2001) further demonstrated that elliptic flow serves as a valuable probe of the initial collision geometry and the properties of the QGP, particularly at SPS and RHIC energies. Beattie et al. (2023) expanded on this by exploring how hard probe path lengths and event-shape engineering influence elliptic flow measurements, emphasizing the need to consider eventspecific geometrical factors. Roland et al. (2014) reviewed advancements in measuring elliptic flow at the LHC, highlighting how high-energy measurements and refined event shape engineering contribute to our understanding of QGP dynamics. Collectively, these studies underscore the significance of elliptic flow in characterizing the QGP and reflect ongoing efforts to enhance measurement precision and theoretical interpretation in heavy-ion collisions, therefore previous studies have significantly advanced our understanding of elliptic flow as a crucial signature of the quark-gluon plasma (QGP).

However, the conditions under which the difference between the major and minor axes reached the peak have hardly been investigated. To better understand the elliptic flow, we analyze the coordinates of the nucleons in results from 75000 real $\mathrm{Pb}-\mathrm{Pb}$ collision events, fitting the position of the center of each nucleus. For each event, the length of the major and minor axes have been measured, and the top 5% with the largest discrepancies are selected for further analysis. These events represent the most asymmetric participant distributions and are of particular interest in understanding the underlying collision mechanics. In this study, five specifications are thought about to have an impact on the pattern of the elliptic flow. The criteria are ellipticity (ϵ_2), triangularity (ϵ_3), event airplane angles $(\psi_2$ and $\psi_3)$, and the effect criterion respectively. Triangularity describes the triangular deformation of the initial accident geometry, which can contribute to the third harmonic element of the flow pattern [14]. ψ_2 and $\psi 2$ are the angles connected with the second and third harmonics of the event aircraft, representing the directions of the elliptic and triangular flow respectively [14]. The effect specification, on the other hand, is the range in between the centers of the two clashing nuclei and plays an important function in identifying the total shape and size of the interaction zone [13]. Together, these specifications help define the complex geometry and characteristics of the crash, influencing the resulting flow patterns observed in high-energy nuclear collisions. A five-dimensional space including these 5 criteria is developed and examined. Besides, the relationship between the major axes length, minor axess length and number of participants also been analyzed in this study. The number of participants as an individual parameter that have influence on the pattern of elliptic flow has some relationship between the impact parameter. Mathematically, the number of participants tends to decrease roughly exponentially or polynomially as a function of the impact parameter(b), depending on the exact geometry of the collision and model used (e.g., Glauber model). Thus, there is a decrease in participants as the collision becomes more peripheral[18]. Because the number of participants affects the intensity and distribution of the collision's energy and momentum, the understanding of how the shape of the participant distribution changes with the number of participants can be improved through analyzing these relationships.

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2. Data and Methods

The data set used in this study consists of 75,000 real high-energy crash occasions, each specified by specific initial conditions in the five-dimensional area. The primary objective of this analysis is to determine and evaluate the leading 5% of occasions that display the largest discrepancy between the lengths of the significant and minor axes of the nucleon distribution of the individual. To achieve this, each event is processed independently, focusing on the participant nucleons. Each nucleus is modeled as a circle with a radius of 6.62 fm , consistent with the parameters of the Glauber model for the lead nuclei (Pb)[8]. The nucleons within these nuclei are represented as smaller circles, with the area of each circle corresponding to the inelastic nucleon-nucleon cross-section. The diameter of small circle is given by:

$$D = \sqrt{\frac{\sigma_{NN}}{\pi}}. \tag{1}$$

[4] Two nucleons will collide with each other if the relative transverse distance between two nucleons is less than this diameter. This makes the visual representation can accurately reflects the physical scale of the collision process. It is important to note that this approach assumes all nucleons (protons and neutrons) have the same size, a simplification that mirrors the assumption made in the Glauber model. In the Glauber model, this uniform size assumption helps simplify the complex interactions between nucleons, enabling easier modeling of nuclear collisions[10]. The center of each nucleus is determined by calculating the mean of the X and Y coordinates of all nucleons within the nucleus. This mean value provides a precise location for the center of the nucleus, which is essential for aligning the coordinate system used in subsequent analyses. To analyze the spatial distribution of participant nucleons, a new coordinate system (X', Y') is established for each event. This reorientation aligns the X' Axis with the line connecting the centers of the two nuclei, while the Y' Axis serves as its perpendicular bisector in the transverse plane. The reorientation of the coordinate system is achieved using a matrix transformation. The code applies a rotation matrix to the original coordinates of the nucleons, effectively rotating the axes to align with the physical geometry of the collision. This transformation ensures that the major and minor axes of the participant zone are properly aligned with the new coordinate system. The rotation matrix $R(\theta)$ used for this transformation is given by:

$$R(\theta) = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \tag{2}$$

where θ is the angle between the original X Axis and the line connecting the nuclei centers (X') [9].

Once the former two steps are finished, graphs are generated for all 75,000 collision events. The spatial distributions of nucleons and participants were plotted. The axis length calculation can be simplified by these graph. Although graphs for every event are plotted, only a typical example is included in the essay to illustrate the methods and key findings. Figure 1 displays the positions of Nucleus A, Nucleus B, and the participant nucleons, with the major and minor axes overlaid of a event which have relative more participants. The different colors are used to differentiate nucleons from the two nuclei and the participants, and the overlapping small circle represent the participants. X' and Y' are represented on the graph by two perpendicular red lines. After the coordinates have been transformed, the major axis is calculated as the distance between the participant nucleons with the maximum and minimum X' coordinates. Similarly, the minor axis is determined by the distance between the participant nucleons with the maximum and minimum Y' coordinates. These calculations are repeated for each of the 75,000 events, providing a comprehensive dataset of axis lengths for further analysis.

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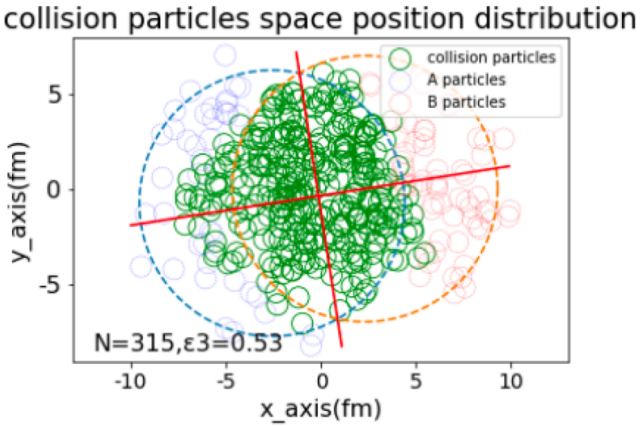


Figure 1: Collision particles space position distribution

When the calculations of the major and minor axes are finished for all events, the absolute difference between these axes is computed. The events are then ranked based on this difference, and the top 5% with the largest discrepancies, are selected for further analysis. The top 5% of events are analyzed within the five-dimensional space defined by $\psi_2, \psi_3, \epsilon_2, \epsilon_3$, and the impact parameter(b). Each event's position in this space is recorded, and correlations between these parameters and the observed axis discrepancies are investigated.After this, the graphs of major axes length against number of participants,minor axes length against number of participants, and the major axes length against minor axes length are plotted by Python respectively. The data set used is same as the former step, and in this process, we consider the number of participants as an individual parameter which is not in the five-dimensional space, and the length of axes in the elliptic flow is the value calculated in the former step.

3. Result

The differences between the major and minor axes in the elliptic flow are influenced by the five factors in the five dimensional space. The top 5% of the collision events, which exhibit the highest values in this difference, have been identified and summarized in Table 1. These top 5% events are characterized by significant spatial anisotropy, a direct consequence of the initial geometric and momentum distributions of the colliding nuclei.

Table 1: Range of Initial Conditions for the Top 5% of Events

Parameter	Minimum 2	Maximum
ϵ_2	0.0046866	0.926646
ϵ_3	0.0004	0.807835
ψ_2	0.000195	3.141129
ψ_3	0.000702	0.2 .094378
Impact Parameter(b)	$3.147549(\mathrm{fm})$	$11.999515(\mathrm{fm})$

Table 1 reveals that the ellipticity (ϵ_2) for these events ranges from 0.0047 to 0.9266 , indicating a pronounced elliptical deformation in the collision zone. Triangularity (ϵ_3) varies between 0.0004

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and 0.8078 , contributing to the complexity of the flow pattern. The event plane angles for elliptic and triangular flow, ψ_2 and ψ_3, are narrowly confined to $0.0002 - 3.1411$ radians and $0.0007 - 2.0944$ radians, respectively, suggesting a consistent alignment of flow patterns across these events. Lastly, the impact parameter falls between 3.1475 and 12.0000 fm , indicating that these events occur in peripheral collisions, where the overlap region is highly asymmetric. These conditions collectively produce significant anisotropy in the elliptic flow, leading to the observed discrepancies.

Among the top 5% of events with the highest discrepancy between the major and minor axes, Event Number 40681 stands out. The specific parameters and geometric measures for this event are detailed in Table 2.

Table 2: Specific Parameters and Geometric Measures for Event 40681

Parameter	**Value**
Ellipticity (ϵ_2)	0.586248
Triangularity (ϵ_3)	0.154744
Event Plane Angle for Elliptic Flow (ψ_2)	0.007850 (radians)
Event Plane Angle for Triangular Flow (ψ_3)	0.827297 (radians)
Impact Parameter	$9.478607(\mathrm{fm})$
Major Axis Length	$7.652(\mathrm{fm})$
Minor Axis Length	$2.986(\mathrm{fm})$
Difference Between Axes	$4.666(\mathrm{fm})$

Event 40681, characterized by a high ellipticity ($\epsilon_2 = 0.586248$) and moderate triangularity ($\epsilon_3 = 0.154744$), exhibits significant deformation in the initial collision geometry. The nearly zero value of ψ_2 (0.007850 radians) suggests a strong alignment along the reaction plane, while ψ_3 (0.827297 radians) contributes higher-order harmonic effects to the flow pattern. The impact parameter of 9.478607 fm places this event in the peripheral collision regime, where the asymmetry in the overlap region is significant. As shown in Table 1, the conditions for Event 40681 are consistent with the range identified for the top 5% of events, characterized by high ellipticity, moderate triangularity, and peripheral collisions. Together, these factors result in substantial anisotropy in the elliptic flow, leading to the large discrepancy observed between the major and minor axes. The relationships between the major and minor axis length and the number of participants are also plotted into graphs respectively. Figure 2 plot displays the relationship between the number of participants and the short axis length.

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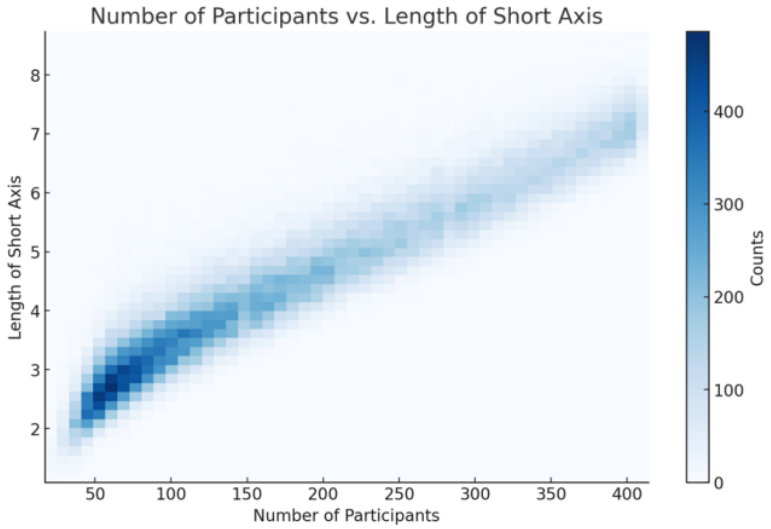


Figure 2: Number of Participants vs. Length of Short Axis

This graph shows that the length of the short axis generally increases as the number of participants increases. When ther is alager participants number, a linear correlation between the number of participants and the length of the short axis can be observed. The concentration of events is higher when the number of participants is around 50 to 150 , where the lengths of the short axis range between 2 and 6 units, therefore the distribution is denser at lower values of the short axis, which indicates that more events occur with a smaller short axis and at lower participant numbers as the result from the data set[15]. The relationship between the number of participants and the length of the long axis is shown in Figure 3. A increasing trend that is similar to the short axis can be observed, though the long axis tends to have slightly larger values. In Figure 4, the major axes's length increases as the minor axes' length increases. The other noticeable observation is that the discrepancies between the two axes decrease as the number of participants increases. This suggests that spatial anisotropy tends to decrease with a larger number of participants in central collisions, so events with higher participant nucleons can result in a weaker elliptic deformation.

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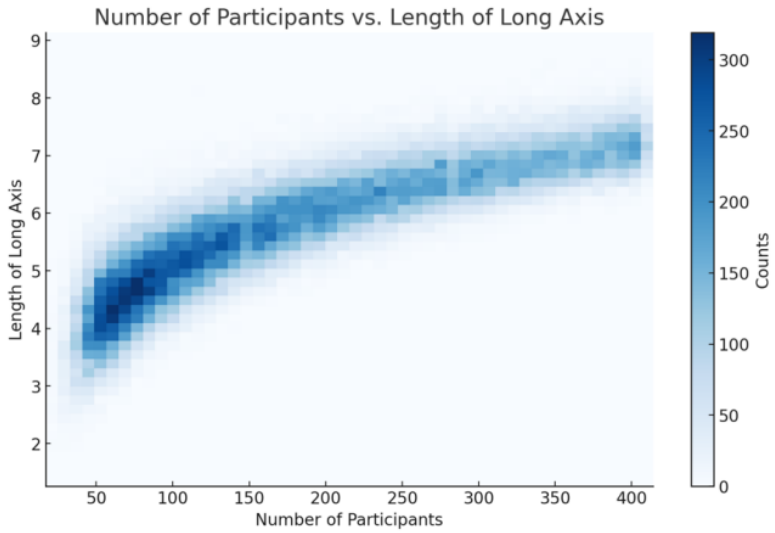


Figure 3: Number of Participants vs. Length of Long Axis

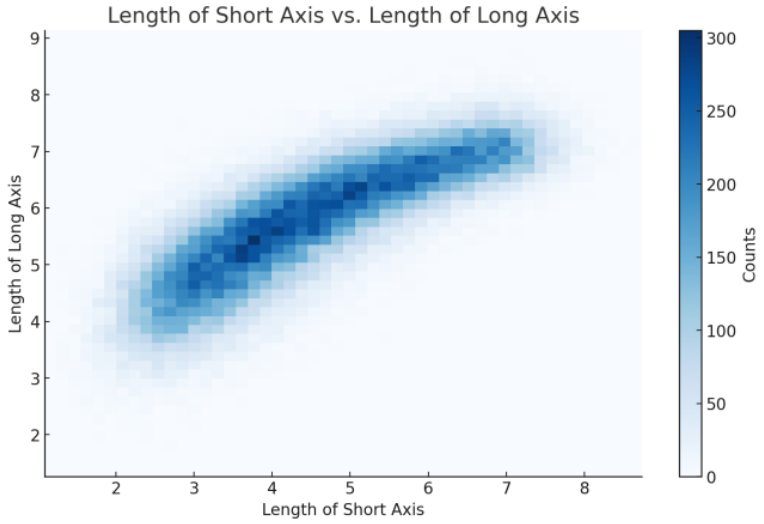


Figure 4: Length of Short Axis vs. Length of Long Axis

4. Discussion

The results of this study can be directly linked to the origins and behavior of flow anisotropies in heavy-ion collisions. Flow anisotropies, such as elliptic flow (v_2) and triangular flow (v_3), arise from the anisotropic pressure gradients in the quark-gluon plasma (QGP), which are a direct consequence of the initial spatial geometry of the collision zone. In high-energy collisions, the initial setup of the overlapping nuclei typically shows spatial eccentricities, where the distribution of participating nucleons is not completely circular. These initial geometric variations relate to the development of flow anisotropies as the QGP expands. The substantial distinctions in between the significant and small axes observed in the selected leading 5% of occasions are indicative of these initial anisotropies. Occasions with bigger disparities recommend more severe initial geometries, which lead to more

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powerful flow patterns. This connection enhances the concept that the observed disparities are not simply random variations but are physically substantial. The geometry of the crash plays a crucial function in developing flow anisotropies. Comprehending the relationship between initial geometry and flow anisotropies is essential for translating the dynamics of the QGP. The findings from this research study contribute to this understanding by demonstrating how particular initial conditions can lead to boosted anisotropic flow, aligning with the theoretical expectations of how (v2) and (v2) develop in reaction to the initial spatial distribution of nucleons [11, 12, 16].

Additionally, the observed relationship between the number of participants and the lengths of the major and minor axes also shows the role of nucleon interactions in shaping the anisotropy of the collision geometry. From the result of our essay, there is a smooth transition from peripheral collisions to central collisions while the number of participants increases, so the ellipticity decreases with the number of participants.

This can be attributed to the geometric evolution of the overlap region between the colliding nuclei. In peripheral collisions, the overlap region is highly asymmetric, resulting in high ellipticity. As the number of participants increases, the collision becomes more central, the overlap region grows and becomes more symmetric, approaching a circular shape. This shift reduces the difference between the major and minor axes, thereby decreasing ellipticity.

5. Conclusion

This research study the particular initial conditions that can trigger more significant difference between the major and minor axes of elliptic flow in high-energyheavy-ionn collision. By analyzing the leading 5% of events within a huge dataset, we find 5 essential parameters including ellipticity, triangularity, event plane angles, and impact parameters play an essential role in shaping the observed flow patterns. The analysis of the relationship between the major axes' length, minor axes' length, and the number of participants in this study can also help us understand nuclear collision which shows that the number of participants as an individual parameter which is related to the impact parameter also have an influence on elliprtic flow. A larger number of participants means the flow pattern is less anisotropic.

In this study, the analysis is based upon a large dataset of 75,000 events, providing an excellent analytical foundation. This great sample size enhances the dependability of the observed patterns and guarantees that the findings are representative of a wide range of accident occasions. By focusing on the five crucial specifications, the research study effectively isolates and analyzes their functions in figuring out anisotropic flow.Furthermore, this study reveals a clear correlation between the number of participants and the lengths of the major and minor axes, showing that a higher participant count amplifies the spatial anisotropy and influences the overall elliptic deformation of the collision zone. Charts are also used in this study to represent information. These visualizationshelp to show the relationships between different criteria and their impact on elliptic flow discrepancies, making complex information more available. In addition, the research study alignment with established models, such as the Glauber design, contributes to its importance. By making comparable assumptions relating to nucleon size, the research study guarantees consistency with commonly accepted theoretical frameworks. This combination not only confirm this study method but also positions its findings within the broader context of nuclear physics research.

However, while the simplification of all the nucleons(protons and neutrons)have the very same size makes the model lines up with established approaches, it may overlook the slight differences in spatial circulations between protons and neutrons. These differences could potentially influence the elliptic flow measurements and the calculated inconsistencies. As a result, this simplification presents

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prospective approximations that may impact the precision of the outcomes. Besides, precisely identifying and confirming the inconsistencies in elliptic flow can be challenging and expensive. High-resolution measurements and advanced experimental equipment are needed to record the exact details of the spatial circulations and flow patterns. The expense and intricacy of such speculative efforts can make it tough to validate theoretical forecasts. This barrier exposes the requirement for ongoing improvements in detection development and techniques to improve the accuracy and scope of measurements.

Looking ahead, there are numerous appealing opportunities for future research study. One important direction is to explore additional aspects and higher-order harmonic circulations to acquire a more extensive understanding of the factors influencing anisotropic flow. Broadening the analysis to consist of a wider variety of collision energies and impact specifications could likewise offer deeper insights into the underlying physics. Moreover, eliminating or modifying assumptions that were made to streamline the issue, such as the consistent size of nucleons, could result in more precise designs that better record the intricacy of nuclear accidents. Evaluating comprehensive spatial circulations of protons and neutrons might reveal impacts that are currently ignored. Another crucial area for future work is the advancement of advanced detection and verification approaches. Since it is hard and pricey to measure flow discrepancies properly, it is important to enhance speculative approaches and information analysis tools to much better confirm theoretical forecasts and expand research possibilities. In summary, while this study can enhance the understanding of anisotropic flow, it likewise highlights the need for future research. By attending to the constraints and structure on the findings presented here, future studies can fine-tune theoretical models and improve experimental approaches.

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Data Availability

The dataset utilized in this study consists of 75,000 high-energy accident occasions, each characterized by specific initial conditions in a five-dimensional space: ψ_2, ψ_3, v_2, v_3, the impact specification and the number of participants. This data is necessary for evaluating the discrepancy in between the significant and small axes of the nucleon distribution in individual nuclei. Due to the nature of the dataset, access is restricted to licensed workers only. For research functions, interested parties might request access through Gunther Roland at MIT, and possible partners should describe their research study goals and requirements.

References

[1] Beattie C, Nijs G, Sas M, et al. Hard probe path lengths and event-shape engineering of the quark-gluon plasma[J]. Physics Letters B, 2023, 836: 137596.
[2] Pruseth P P. Role of Elliptic ffow in Quark Gluon Plasma[J].
[3] Voloshin S A, Poskanzer A M. The physics of the centrality dependence of elliptic ffow[J]. Physics Letters B, 2000, 474(1-2): 27-32.

DOI: 10.54254/2753-8818/2025.23341

[4] Alver B, Baker M, Loizides C, et al. The phobos glauber monte carlo[J]. arXiv preprint arXiv:0805.4411, 2008.
[5] Stoecker H. Collective ffow signals the quark-gluon plasma[J]. Nuclear Physics A, 2005, 750(1): 121-147.
[6] Teaney D, Lauret J, Shuryak E V. Flow at the SPS and RHIC as a quark-gluon plasma signature[J]. Physical Review Letters, 2001, 86(21): 4783.
[7] Roland G, Šafařík K, Steinberg P. Heavy-ion collisions at the LHC[J]. Progress in Particle and Nuclear Physics, 2014, 77: 70-127.
[8] Mäntysaari H, Salazar F, Schenke B. Nuclear geometry at high energy from exclusive vector meson production[J]. Physical Review D, 2022, 106(7): 074019.
[9] Weisstein E W. Rotation matrix[J]. https://mathworld. wolfram. com/, 2003.
[10] Mifsud J. Optical Glauber Modeling in high-energy nuclear collisions[R]. 2013.
[11] Mallick N, Prasad S, Mishra A N, et al. Estimating elliptic ffow coefffcient in heavy ion collisions using deep learning[J]. Physical Review D, 2022, 105(11): 114022.
[12] Snellings R. Elliptic ffow: a brief review[J]. New Journal of Physics, 2011, 13(5): 055008.
[13] Das S J, Giacalone G, Monard P A, et al. Relating centrality to impact parameter in nucleus-nucleus collisions[J]. Physical Review C, 2018, 97(1): 014905.
[14] Bierlich C, Buckley A, Christensen C H, et al. Confronting experimental data with heavy-ion models: RIVET for heavy ions[J]. The European Physical Journal C, 2020, 80: 1-15.
[15] Wang E Q, Wei H R, Li B C, et al. Dependence of elliptic ffow on participant nucleon number in Au+ Au collisions at s NN= 200 GeV[J]. Physical Review C—Nuclear Physics, 2011, 83(3): 034906.
[16] Gardim F G, Grassi F, Luzum M, et al. Mapping the hydrodynamic response to the initial geometry in heavy-ion collisions[J]. Physical Review C—Nuclear Physics, 2012, 85(2): 024908.
[17] Pandey S, Singh B K. Predicting quadrupole deformation via anisotropic ffow and transverse momentum spectra in isotopic 54 128-135 Xe collisions at LHC[J]. The European Physical Journal C, 2024, 84(5): 511.
[18] Gorenstein M I, Gaździcki M, Greiner W. Critical line of the deconffnement phase transitions[J]. Physical Review C—Nuclear Physics, 2005, 72(2): 024909.