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Dynamic Modeling of Rolling Bearings under Speed Fluctuation Conditions

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Abstract. This article proposes a nonlinear dynamic model to study the effect of spindle speed fluctuations on the dynamic vibration behavior of the bearing. Establishing a two freedom degree dynamic model of the bearing, a speed fluctuation function and a local defect additional displacement function were established. The Runge Kutta method was used to solve motion differential equations. Simulating and analyzing the effect of speed fluctuations and amplitude frequency changes on vibration acceleration was also done.The results indicate that fluctuations in spindle speed can affect the amplitude changes of vibration acceleration. An increase in the amplitude of speed fluctuations will synchronously increase the amplitude fluctuation range of vibration acceleration. An increase in the frequency of speed fluctuations will not only affect the amplitude fluctuation range of acceleration, but also increase the frequency of acceleration amplitude fluctuations.

Keywords: Rolling bearing, Dynamics, Speed fluctuation, Time-varying motivation

1. Introduction

As a critical fundamental component, rolling bearings are widely employed in essential machinery across sectors such as energy, defense, industry, and transportation, owing to their low frictional resistance, responsive startup, high efficiency, and ease of installation and maintenance. Speed fluctuations, which often occur during the operation of equipment like wind turbines, can modify the contact force between rolling elements and raceways. This may result in abnormal vibration and fatigue damage to both the bearings and the rotor system, and could potentially lead to major safety incidents. Hence, it is essential to perform timely and accurate quantitative diagnosis and identification of bearings under fluctuating speed conditions. A thorough understanding of the dynamic behavior of bearings during speed variations can provide valuable references for condition monitoring and quantitative fault diagnosis, while also helping improve their accuracy. However, due to influencing factors such as variable loads, nonlinear contact characteristics, and interactions with lubricating oil films, the excitation mechanisms and vibration features of defective rolling bearings under speed fluctuation conditions are not yet fully understood. Therefore, investigating the dynamic modeling and vibration characteristics of rolling bearings under variable speeds carries both theoretical significance and practical engineering value.

Extensive research on the dynamic modeling and vibration properties of local faults in ball bearings has been performed by numerous scholars [1]. Sassi et al. [2] proposed a rotor bearing

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coupled three degree of freedom system model and compiled it into the MATLAB toolbox BEAT. Patil et al. [3] considered the size of various defects and proposed a 2-degree-of-freedom lumped parameter model to study the influence of defect size on the vibration response of deep groove ball bearings. Nakhaeinejad et al. [4] used bond graphs and complex dynamic models to study the effect of defects on bearing vibration. Reference [5-8] conducted dynamic simulation analysis on defective bearings using the Runge-Kutta numerical integration method. A dynamic model of rolling ball bearings with local defects, based on the GUPTA 6- freedom -degree bearing model, was established in Niu et al. [9]. Based on the existing dynamic model and Hertz contact theory, liu et al. [10] considers the curved fault edge after elastic-plastic deformation and further peeling as a cylindrical edge, and studies the vibration response characteristics of ball bearings under local fault excitation of the outer ring of the cylindrical edge type. In practical applications, the rotational speed of bearings is not fixed and often fluctuates. However, the above research is mostly focused on dynamic modeling under stable rotational speed. In response to this problem, tu et al. [11] have studied the dynamic interaction between rolling elements and cages of healthy bearings, and proposed a nonlinear dynamic model of rolling bearings for contact force and clearance, but did not consider the situation of defective bearings, as well as the influence of rotational speed fluctuation amplitude and frequency on bearings. Wang et al. [12] considers the rough surface caused by bearing extension, establishing a mathematical model of extension defects that is used to assess the bearing raceway in rotating machinery. Wang et al. [13] proposed a function for analyzing the bias of ball bearings and obtained the vibration response by solving a 5-degree-of-freedom dynamic model. Mufazzal et al. [14] proposed a bearing model with outer ring expansion defects, introducing stiffness and impact force changes as excitation models. Atef er al. [15] established a 6-degree-of-freedom dynamic model, which is used to accurately study the vibration behaviors of bearings that have multi-point defects. Simulation outcomes demonstrated that the vibration response of bearings is influenced by two factors: the number of defects and the angle between individual defects. Wu et al. [16] proposed a composite displacement excitation model that considers the influence of roughness and analyzes the high-speed effect and transmission path.

By analyzing the time-varying contact relationship between the ball and the raceway edge under speed fluctuation conditions, this article establishes a nonlinear contact stiffness model derived from Hertz contact theory, which accurately reflects the elastic deformation and stress distribution at the contact interface. Concurrently, a time-varying displacement excitation model is developed to characterize the dynamic behavior of the ball under fluctuating rotational speeds, accounting for periodic and non-periodic variations in kinematic conditions. Furthermore, by incorporating the influence of the elastohydrodynamic lubricating (EHD) oil film—considering its thickness, pressure distribution, and damping effect—a comprehensive dynamic model of rolling bearings coupled with time-varying impact excitation is proposed. This integrated model effectively captures the interactions between mechanical impacts, structural dynamics, and lubricating conditions, thereby providing a more accurate and realistic simulation of real-world bearing operation. The vibration response characteristics of both healthy and defective bearings were thoroughly investigated using this proposed model through numerical simulations and time-frequency analysis. The results enable a detailed comparison and analysis of the vibrational features, such as amplitude modulation, spectral structure, and transient impulses, between the two types of bearings.Offering a theoretical basis for fault diagnosis and life prediction in practical applications, these discoveries provide a more profound understanding of the consequences of localized flaws on bearing performance in varying operational circumstances.

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2. Dynamic modeling method for bearings under speed fluctuation conditions

2.1. Differential equation of motion for defective rolling bearings

Based on a lumped-parameter spring-mass approach and Newton's second law, a two-degree-of-freedom dynamic model for a ball bearing system is rigorously developed. In this model, each rolling element is idealized as a discrete mass, and the interactions between rolling elements and raceways are modeled using nonlinear springs based on Hertzian contact theory, incorporating elastic deformations at the contact interfaces. The resulting system of governing differential equations accounts for phenomena such as varying compliance vibration, internal clearance, and applied external loads. The general form of the nonlinear equations of motion for the rolling bearing system is expressed as follows:

$$\begin{cases} m\ddot{x} + c\dot{x} + K_e \sum_{j=1}^{Z} \lambda_j \delta_j^n cos\theta_j = \omega_x \\ m\ddot{y} + c\dot{y} + K_e \sum_{j=1}^{Z} \lambda_j \delta_j^n sin\theta_j = \omega_y \end{cases} \tag{1}$$

In the formula: m is the total mass of the bearing inner ring; C is the damping coefficient between the bearing inner ring and the surrounding environment during movement; x and y represent the vibration displacement of the bearing inner ring in the x and y directions, respectively; $\dot{x}$ and $\dot{y}$ are the vibration velocities of the bearing inner ring in the x and y directions, respectively; $\ddot{x}$ and $\ddot{y}$ represent the vibration acceleration of the inner ring of the bearing in the x and y directions, respectively; ω_x and ω_y are the radial forces borne by the inner ring of the bearing in the x and y directions, respectively; For the total contact stiffness of the rolling element and the inner/outer raceway surfaces, λ is contact state coefficient (1 represents contact, 0 represents separation).

c is the damping contact coefficient inside the bearing, which can be expressed:

$$c = \frac{\eta_b K_b}{\omega_{ext}} \tag{2}$$

Among them: η_b is the loss coefficient; K_b is the bearing contact stiffness; ω_{ext} is incentivize frequency.

Speed fluctuation generally refers to the non-uniform variation of the rotational speed of the inner or outer rings of the bearing over time. After setting the ideal speed, additional speed fluctuations are usually generated on the basis of the ideal speed due to unstable power sources, transmission system errors, and sudden changes in external loads. The mathematical description of spindle speed fluctuations is usually:

$$n_s(t) = n_0 + Asin(2\pi ft) \tag{3}$$

Among them: n_0 is the ideal speed; A is the amplitude of the speed fluctuation, which represents the maximum amount of fluctuation generated by the speed fluctuation; f is the frequency of the speed fluctuation, which represents the number of times the speed generates a sinusoidal signal fluctuation within 1 second.

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2.2. Additional displacement excitation model for rolling bearings

The contact stress at the interface causes contact deformation when the rolling element makes contact with the raceway. This localized deformation is a critical factor influencing the dynamic behavior and load distribution within the bearing assembly. According to Hertz contact theory, which provides an analytical framework for evaluating the elastic deformation between two curved surfaces in contact, the formula for calculating the resulting contact deformation can be expressed:

$$\delta_j = xcos\phi + ysin\phi - C_r - \beta H \tag{4}$$

Among them, x is the horizontal vibration displacement of the bearing inner ring; y represents the vertical vibration displacement of the inner ring of the bearing; C_r is the initial radial clearance between the rolling element and the inner ring; β is the defect contact marker (1 indicates contact with a local defect, 0 indicates contact with a healthy raceway).

φ is position angle of the rolling element, expressed as follows:

$$\phi = \omega_c t + 2\pi\frac{(i-1)}{N_b} \tag{5}$$

Among them, i is the ith number of rolling element, and N_b is the total number of rolling elements. The expression for calculating the angular velocity of the cage is:

$$\begin{cases} \omega_c = 0.5\omega_s\left(1 - \dfrac{d}{d_m}\right) \\ \omega_s = \dfrac{2\pi n_s}{60} \end{cases} \tag{6}$$

Among them, d is the diameter of the rolling element, dm is the diameter of the pitch circle, n_s is the spindle speed (r/min), and ω_s is the spindle angular velocity (rad/s). H_{dt} is the dynamic depth at the local defect, expressed as:

$$H_{dt} = H_{max}\sin\left(\frac{\pi}{\phi_d}(\phi_t - \phi_0)\right) \tag{7}$$

Among them, φ_0 represents the angular position of the local defect center relative to the center of the rolling bearing, which is generally located below the vertical direction of the bearing; The central angle corresponding to the defect with diameter d can be written as:

$$\phi_d = 2\arcsin\left(\frac{L_d}{d_o}\right) \tag{8}$$

In the formula, L_d is the defect length and d_0 is the outer raceway diameter. H_{max} is the maximum depth of the defect, which can be expressed:

$$H_{max} = \frac{d}{2} - \sqrt{\left(\frac{d}{2}\right)^2 - \left(\frac{L_d}{2}\right)^2} - r_o + \sqrt{r_o^2 - \left(\frac{L_d}{2}\right)^2} \tag{9}$$

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3. Result analysis

The Runge-Kutta technique is commonly employed to address the displacement, velocity, and acceleration of bearing motion by resolving the differential equations. Set the simulation time step to 1×10^{-6}s and the simulation time to 0.5s. At the outset, displacement and velocity in both the horizontal and vertical directions were 0m and 0m/s respectively. The local defect was 0.5mm in length and 0.2mm in depth. The greatest contact force was found directly beneath the bearing; due to this, the defect was located precisely vertically below it. The ideal spindle speed is 2000rpm, with a horizontal external load of 0N and a vertical external load of 100N. The raceway's contact stiffness with the rolling elements is 1.39×1010N/m, while the bearing inner ring's contact damping is 200N·s/m. Table 1 presents the geometric parameters of the rolling bearings necessary for constructing the dynamic model.

Table 1. Geometric parameters of rolling bearings

Parameter	Value	Parameter	Value
Diameter of inner raceway(mm)	49.912	Diameter of outer raceway(mm)	80.088
Diameter of pitch circle(mm)	65	Diameter of rolling element(mm)	15.081
Number of scroll elements	8	Radial clearance of the inner raceway(μm)	1
Quality of the inner ring and the rotating shaft(Kg)	0.6	Contact Angle	0 °

3.1. Comparison of vibration characteristics of healthy bearings

To investigate the influence of spindle speed fluctuations on bearing vibration, this study begins by analyzing the vibrational characteristics of healthy bearings without defects. A dynamic model that excludes defects is developed, and the system's equations of motion are numerically solved using the Runge–Kutta method. By comparing vibrational responses under constant and fluctuating speed conditions, vibration acceleration time-domain waveforms are derived, as shown in Figure 1.

The acceleration time-domain profile exhibits an initial transient phase followed by stabilization. In the steady phase, the impact of speed fluctuations remains discernible yet relatively moderate. A magnified view of the signal indicates that under constant speed, the vibration displays periodic stability, whereas under fluctuating speed, non-periodic oscillations and greater variability emerge. These non-stationary features align closely with real-world healthy bearing signals, which often display comparable irregular fluctuations under varying operational conditions.Thus, integrating speed fluctuations into the bearing dynamic model improves its fidelity to real operating environments and offers more reliable data for condition monitoring and predictive maintenance.

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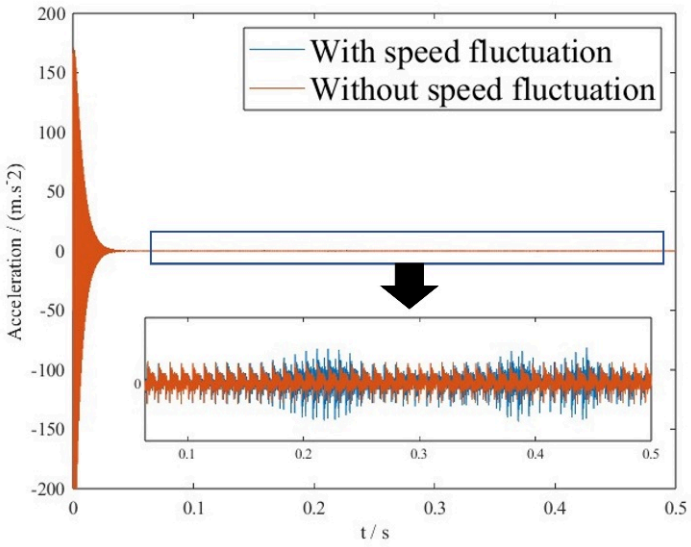


Figure 1. Effect of speed fluctuation on vibration acceleration of healthy bearings

3.2. Comparison of vibration characteristics of defective bearings

In the dynamic model for calculating contact deformation, the effect of additional displacement caused by local defects is incorporated. This allows for the derivation of the acceleration time-domain response of the bearing under defect influence, as depicted in Figure 2.

A comparison with vibration signals from healthy bearings reveals that defective bearings significantly alter signal amplitude, demonstrating that acceleration signals can serve as a strong indicator of rolling bearing health. Moreover, under localized defects, the vibration acceleration amplitude at ideal speed varies periodically within a certain interval, remaining stable between –100 m/s² and 100 m/s². However, when a speed fluctuation amplitude of 10 rpm and fluctuation frequency of 5 Hz are introduced, the acceleration signal continues to exhibit periodic patterns, yet its amplitude becomes unstable. This behavior aligns with the irregular signal responses observed in actual tests under speed variation conditions, confirming that speed fluctuations contribute to more realistic simulation of bearing vibration signals.

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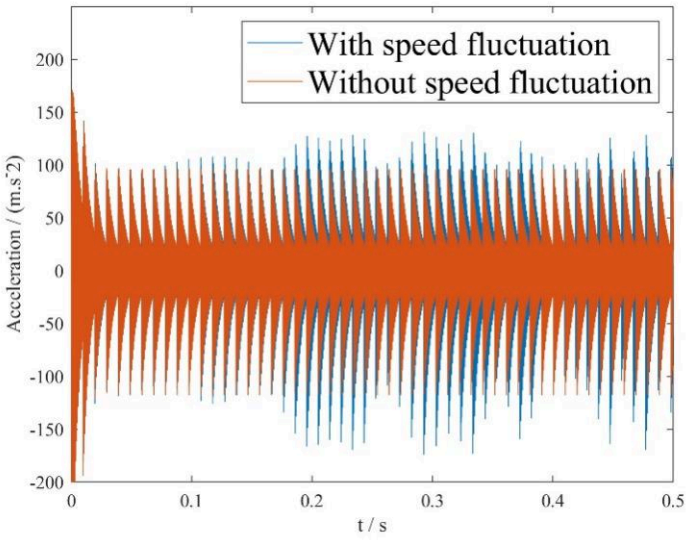


Figure 2. Effect of speed fluctuation on vibration acceleration of defective bearings

3.3. Analysis of the influence of speed fluctuation on bearing vibration

Systematically investigating and discussing the effects of varying speed fluctuation amplitudes and frequencies separately is necessary to gain a more thorough understanding of the effect of spindle speed fluctuations on the vibration acceleration of rolling bearings. Figure 3 depicts how the amplitude of rotational speed fluctuation affects the vibration acceleration response of the bearing. Specifically, Figure 3(a)-(e) illustrate the trend in vibration acceleration under conditions where the speed fluctuation amplitude is progressively increased. These subfigures clearly show the time-domain responses under each amplitude setting, highlighting the dynamic behavior changes induced by increasing fluctuation intensity. Figure 3(f) provides a summarized analysis of the overall influence of speed fluctuation amplitude on vibration performance. The figure reveals that, despite the acceleration fluctuation's uniformity and lack of variation with increasing amplitude, the vibration acceleration in the faulty bearing experiences much more extensive fluctuations. This suggests that larger speed fluctuations exacerbate the vibration intensity in defective bearings without altering the fundamental periodicity of the excitation, underscoring the critical role of controlling rotational stability in practical applications to mitigate vibration-related failures.

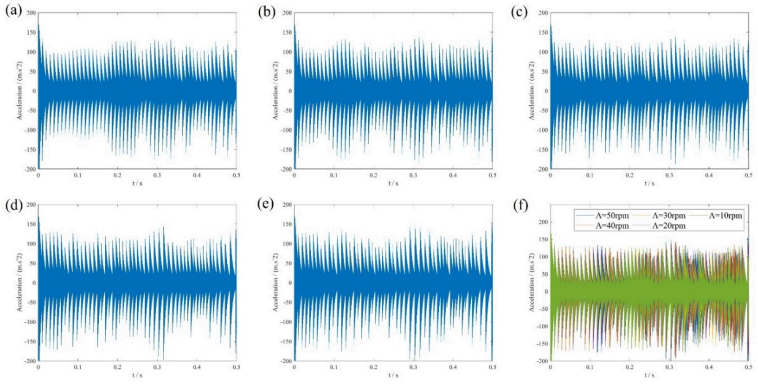


Figure 3. Effect of speed fluctuation amplitude on bearing vibration acceleration, (a) A=10rpm, (b) A=20rpm, (c) A=30rpm, (d) A=40rpm, (e) A=50rpm, (f) Summary chart

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Figure 4 illustrates the effect of rotational speed fluctuation frequency on the vibration acceleration response of the bearing system. Subfigures 4(a) to 4(e) show the variation in vibration acceleration as the fluctuation frequency is gradually increased. These plots present the corresponding time-domain waveforms and demonstrate how elevated fluctuation frequencies modify the system's dynamic behavior. Figure 4(f) offers a comprehensive summary of the overall impact of fluctuation frequency on vibration characteristics. As evident from the results, higher fluctuation frequencies not only raise the oscillation rate of the vibration acceleration, but also considerably widen the amplitude range of the vibration response in defective bearings. This suggests that increased fluctuation frequencies amplify both frequency and amplitude modulation within the vibration signal, especially in the presence of defects. These effects can be explained by more frequent and severe interactions between rolling elements and the defective area, leading to stronger transient vibrations and higher energy responses. Hence, managing the frequency of speed fluctuations is crucial for reducing vibration-induced damage in bearing systems.

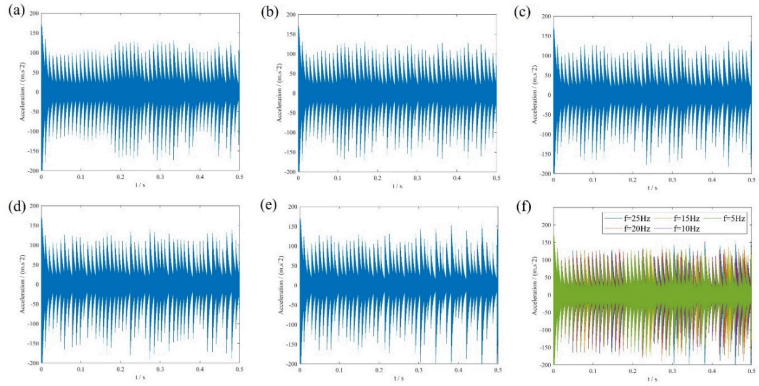


Figure 4. Effect of speed fluctuation frequency on bearing vibration acceleration, (a) f=5Hz, (b) f=10Hz, (c) f=15Hz, (d) f=20Hz, (e) f=25Hz, (f) Summary chart

4. Conclusion

This study investigates the influence of spindle speed fluctuations on the vibration acceleration of rolling bearings. A speed fluctuation function and an additional displacement function representing local defects were developed and incorporated into the dynamic motion equations of the bearing to simulate vibration signals under various operating conditions. Simulation results reveal that vibration acceleration signals under speed fluctuation conditions exhibit irregular variations in amplitude a behavior that corresponds more closely to real-world observations. The effects of both the amplitude and frequency of speed fluctuations were further analyzed: larger fluctuation amplitudes lead to broader variations in vibration acceleration amplitude, while higher fluctuation frequencies not only widen the amplitude range but also increase the rate of acceleration oscillations. Future research should focus on monitoring actual spindle speed variations and vibration acceleration signals, which could then be compared with dynamic model simulations to further elucidate the impact of speed fluctuations.

References

[1] Zhou Zewen, Hu Tianqing, Li Zhinong, etc. Summary of local defect modeling of Rolling bearings [C]// Chinese Society of Vibration Engineering. Proceedings of the 13th National Conference on Vibration Theory and Application. Key Laboratory of Non-destructive Testing Technology, Ministry of Education, Nanchang University

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of Aeronautics and Astronautics; Key Laboratory of Comprehensive Equipment Assurance Technology, National Defense University of Science and Technology; , 2019: 209-216

[2] Sassi S, Badri B, Thomas M. A numerical model to predict damaged bearing vibrations [J]. Journal of Vibration and Control - J VIB CONTROL, 2007, 13: 1603-1628.

[3] Patil M, Mathew J, Rajendrakumar P, et al. Atheoretical model to predict the effect oflocalized defect on vibrations associated withball bearing [J]. International Journal ofMechanical Sciences, 2010, 52: 1193-1201.

[4] Nakhaeinejad M, Bryant M D. Dynamic modeling of rolling element bearings with surface contact defects using bond graphs [J]. Journal of Tribology-Transactions of the Asme, 2011, 133(1).

[5] Chen Guo. Dynamic analysis of rolling bearing failure in rotary-rolling-chassis coupling system [J]. Journal of Vibration Engineering, 2008, 21(06): 577-587.Chen Guo.Dynamic analysis of ball bearing failure in rotor-ball bearing-stator coupling system [J]. Journal of Vibration Engineering, 2008, 21(06): 577-587.

[6] Guan Zhenzhen, Zheng Haiqi, Wang Yangang. There are all of them. Vibration testing and diagnosis, 2012, 32(06): 950-955.Guan Z, Zheng H, Wang Y.Dynamic fault modeling and simulation of locally defective rolling bearings [J]. Journal of Vibration, Measurement and Diagnosis, 2012, 32(06): 950-955.

[7] Zhang Yaoqiang, Chen Jianjun, Tang Liuding Nonlinear dynamic characteristics of rolling bearings considering local defects in the outer ring [J]. Acta Aeronautica Sinica, Zhang Y, Chen J, Tang L. Nonlinear dynamic characteristics of rolling element bearing with localized defect on outer ring [J]. Acta Aeronautica et Astronautica Sinica, 2009, 30(4): 751-756.

[8] Zhang Jianjun, Wang Zhongsheng, Lu Yuhua Nonlinear dynamic modeling for localized defects in a rolling element bearing based on nonlinear dynamics [J]. Vibration and Shock, 2010, 29 (11): 30-34. Zhang J, Wang Z, Lu Y. Nonlinear dynamic modeling for localized defects in a rolling element bearing [J]. Journal of Vibration and Shock, 2010, 29 (11): 30-34

[9] Niu Linkai, Cao Hongrui, He Zhengjia Study on the dynamic modeling of local surface damage of rolling ball bearings considering three-dimensional motion and relative sliding [J]. Journal of Mechanical Engineering, 2015, 51 (19): 53-59

[10] Liu Jing, Shi Zhifeng, Shao Yimin Vibration characteristics of ball bearings considering local fault edge morphology [J]. Vibration Testing and Diagnosis, 2017, 37 (04): 807-813+848. DOI: 10.16450/j.cnki. issn. 1004-6801.2017.04.027

[11] Tu, W., Luo, Y., and Yu, W. (July 15, 2019). "Dynamic Interactions Between the Rolling Element and the Cage in Rolling Bearing Under Rotational Speed Fluctuation Conditions." ASME. J. Tribol. September 2019; 141(9): 091101. https: //doi.org/10.1115/1.4044082

[12] Wang P, Zhao X, Yang Y, et al. Dynamic modeling and analysis of two-span rotor-pedestal system with bearing tilt and extended defect: Simulation and experiment [J]. Applied Mathematical Modelling, 2024, 125(1-28)

[13] Wang Z, Li G, Zhou X, et al. Dynamic analysis of deep groove ball bearing with localized defects and misalignment [J]. Journal of Sound and Vibration, 2024, 568

[14] Mufazzal S, Muzakkir S M, Khanam S. A dynamic model for simulating vibration response of ball bearings with extended outer race defects for precise spall size estimation [J]. Journal of Sound and Vibration, 2023, 552

[15] Atef M M, Khair-Eldeen W, Yan J, et al. Investigating the Combined Effect of Multiple Dent and Bump Faults on the Vibrational Behavior of Ball Bearings [J]. Machines, 2022, 10(11):

[16] Wu X, Qin Y, Luo J, et al. Fault dynamic model of high-speed rolling bearing by a compound displacement excitation function considering the effect of defect roughness [J]. Mech Mach Theory, 2022, 177