Proceedings of the 3rd International Conference on Applied Physics and Mathematical Modeling
DOI: 10.54254/2753-8818/2025.DL27127

A Study of Short Sale Constrained Portfolios Based on the Mean-Variance Model in the U.S. Stock Market

Yaming Liu

Beijing No.80 High School, Beijing, China
482774679@qq.com

Abstract. With the continuous development of the stock market, achieving a balance between return and risk through the optimization of investment portfolios has become one of the core concerns for investors. This paper studies the problem of optimal stock portfolio selection using the mean-variance framework. Building on the mean-variance model proposed by Markowitz, numerous subsequent studies have extended this foundation, including the introduction of solution-based approaches and the establishment of mean-variance models under novel risk perceptions. This study selects 12 U.S. stocks (ABBV, ADBE, AMD, etc.) and uses the closing prices on the first trading day of each month from January 3, 2022, to February 3, 2025, as sample data. By calculating monthly stock growth rates, expected returns, and the covariance matrix, and applying the solution algorithm of the mean-variance model, the efficient frontier of the investment portfolio is derived. Under the practical market constraint of a short-sale prohibition, iterative calculations are performed. The optimal frontier portfolio corresponding to an expected return of 0.013 is obtained, providing a quantitative reference for investors to balance return and risk and to formulate investment strategies in actual stock investment.

Keywords: Mean-variance Model, Investment Portfolio, Stock Investment, Efficient Frontier

1. Introduction

In China, stock investment can be traced back to the establishment of the Chinese stock market in 1990, when the market was small and the trading mechanisms were simple. During the development of the stock market, policies gradually improved, and with technological advancements, online stock trading was introduced. Since then, the number of individuals participating in stock investment has continued to grow. Today, there are over 5,000 listed companies in China, with a total market capitalization approaching 80 trillion yuan and more than 200 million investors. To achieve higher returns, an increasing number of people are paying attention to the optimal allocation of stock portfolios.

In the stock market, returns and volatility are closely linked. For investors seeking maximum returns, the ability to accurately select stocks to achieve relatively high returns while minimizing risk is crucial. The mean-variance model provides a quantitative framework for obtaining optimal stock portfolios. It enables precise calculation of expected returns and variance, which represents risk, thereby effectively optimizing decision-making and reducing the risk of blind investment.

© 2025 The Authors. This is an open access article distributed under the terms of the Creative Commons Attribution License 4.0 (https://creativecommons.org/licenses/by/4.0/).

DOI: 10.54254/2753-8818/2025.DL27127

The research approach of this paper is divided into four main parts. First, the literature review section summarizes previous studies on the mean-variance model. In the Methodology section, relevant theories, idealized assumptions, the modeling process, and the solution algorithm of the mean-variance model are presented. In the Results section, closing prices on the first trading day of each month for 12 U.S. stocks are selected as sample data, and iterative calculations are performed using the model. Finally, the efficient frontier and frontier portfolio schemes are obtained, providing conclusions and quantitative references for practical investment behavior.

2. Literature review

The mean-variance model first appeared in 1952, when American economist Markowitz [1] conducted a quantitative analysis of risk and return to derive optimal investment strategies. Due to certain limitations of Markowitz's mean-variance model, numerous subsequent studies have sought to extend and improve it. Wang, X. G., and Qiu, W. H [2], within the mean-variance framework, considered investors' mean-variance preferences and downside risk control requirements. By constructing a dynamic portfolio decision model, they derived the optimal strategy for dynamic investment portfolios. Yue, Y. J., et al.[3] employed the primal-dual interior-point algorithm for convex quadratic programming problems to solve the mean-variance model. Wang, X. G., and Wang, Y. D [4]. also applied convex duality methods and martingale theory to study continuous-time dynamic portfolio optimization under expected loss risk constraints, obtaining analytical expressions for optimal wealth and optimal investment strategies.

Zhang, S. P., and Wu, C. F [5]. established a mean-variance model under novel risk perceptions, extending the explanatory power of the traditional model and further elucidating the economic phenomenon of investors exhibiting different behaviors under identical investment opportunities. Xu, Y. H., and Li, Z. F [6]. embedded the indivisible dynamic mean-variance model into a divisible quadratic utility problem, taking into account the serial correlation of risky asset returns, and studied the optimal portfolio selection problem, obtaining the optimal investment strategy. Huo, Y. L., and Zhang, H. T [7]. extended the general mean-variance model and proposed the solution-equation approach. Liu, X. D [8]. conducted an empirical study and, based on the China Insurance Regulatory Commission's requirements for the use of insurance company funds and the mean-variance model, obtained the optimal investment portfolio for insurance companies.

Wang, L., and Xu, M. P [9]. applied the mean-variance model and maximized the Sharpe ratio, using a multivariate t-distribution to fit daily return data with leptokurtosis and fat tails, obtaining the optimal investment proportions and the risk value, expected loss, and median loss of the optimal portfolio at different confidence levels. These three indicators can help investors monitor risk. Hu, S. C., et al.[10] considering the actual situation in China, developed an improved mean-variance portfolio model that accounts for both transaction costs and risk-free assets, and designed an interactive decision-making process for this improved model, enhancing the practical applicability of portfolio investment schemes. Li, S. L [11]. established a Markowitz portfolio model in which both mean and variance vary simultaneously, and compared it with the traditional Markowitz model, deriving the efficient frontier and examining the impact of changes in mean and variance on the frontier curve. Li, X. Y., and Wang, B. H [12]. combined robust statistical concepts with the traditional mean-variance model to construct a robust mean-variance model aimed at reducing or eliminating the influence of outliers on model calculations.

DOI: 10.54254/2753-8818/2025.DL27127

3. Methodology

Based on the closing prices of n stocks on the first trading day of each month, the growth rate R is calculated using the following formula:

$$R = \frac{P_t - P_{t-1}}{P_{t-1}} \tag{1}$$

Where P_t is the closing price on the current day, and P_{t-1} is the closing price on the first trading day of the previous month.

For the growth rate of each stock, the expected return E is calculated.

$$E = \frac{1}{n} \sum_{i=1}^{n} R_i \tag{2}$$

The covariance is calculated according to the formula

$$Cov(R_i, R_j) = E[(R_i - E[R_i])(R_j - E[R_j])] \tag{3}$$

This yields an n×n variance-covariance matrix V.

$$V = \begin{pmatrix} Cov(R_1, R_1) & \cdots & Cov(R_1, R_{12}) \\ & \ddots & \\ Cov(R_{12}, R_1) & \cdots & Cov(R_{12}, R_{12}) \end{pmatrix} \tag{4}$$

When the differences among portfolios with the same expected return are minimized, the portfolio is considered an efficient frontier portfolio, that is, the solution.

$$min \frac{1}{2} w^T V w \tag{5}$$

The constraints are as follows

$$w^T e = E[r_p] \tag{6}$$

and

$$w^T 1 = 1 \tag{7}$$

Where $\mathrm{E}[\mathrm{r_p}]$ is the expected return, and 1 represents an N×1 vector of ones.

Solving the model yield

$$w_p = g + hE[r_p] \tag{8}$$

and the following expression

$$g = \frac{1}{D} \left[B(V^{-1}1) - A(V^{-1}e) \right] \tag{9}$$

$$h = \frac{1}{D} \left[C(V^{-1}e) - A(V^{-1}1) \right] \tag{10}$$

$$A = 1^T V^{-1} e \tag{11}$$

DOI: 10.54254/2753-8818/2025.DL27127

$$B = e^T V^{-1} e \tag{12}$$

$$C = 1^T V^{-1} 1 \tag{13}$$

$$D = BC - A^2 \tag{14}$$

and the expression of the efficient frontier curve can also be obtained.

$$\frac{\sigma^2(r_p)}{\frac{1}{C}} - \frac{\left(E(r_p) - \frac{A}{C}\right)^2}{\frac{D}{C^2}} = 1 \tag{15}$$

4. Results

Twelve stocks were selected from the U.S. stock market, namely ABBV, ADBE, AMD, AMT, AMZN, BA, BAC, BLK, CAT, COP, CRM, and CVX. The data were obtained from the Kaggle database. Closing prices on the first trading day of each month from January 3, 2022, to February 3, 2025, were chosen as sample data for these 12 stocks. The data were preprocessed, and, according to formulas (1–4), the monthly growth rates of each stock, the expected return vector, and the covariance matrix were calculated.

Based on these data, calculations according to formulas (9–14) yielded A=5.5931, B=0.1855, C=709.9158,D=100.3986, as well as the parameters g and h.

Table 1. Vectors g and h

g	h
0.3243	10.5729
0.0635	-8.7336
0.0028	-4.4177
0.4145	-24.1104
0.0417	18.0543
0.0674	-3.3404
0.4017	-19.7603
-0.4354	15.7191
-0.2980	23.4033
0.0130	9.0856
0.0642	0.9797
0.3403	-17.4526

According to formula (15), the expression of the efficient frontier curve was obtained, and the corresponding graph was plotted, with the horizontal and vertical axes representing $\sigma(r_p)$ and $E[r_p]$, respectively.

DOI: 10.54254/2753-8818/2025.DL27127

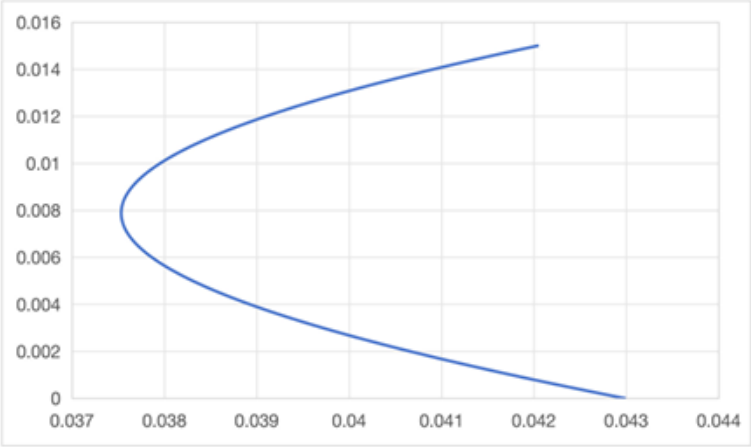

Figure 1. Efficient frontier curve

To ensure the validity of the results, $E[r_p]$ =0.01 was chosen above A/C. According to formula (8), the frontier portfolio scheme w_1 was obtained.

Table 2. Frontier portfolio scheme w_1

ABBV	43.01%
ADBE	-2.38%
AMD	-4.13%
AMT	17.34%
AMZN	22.22%
BA	3.40%
BAC	20.41%
BLK	-27.82%
CAT	-6.40%
COP	10.38%
CRM	7.40%
CVX	16.57%

In the actual stock market, short selling is not allowed. Therefore, the four stocks with negative weights were removed, and iterative calculations were performed. Recalculations based on formulas (9–14) yielded new values: A=6.6274, B=0.1449, C=581.9490, D=40.4284, A/C=0.0114, along with the updated vectors g and h.

DOI: 10.54254/2753-8818/2025.DL27127

Table 3. Iterated vectors g and h

g	h
0.0523	31.0604
0.5936	-42.8175
-0.0071	17.3968
0.0493	-5.8273
0.1421	-4.2177
0.0147	8.1404
-0.0972	7.2914
0.2524	-11.0265

At this point, A/C became 0.0114, which is greater than the previously selected $E[r_p]$. Therefore, $E[r_p]$ =0.013 was chosen as the new expected return.

The frontier portfolio scheme w_2 was then obtained.

Table 4. Frontier portfolio scheme w_2

ABBV	45.61%
AMT	3.70%
AMZN	21.90%
BA	-2.65%
BAC	8.72%
COP	12.06%
CRM	-0.24%
CVX	10.90%

After removing the two stocks with negative investment weights, iterative calculations were performed again. Based on formulas (9–14), the new values obtained were A=6.6058, B=0.1423, C=580.7681, D=38.9833, A/C=0.0114, along with the updated vectors g and h.

Table 5. Vectors g and h after the second iteration

g	h
0.0428	31.5406
0.6222	-45.8008
-0.0628	21.2009
0.1371	-4.9780
0.0175	8.5389
0.2431	-10.5016

The new frontier portfolio scheme w_3 was obtained.

DOI: 10.54254/2753-8818/2025.DL27127

Table 6. Frontier portfolio scheme

ABBV	45.29%
AMT	2.68%
AMZN	21.28%
BAC	7.24%
COP	12.85%
CVX	10.66%

All stocks now have positive investment weights, which aligns with practical market conditions. Therefore, under the original market constraints, the frontier portfolio scheme w corresponding to E=0.013 was obtained.

Table 7. Frontier portfolio scheme w

ABBV	45.29%
ADBE	0.00%
AMD	0.00%
AMT	2.68%
AMZN	21.28%
BA	0.00%
BAC	7.24%
BLK	0.00%
CAT	0.00%
COP	12.85%
CRM	0.00%
CVX	10.66%

5. Discussion

Based on the closing prices of the twelve U.S. stocks on the first trading day of each month from January 3, 2022, to February 3, 2025, this study, through model calculations, obtained the frontier portfolio scheme corresponding to E=0.013. The twelve selected stocks span multiple industries, including technology, pharmaceuticals, and industrials. Diversified investment helps to mitigate risk and effectively reduces the impact of volatility in a single industry on the overall portfolio, making it suitable for the application of this model.

The dataset covers approximately three years of monthly data, representing a medium-term time window, which helps avoid interference from short-term fluctuations. The experimental design, which iteratively removes stocks with negative weights to compress the total investment, can be generalized to other large stock markets. In the transition from theoretical research to practical application, the model employs real-number programming, whereas actual investments require integer programming. The larger the investment scale, the smaller the difference between the optimal solutions of integer and real-number programming, making the experimental results more applicable to large-scale investment scenarios.

DOI: 10.54254/2753-8818/2025.DL27127

6. Conclusion

Against the backdrop of the continuous development of the Chinese stock market, optimizing stock portfolios to balance return and risk has become an important concern for many investors. Using the mean-variance model as a framework, this study quantified return and risk through calculations of expected returns and variance, and, combined with the covariance matrix, focused on the problem of optimal portfolio selection.

Analyzing the closing prices on the first trading day of each month from January 3, 2022, to February 3, 2025, and applying the mean-variance model, the efficient frontier portfolio corresponding to an expected return of E=0.013 was obtained. In this portfolio, ABBV accounts for 45.29%, AMT 2.68%, AMZN 21.28%, BAC 7.24%, COP 12.85%, and CVX 10.66%. This portfolio avoids short selling, aligns with actual stock market conditions, and achieves a balance between expected return and risk.

The limitations of this study are mainly as follows. First, the data sample includes only 12 U.S. stocks, which limits the generalizability of the conclusions. Second, the model employs real-number programming, whereas actual investment decisions use integer programming, leading to a gap between the theoretically optimal portfolio and practical investment situations. Future research could expand the sample size and cover additional markets to increase the applicability of the study.

References

[1] Markowitz, H. M. (1952). Portfolio selection. Journal of Finance, 7(1).
[2] Wang, X. G., & Qiu, W. H. (2005). Dynamic portfolio decision model under mean-variance preference and downside risk control. Journal of Quantitative & Technical Economics, 22(12), 9.
[3] Yue, Y. J., Cai, X. Z., He, B. J., & Wang, G. Q. (2007). Primal-dual interior-point algorithm for the Markowitz mean-variance model. Proceedings of the National Conference on Decision Science / Multi-Objective Decision Making.
[4] Wang, X. G., & Wang, Y. D. (2012). Dynamic portfolio selection under mean-variance preference and expected loss risk constraints. Statistics and Management, 31(3), 9.
[5] Zhang, S. P., & Wu, C. F. (2007). Mean-variance model under novel risk perceptions: Model and empirical study. Journal of Shanghai Jiao Tong University, 41(12), 7.
[6] Xu, Y. H., & Li, Z. F. (2008). Dynamic portfolio selection based on serial correlation of returns: A dynamic mean-variance model. Systems Engineering Theory & Practice, 28(8), 9.
[7] Huo, Y. L., & Zhang, H. T. (2009). Study on mean-variance models. Science, Technology and Engineering, 4, 4.
[8] Liu, X. D. (2013). Research on insurance fund portfolios based on the mean-variance model. Journal of Chongqing Technology and Business University: Natural Science Edition.
[9] Wang, L., & Xu, M. P. (2016). Portfolio risk analysis based on multivariate t-distribution and mean-variance model. Journal of Ludong University: Natural Science Edition.
[10] Hu, S. C., Yang, Y., Liu, T. J., & School of Economics and Management, Harbin Institute of Technology, Heilongjiang, Harbin. (2016). Interactive decision-making method for an improved mean-variance portfolio model. China Modernization Management Association. China Modernization Management Association.
[11] Li, S. L. (2020). Study on Markowitz portfolio model with simultaneous mean and variance changes. Bohai Rim Economic Outlook, (2), 2.
[12] Li, X. Y., & Wang, B. H. (2020). Construction and comparative study of a robust mean-variance model. Statistics and Decision, (13), 6.