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Analysis of Galactic Rotation Curves: Newtonian Predictions vs. Observations and the Role of Dark Matter

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Abstract. This study examines the rotational path of spiral galaxies to test Newtonian predictions against observations, and assesses the presence of dark matter. It uses the equation of expected orbital velocity and compares to observed rotation curves for the Milky Way, Andromeda (M31), and NGC 3198, followed by an application of Virial Theorem to estimate total galactic mass from stellar velocity dispersions. The observations demonstrate that the rotation path for spiral galaxies is flat at large radii, in sharp contrast to the declining profiles predicted by Newtonian gravity. This conflict demonstrates that a dark matter halo (e.g., Navarro–Frenk–White profile) naturally produces the additional gravitational pull, resulting in flat rotation curves as observed. Lastly application of Gravitational lensing observations and Modified Newtonian Dynamics (MOND) to simulate flat rotation curves without dark matter is discussed.

Keywords: spiral galaxies, Newtonian predictions, dark matter, galactic mass, dark matter halo

1. Introduction

Observations of spiral galaxies have revealed a major discrepancy with simple Newtonian dynamics [1,2]. Vera Rubin and collaborators measured Doppler shifts across galaxy disks and observed that outer stars orbit much faster than predicted by the gravitational pull of the luminous mass. The authors suggested that a tremendous amount of unseen matter in the galactic halos is one of the reasons for this inconsistency [3]. Quantitatively, the authors observed that typical spiral galaxies contain about ten times as much dark mass as visible matter – at least 90% of the total mass is nonluminous. In modern terms, this unseen component is called dark matter.

The goal of this paper is to analyse galactic rotation curves and demonstrate the need for dark matter (or modified gravity). Newtonian gravity concepts are applied to compute the expected velocity as a function of radius for several galaxies and are thereafter compared to measured rotation curves. Thereafter, an approximate virial-mass argument is used to estimate total mass from stellar motions. By contrasting predictions based on visible (baryonic) mass with observed dynamics, the findings indicate the need to consider an additional mass component. Lastly, the Navarro–Frenk–White (NFW) halos as a common dark-matter model are discussed with an emphasis on the presence of massive, extended dark-matter halos in galactic mass.

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2. Galactic rotation curves and Newtonian predictions

2.1. Newtonian mechanics and keplerian motion

In Newtonian gravity, a star orbiting at a distance r from a galaxy's centre has an orbital speed determined by the enclosed mass $M(r)$. Setting the centripetal force equal to gravity gives

$$v(r) = \sqrt{\frac{G\,M(r)}{r}} \tag{1}$$

where G is Newton's gravitational constant. If beyond some radius r the enclosed mass $M(r)$ is approximately constant (i.e.\ no new mass is added), then $v(r) \propto r^{-1/2}$. This Keplerian fall-off means that at large radii, v is expected to decrease as $r^{-0.5}$. In practice, one states that without additional mass, the rotation curve should decline (much like outer planets slow down).

However, observations contradict this expectation. For typical spiral galaxies, the rotation speed does not decline at large r, it remains roughly constant. For example, if only the visible stars and gas contributed, the rotational speed of the Milky Way would drop beyond the Sun's orbit. Instead, the Milky Way's rotation curve is flat at about 220 km/s far beyond the Solar radius [4,5]. This discrepancy – flat (Vr) instead of Keplerian decay is the classic dark matter problem.

2.2. Observed rotation curves

Empirically, the rotation curves of well-studied spirals are characteristically flat or slowly declining in their outer regions, contrary to the $r^{-1/2}$ decay. This inconsistency with Newtonian predictions from luminous mass alone has been documented in many galaxies [5,3]. For instance:

• Milky Way: Beyond the Solar orbit (~8 kpc), the rotation speed stays near ~220 km/s, with little Keplerian decline [4,5].

• Andromeda (M31): Recent data show M31's rotation curve is roughly constant at ≈220 km/s out to ~25 kpc, then gently declines to ~170 km/s further out [1].

• NGC 3198: This classic case has a rotation speed of ≈150 km/s that remains flat beyond ~20 kpc [6] (over ~10 optical scale lengths).

In each case, the observed speeds far exceed the decrease predicted from the visible mass distribution, clearly indicating additional (nonluminous) mass. This can be depicted using the galaxy rotation curve shown in Figure 1. The curve indicates that star velocities (yellow) stay flat at large radii in contrast to the decline expected from visible matter (green dashed).

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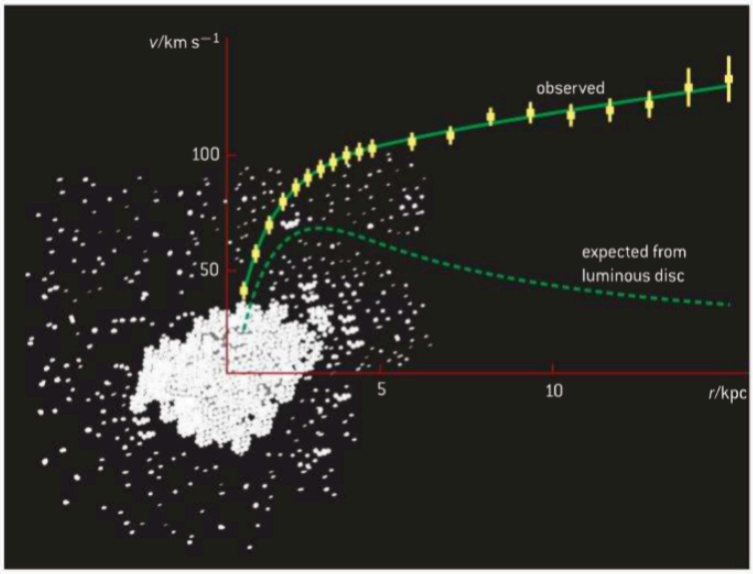


Figure 1. Comparison of Newtonian predictions (dashed line) and observed rotation curves (solid line) for a typical spiral galaxy [8]

3. Virial theorem

3.1. Basic form of the virial theorem

For a self-gravitating system in steady state (equilibrium), the Virial Theorem relates kinetic and potential energy. In particular, one obtains

$$2\langle T\rangle + \langle U\rangle = 0 \tag{2}$$

where $\langle T\rangle$ is the time-averaged total kinetic energy and $\langle U\rangle$ the time-averaged gravitational potential energy [7]. Equivalently, this implies $2T + U = 0$ for the system as a whole. In words, a bound gravitational system's total kinetic energy is (on average) $-1/2$ times its potential energy.

3.2. Estimating dark matter mass

The virial theorem is used to estimate the total mass M_{tot} of a galaxy from stellar motions, then infer the dark mass M_{DM}. Consider a galaxy of radius R containing visible matter (mass M_{vis}) and dark matter (M_{DM}). If the visible stars have an isotropic velocity dispersion σ, then their kinetic energy is roughly

$$T \approx \frac{3}{2}\, M_{vis}\ \ \sigma^2 \tag{3}$$

The total gravitational potential energy is approximately

$$U \approx \frac{G\, M_{vis}\, M_{tot}}{\mathrm{R}} \tag{4}$$

where $M_{tot} = M_{vis} + M_{DM}$ is the total mass. Applying $2T + U = 0$ gives

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$$3M_{vis} \quad \sigma^2 \approx \frac{G\ M_{vis}\ M_{tot}}{\mathrm{R}}$$

The factor M_{vis} cancels, yielding

$$M_{tot} \ = \frac{3\sigma^2\ R}{\mathrm{G}} \tag{5}$$

Thus, the total mass enclosed within the radius R can be estimated directly from $\sigma^2 R/G$. The dark matter mass is then $M_{DM} = M_{tot} - M_{vis}$. In practice, one finds M_{tot} is much larger than $M_{vim,}$ implying a large dark component. For brevity, detailed steps have been removed; this simple application of the virial theorem is standard in galactic dynamics [7].

4. Dark matter hypothesis

4.1. Evidence for dark matter

The failure of Newtonian gravity with visible mass to explain flat rotation curves can be resolved by positing a dark-matter halo enveloping each galaxy [5]. In this view, the total mass profile is

$$Mtot(r) = M_{vis}(r) + M_{DM}(r) \tag{6}$$

where $M_{\mathrm{DM}}(r)$ represents the cumulative dark matter. A commonly used model for dark halos is the Navarro–Frenk–White (NFW) profile, with density

$$\rho(r) = \frac{\rho_0}{(r/r_s)(1+r/r_s)^2} \tag{7}$$

where ρ_0 and r_s are scale density and radius [2]. This profile yields extra mass at large radii.

The extended halo of dark matter provides the additional gravitational pull needed to sustain high orbital speeds. As Teodoro et al. (2021) observe, spiral galaxy observations demonstrate that rotation curves flatten out at large radii, with rotational velocities remaining constant rather than decreasing with distance. This phenomenon requires the presence of unseen mass, since without additional gravitational sources, the rotational velocity of the galaxy would decline significantly at large distances from the galactic centre [5]. In short, galaxies appear to be embedded in large, roughly spherical halos of dark matter (not directly visible), which dominate the mass.

Quantitatively, dark matter often outweighs baryonic matter by an order of magnitude in galaxies. Vera Rubin estimated that galaxies contain about ten times more dark mass than light-emitting mass, so that at least 90% of a galaxy's mass is invisible [3]. The author emphasised that the stellar disk is only the "inner tracer" of a much larger dark matter distribution: the luminous stars occupy the inner regions, while an enormous spherical 'halo' of unseen dark matter comprises the bulk of the mass [3]. In practical terms, including such a halo in the mass model allows the theoretical rotation curve to match the observed flat profile.

Thus, all evidence from rotation curves and related dynamics strongly supports a dominant dark matter component. Whether one models it as an NFW halo or some other distribution, the key result is that the gravitational influence of dark matter explains the observed orbital speeds. Alternative

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explanations (e.g. modified gravity) exist, but the dark matter hypothesis has been the simplest and most widely accepted interpretation of the data [5,3].

4.2. Successes of the dark matter model

The dark matter hypothesis successfully explains several observations that are difficult to reconcile with visible matter alone. In particular, it accounts for (1) flat rotation curves of spiral galaxies, (2) gravitational lensing signals that imply excess mass, and (3) the dynamics of galaxy clusters (e.g., the Bullet Cluster). These successes are described in the following subsections.

4.2.1. Spiral galaxy rotation curves

In a galaxy with only visible matter, Newtonian dynamics gives $v(r) = \sqrt{GM_{\text{vis}}(r)/r}$ for the orbital velocity of stars. Beyond the luminous disk, $M_{\text{vis}}(r)$ saturates, so this predicts $v \propto r^{-1/2}$ at large r. However, observed rotation curves remain flat instead of declining. Introducing a dark matter halo fixes this: for an NFW halo, the total mass is given in equation 6. Considering

$$M_{\text{DM}}(r) = 4\pi \int_0^r \rho_{\text{DM}}(r') \, r'^2 \, dr' \tag{8}$$

The corrected rotation velocity is:

$$v^2\left(\mathrm{r}\right) = \frac{G\left(M_{vis}(r) + M_{DM}(r)\right)}{\mathrm{r}} \tag{9}$$

The orbital velocity then becomes $v(r) = \sqrt{G\,M_{\text{tot}}(r)/r}$. Because the halo mass continues to rise with radius, $v(r)$ remains roughly constant at large r, matching the flat rotation curves seen in data.

4.2.2. Gravitational lensing and excess mass

Gravitational lensing by galaxies and clusters provides a way to measure mass independently of light. The deflection angle α for light passing a mass M at the impact parameter ξ is $\alpha = 4GM(\xi)/\left(c^2\xi\right)$. In strong lensing, such as an Einstein ring, the angular Einstein radius θ_E satisfies

$$\theta_E = \sqrt{\frac{4G\,M(\xi)\,D_{LS}}{c^2\,D_L\,D_S}} \tag{10}$$

where D_L, D_S, and D_{LS} are the angular diameter distances to the lens, to the source, and between the lens and source, respectively. Observations of θ_E allow one to infer the lensing mass $M(\xi)$. In many cases, the inferred M_{tot} far exceeds the luminous mass, implying a dominant dark component.

For example, weak lensing maps of the Bullet Cluster show that most of the gravitational mass (inferred from lensing) is spatially offset from the hot X-ray–emitting gas. This indicates that a collisionless dark component constitutes the bulk of the mass, while the baryons have been slowed by the cluster collision. Such lensing evidence (both strong and weak) directly confirms the presence of dark matter in galaxies and clusters.

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4.2.3. Galaxy cluster dynamics (e.g., bullet cluster)

The Virial Theorem deduced in equation 5 provides an estimate of a cluster's mass from the motions of its galaxies. Here, $\sigma = \sigma_v$ is the velocity dispersion of member galaxies and R is a characteristic radius. Similarly, the temperature T of the intracluster gas reflects the gravitational potential, with

$$k_B T \approx \mu m_p \sigma_v^2 \tag{11}$$

(where μ is the mean molecular weight and m_p the proton mass). In both cases, using only the visible mass ($M_{\text{gas}} + M_{\text{stars}}$) underpredicts the observed σ_v and T. Introducing dark matter ($M_{\text{DM}} \gg M_{\text{vis}}$) ensures $M_{\text{tot}} \approx M_{\text{DM}}$, matching the observed σ_v and T.

The Bullet Cluster provides a quantitative example: X-ray observations indicate $M_{\text{gas}} \sim 10^{14} M_{\odot}$, while weak lensing reveals $M_{\text{tot}} \sim 10^{15} M_{\odot}$. Thus, $M_{\text{DM}} \approx M_{\text{tot}} - M_{\text{gas}} - M_{\text{stars}}$ is about five times the visible mass. This calculation confirms that dark matter dominates the cluster's mass budget, consistent with both lensing and dynamical data.

5. Alternative theories: Modified Newtonian Dynamics (MOND)

5.1. Basics of MOND

Modified Newtonian Dynamics (MOND), proposed by Milgrom, is a modified form of Newton's law at accelerations below a critical scale $a_0 \approx 1.2 \times 10^{-10}\,\text{m/s}^2$. It introduces an interpolation function $\mu(x)$ such that $\mu(a/a_0)\,a = a_N$, where a_N/r^2 is the Newtonian acceleration. For $a \gg a_0$, $\mu \approx 1$ and standard dynamics are recovered; for $a \ll a_0$, $\mu(a/a_0) \approx a/a_0$. In the deep-MOND regime ($a \ll a_0$), one obtains $a \approx \sqrt{a_0\, a_N}$, which yields asymptotically constant orbital velocities ($v^4 \approx G M_{tot}\, a_0$) without requiring dark matter.

5.2. Comparison with dark matter

• Advantages of MOND: MOND naturally predicts flat rotation curves and the empirical Tully–Fisher relation ($v^4 \propto M_{tot}$) without invoking dark matter. It can match the detailed rotation profiles of many galaxies and explains some small-scale dynamics better than a generic dark matter halo.

• Limitations of MOND: MOND fails to explain the gravitational lensing observed in galaxy clusters and the high gas temperatures without additional unseen mass. It also struggles to fit cosmological data (such as the cosmic microwave background spectrum) without an equivalent dark component. In cluster-scale systems and cosmology, MOND effectively requires additional matter or new physics.

Overall, MOND remains an intriguing alternative on galactic scales, but it cannot reproduce the full range of evidence that is well explained by dark matter.

6. Conclusion

Galactic rotation curve analysis demonstrates a consistent discrepancy between Newtonian prediction and observation, heavily in favour of there being unseen mass. The dark matter paradigm accounts for it all: it naturally produces flat rotation curves due to extended halos and accounts for

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gravitational lensing and cluster dynamics, such as in the Bullet Cluster. These lines of evidence point directly to the total mass being many orders of magnitude greater than luminous mass in galaxies and clusters, heavily verifying the dark matter hypothesis over alternatives. Future observations (e.g., with the Euclid space telescope) will further test these models and further refine our understanding of the unseen mass.

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