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Stability Analysis of Predator-Prey Model

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Abstract. This paper analyzed the stability of the predator-prey model by using the original Lotka-Volterra predator-prey model, which consists of two ordinary differential equations and describes the interaction between predator populations and prey populations. This study solved the equilibrium points and analyzed the stability on this basis. The specific steps were constructing a Jacobian matrix to replace nonlinear systems with linear approximations and substituting the equilibrium points into it. After obtaining the Jacobian matrix with specific values, we calculated its eigenvalue. According to the theory of linear stability, which says the sign of the real part of the eigenvalue determines the type of equilibrium points, we performed a stability analysis. The result of this study proved that the trivial equilibrium point is a saddle point and thus is unstable. In contrast, the non-trivial equilibrium point is neutrally stable, for it is a center. By using the method of numerical simulation, we verified the conclusion and visually showed the nature of the periodic oscillation of the Lotka-Volterra system.

Keywords: predator-prey, Lotka-Volterra, equilibrium points, stability analysis, Jacobian matrix

1. Introduction

The Lotka-Volterra model was independently proposed by two mathematicians, Alfred J. Lotka in 1925 and Vito Volterra in 1926, based on different inspirations [1,2]. This set of differential equations provides a simple and basic mathematical model for interaction between predators and prey. It was one of the representatives of the earliest models in the field of mathematical ecology and thus had a great influence on the latter analysis of the stability of ecosystems [3]. However, the model was based on many idealized assumptions, making it less suitable to be applied to the complex reality directly. For example, it assumed that the reproduction was continuous and immediate but ignored the time lag between predation and breeding [4].

Although most of the current studies have taken these limitations into account, conducting stability analysis on the original Lotka-Volterra model by using realistic parameters is still worth being studied.

Therefore, this paper explores the effectiveness of the original Lotka-Volterra model in practical application, that is, to find whether the model can predict the practical population dynamics or will get wrong results due to its simplified structure when real data is used to set the parameters of the model. For this purpose, this study adopts the method of linear stability analysis based on the

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Jacobian matrix. By calculating the eigenvalues of the Jacobian matrix at the equilibrium points of the system, people can classify the stability of the equilibrium points. This work demonstrates how to obtain stability conclusions from mathematical models and tests the model by using real parameters to show the predictive ability and limitations of the classic L-V framework.

2. Related works

The basic Lotka-Volterra model is the earliest model that describes the relationship between predator and prey populations, so there are also many idealized assumptions in it. These idealizations have made many studies focus on exploring how to make the model closer to reality [5,6]. In this part, the kind of research mentioned above that expands the basic model by introducing other realistic factors will be reviewed.

Most researchers have added randomness to the original model by introducing stochastic differential equations, for the extinction phenomenon that they focused on is a random event. For example, Vadillo, F., calculated the mean time for extinction of population by three different models, which randomize the simple one, solving the Backward Kolmogorov Equation [7]. The results in that paper show that the model based on demographic variability is more consistent with the biological reality, for its mean extinction time is the nearest to the results of existing research. Shaikhet L. and Korobeinikov A. are dedicated to solving the extinction problem in the Lotka-Volterra predator-prey model under random disturbances as well, but their research paradigms, methods and conclusions are different [8]. This paper explores the sufficient conditions for species extinction and the necessary conditions for survival, and employs a randomization method that adds linear white noise proportional to population size to the growth rate parameter, which is the third method in [7]. This article provides computable critical conditions for the random extinction of species in the Lotka-Volterra model disturbed by linear multiplicative noise. The randomness is discussed in the research from Bondesan et al. [9], who also introduce intraspecific competition. They started from microscopic individual interactions, formulated a Boltzmann-type kinetic model, and derived a macroscopic Fokker-Planck system. Their analysis proved that the local equilibrium of this system is a gamma distribution.

The research mentioned above enriches our understanding of the noise in its population dynamics by introducing stochastic processes. Based on it, as another key factor influencing population dynamics, the periodic changes in environmental variables have been paid large attention in recent years. Swailem and Tauber combined the random space Lotka-Volterra model with periodically varying carrying capacity to explore the impact of seasonal resource fluctuations on species survival in their research [10]. This study indicates that the periodic changes in the environment can not only effectively expand the coexistence area but also stabilize the spatial pattern by resonating with the inherent fluctuations of the system. This illustrates the significance of randomness, spatial structure and environmental drive in an ecosystem with spatio-temporal variations.

In addition, there are also studies dedicated to introducing time delay into the model to describe lag effects such as generation time. In 2018, Yuzba and Karacay developed a numerical method based on inner product operations for a class of LV models with time delays to obtain their polynomial approximate solutions [11]. Their work provides a new computing method for solving such complex time-delay models.

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3. Methodology

The original Lotka-Volterra predator-prey model is a pair of ordinary differential equations, consisting of two formulas that describe the rate of changes in the amount of populations:

$$\frac{dx}{dt} = \alpha x - \gamma xy \tag{1}$$

$$\frac{dy}{dx} = -\beta y + \delta xy \tag{2}$$

Among them, x(t) and y(t) represent the amounts of prey and predator populations at time t respectively. Four parameters α, β, γ, δ are all positive real numbers. Parameter α represents the growth rate when there are no predators, and β represents the death rate of predators. Predators make the growth rate of prey reduce, and the number reduced is directly proportional to the number of predators. The ecological meaning of this ratio is predation rate, represented by γ. Besides, prey reduce the death rate of predators, and the reduction is directly proportional to the number of prey. The meaning of this ratio is the predation conversion efficiency, denoted by δ.

4. Results

4.1. Stability analysis

To study the stability of the system, we first need to solve for the equilibrium points. The system will become stable at these points, meaning that the rate of change of the population size will be zero. Thus, we can obtain the following set of equations:

$$\frac{dx}{dt} = \alpha x - \gamma xy = (\alpha - \gamma y)x = 0 \tag{3}$$

$$\frac{dy}{dt} = -\beta y + \delta xy = (-\beta + \delta x)y = 0 \tag{4}$$

Through this pair of formula, we solve for two equilibrium points: one of them is the trivial equilibrium point (0, 0), representing that the predator and the prey populations will extinct in this situation; the other one is non-trivial equilibrium point $\left(\frac{\beta}{\delta}, \frac{\alpha}{\gamma}\right)$, showing that the two populations can become stable with both of them existing at the same time. The subsequent analysis will focus more on the non-trivial equilibrium point, for it has a greater biological significance.

Then, this study substitutes the points into the Jacobian matrix and calculate them. The Jacobian matrix is a matrix that consists of all the first-order partial derivatives of a multivariate function, and it shows the local linear behavior of the system near the equilibrium point. We let $f(x, y) = \alpha x - \gamma xy$, $g(x, y) = -\beta y + \delta xy$ and calculate the matrix:

$$J\left(x, y\right) = \begin{pmatrix} \frac{df}{dx} & \frac{df}{dy} \\ \frac{dg}{dx} & \frac{dg}{dy} \end{pmatrix} = \begin{pmatrix} \alpha - \gamma y & -\gamma x \\ \delta y & -\beta + \delta x \end{pmatrix} \tag{5}$$

Next, we need to calculate the Jacobian matrix at the equilibrium point and find the eigenvalues, for the sign of the real part of the eigenvalues is the most crucial basis for judging the stability. We

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analyze the equilibrium points (0, 0) and $\left(\frac{\beta}{\delta}, \frac{\alpha}{\gamma}\right)$ respectively.

For (0, 0):

$$J\left(0,0\right) = \begin{pmatrix} \alpha - \gamma \cdot 0 & -\gamma \cdot 0 \\ \delta \cdot 0 & -\beta + \delta \cdot 0 \end{pmatrix} = \begin{pmatrix} \alpha & 0 \\ 0 & -\beta \end{pmatrix} \tag{6}$$

The eigenvalue λ satisfies the following condition:

$$|J - \lambda \cdot I| = \begin{vmatrix} \alpha - \lambda & 0 \\ 0 & \beta + \lambda \end{vmatrix} = 0 \tag{7}$$

According to formula (7), it can be found that $\lambda_1 = \alpha$, $\lambda_2 = -\beta$. As α and β are both positive real numbers and the real parts of eigenvalues are one positive and one negative, the type of equilibrium point (0, 0) is a saddle point, indicating that it is unstable.

For $\left(\frac{\beta}{\delta}, \frac{\alpha}{\gamma}\right)$:

$$J\left(\frac{\beta}{\delta}, \frac{\alpha}{\gamma}\right) = \begin{pmatrix} \alpha - \gamma \cdot \frac{\alpha}{\gamma} & -\gamma \cdot \frac{\beta}{\delta} \\ \delta \cdot \frac{\alpha}{\gamma} & -\beta + \delta \cdot \frac{\beta}{\delta} \end{pmatrix} = \begin{pmatrix} 0 & -\frac{\beta \cdot \gamma}{\delta} \\ \frac{\alpha \cdot \delta}{\gamma} & 0 \end{pmatrix} \tag{8}$$

$$|J - \lambda \cdot I| = \begin{vmatrix} -\lambda & -\frac{\beta \cdot \gamma}{\delta} \\ \frac{\alpha \cdot \delta}{\gamma} & -\lambda \end{vmatrix} = 0 \tag{9}$$

From formula (9), it can be found that $\lambda^2 + \alpha\beta = 0$, therefore $\lambda_1 = i \cdot \sqrt{\alpha\beta}$, $\lambda_2 = -i \cdot \sqrt{\alpha\beta}$. The eigenvalues are conjugate complex roots with a real part of zero. The stability of the system it determines near the equilibrium point is neutrally stable.

4.2. Numerical simulation

In order to prove the results of theoretical analysis above, this part conducts a numerical simulation to the Lotka-Volterra model. Through different initial conditions, the images show directly how population change over time and how the initial condition makes an impact.

For a set of value: $\alpha = 1.0$, $\beta = 1.5$, $\gamma = 0.1$, $\delta = 0.075$, following equations can be obtained:

$$\frac{dx}{dt} = x - 0.1xy \tag{10}$$

$$\frac{dy}{dt} = -1.5y + 0.075xy \tag{11}$$

We conducted two simulations for observation: In the first simulation, the initial number of predators was set at 5 and the prey was 20; The second time was the opposite, with 20 predators and 5 preys. Figures 1(a) and (b) respectively illustrate changes in population size in these two simulations calculated based on the model.

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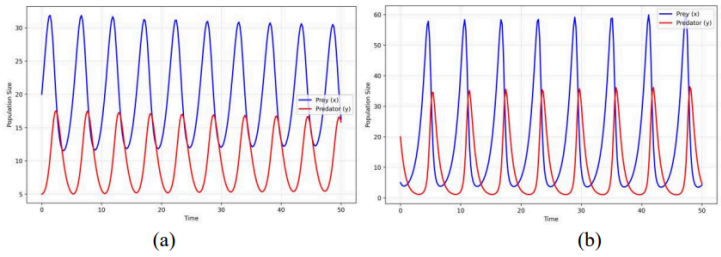


Figure 1. Dynamics of Lotka-Volterra model: (a) x=20; y=5; (b) x=5; y=20

It can be observed that under the two initial conditions, the populations of both the prey and the predator show continuous and regular periodic oscillations and the period is almost the same, but the amplitude of the oscillation strongly depends on the initial conditions.

5. Conclusion

This study discusses the stability of the classical Lotka-Volterra predator-prey model by methods including theoretical analysis and numerical simulation. Firstly, by calculating the Jacobian matrix and finding its eigenvalues, we found and analyzed the equilibrium point and stability of the system. The results show that the origin equilibrium point (0, 0) is a saddle point and is unstable. The non-trivial equilibrium point $\left(\frac{\beta}{\delta}, \frac{\alpha}{\gamma}\right)$ is a central point, with the system presenting neutral stability. Subsequently, we visually verified the conclusion through numerical simulation. Under a set of parameters, simulations were conducted on two different initial conditions, and it was concluded that the population sizes of both predators and preys showed continuous and regular periodic oscillations in both cases. The simulation results confirm the conclusion of the theoretical analysis: the oscillation period of the system is the same, but its amplitude depends on the initial conditions. However, this article has some deficiencies in its analysis, such as the stability analysis not being deep enough and lacking a strict proof of the stability of the center of nonlinear systems. Besides, the parameters selected for numerical simulation are too simple, resulting in a lack of systematicness in the simulation. Based on the above deficiencies, the author's future research work will mainly focus on expanding the model to make it closer to reality, like increasing the amount of prey or introducing the Holling Type II functional response. As expected, the model can ultimately be used on realistic data, the author also can use real data to calibrate and verify the improved model.

To summarize, this study demonstrates that the Lotka-Volterra model has neutral stability at non-zero equilibrium points, revealing that the model actually can show the periodic oscillation mechanism in interactions between predators and prey. However, the final state of the system is determined by the initial conditions, which also provides a mathematical explanation for its limitations in practical applications.

References

[1] Lotka, A. J. (1925). Elements of physical biology. Williams & Wilkin, pp.341–343.
[2] Volterra, V. (1926). Variazioni e fluttuazioni del numero d'individui in specie animali conviventi. Memorie della R. Accademia Nazionale dei Lincei, Classe di Scienze Fisiche, Matematiche e Naturali, 6(2), 31–113.
[3] Hoppensteadt, F. (2006). Predator-prey model. Scholarpedia, 1(10), 1563.
[4] Smith, J. M., & Slatkin, M. (1973). The stability of predator-prey systems. Ecology, 54(2), 384-391.
[5] Diz-Pita, É., & Otero-Espinar, M. V. (2021). Predator–prey models: A review of some recent advances. Mathematics, 9(15), 1783.

DOI: 10.54254/2753-8818/2025.DL27923

[6] Berryman, A.A. (1992). The orgins and evolution of predator-prey theory. Ecology, 73(5), 1530-1535.
[7] Vadillo, F. (2019). Comparing stochastic Lotka–Volterra predator-prey models. Applied Mathematics and Computation, 360, 181-189.
[8] Shaikhet, L., & Korobeinikov, A. (2024). Persistence and stochastic extinction in a Lotka–Volterra predator–prey stochastically perturbed model. Mathematics, 12(10), 1588.
[9] Bondesan, A., Menale, M., Toscani, G., & Zanella, M. (2025). Lotka–Volterra-type kinetic equations for interacting species. Nonlinearity, 38(7), 075026.
[10] Swailem, M., & Täuber, U. C. (2023). Lotka-Volterra predator-prey model with periodically varying carrying capacity. Physical Review E, 107(6), 064144.
[11] Yüzbaşı, Ş., & Karaçayır, M. (2018). A numerical method for solutions of Lotka–Volterra predator–prey model with time-delay. International Journal of Biomathematics, 11(2), 1850028.