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Investigating the Effect of Different Dark Matter Models on the Linear Matter Power Spectrum

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Abstract. This study investigates the impact of different dark matter models on the linear matter power spectrum $P(k)$, a key measure of cosmic structure formation across scales. Using the Boltzmann solver CLASS, three cosmological models are simulated and compared: the standard Cold Dark Matter (ΛCDM) model, a pure Warm Dark Matter (WDM) model with a 2 keV thermal relic, and a Mixed Dark Matter (MDM) model composed of 80% CDM and 20% WDM (3 keV). The results show that all models behave identically on large scales (low k), but significant differences emerge on small scales (high k). The WDM model exhibits strong suppression of power due to free-streaming effects, while the MDM model shows intermediate suppression. These suppression features are already present at high redshift ($z = 3$) and persist to the present day ($z = 0$), indicating that dark matter properties imprinted early in cosmic history remain observable. This work highlights how the matter power spectrum serves as a sensitive probe of dark matter physics, bridging particle properties and large-scale structure.

Keywords: Dark Matter Models, Linear Matter Power Spectrum, Cold Dark Matter (CDM), Warm Dark Matter (WDM), Cosmological Simulations

1. Introduction

1.1. Why care about dark matter

We all know its name. Dark matter makes up about 85% of the universe's total matter, yet it cannot be observed directly, and its presence can only be inferred through how it interacts with visible matter and light via gravity. Galaxies tend to rotate faster if they were only made up of baryonic matter, so fast that it would rip itself apart. Large galaxy clusters bend light coming from background galaxies more strongly than what is expected if they were only composed of baryonic matter. It is a widely known fact that dark matter plays a crucial role in shaping the formation of cosmic structures on all scales, from giant superclusters to dwarf galaxies. Because of its nature, even slight differences in the properties of dark matter can drastically alter the way structures are formed. This begs the question: if dark matter particles were lighter and moved faster than in the current model, how would cosmic structures have formed differently in the universe? Would reality, as we know it, still be the same [1-3]?

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1.2. Research question

This project investigates how different types of dark matter affect the linear matter power spectrum of the universe, which describes how structure is clustered on different scales. Three models will be compared: a standard Cold Dark Matter (CDM) model that reflects the current state of the universe according to observations, a Warm Dark Matter (WDM) model where DM particles moved closer to the speed of light and carried more energy, and a Mixed Dark Matter (MDM) model that is a mix of the previous two models, with partial CDM and partial WDM. To run the simulation, I used CLASS, a Boltzmann solver widely used in cosmology, to simulate each model and extract their linear matter power spectra. By comparing the results, I aim to see how the different models affect the scales on which structure can form [4-8].

1.3. CDM vs WDM vs mixed

Each model has its own properties that dictate how structure forms on different scales:

Cold Dark Matter (CDM): Particles are heavy and slow-moving, so they cluster on all scales. This is the standard assumption in cosmology.

Warm Dark Matter (WDM): Particles are lighter and move at faster speeds, so they tend to travel further before slowing down. This eliminates small-scale clustering, eliminating some of the structure

Mixed Dark Matter (MDM): A combination of cold and warm components. Its predictions are intermediate: small-scale structures are suppressed, but not as strongly as in pure WDM [9].

1.4. Why P(k)

The linear matter power spectrum, usually written as P(k), is the main tool for quantifying and comparing these models. It describes how strongly matter clusters on a given scale, with small values of k corresponding to large scales and large values of k corresponding to small scales. Theory predicts that WDM and MDM should suppress clustering on small scales compared to CDM. The linear matter power spectrum is ideal for this study because it is straightforward to compute with CLASS and provides a clear, universal way of comparing clustering across models [10, 11].

2. Background

2.1. From correlation function to matter power spectrum

To understand the matter power spectrum, let's start with the idea of density fluctuations in the universe. At any point in space, the density of matter can be compared to the average density. The difference is described by the over density field,

$$\delta(x,t) = \frac{\rho(x,\ t) - \bar{\rho}\left(t\right)}{\bar{\rho}\left(t\right)}$$

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In plain English, δ tells us how much denser or emptier a location is compared to the cosmic average.

The simplest way to describe clustering is with the two-point correlation function, ξ(r). This function tells us the actual probability of finding two galaxies separated by a distance

r compared to what would be expected in a random, uniform distribution. In very simple terms, ξ(r) provides a way to quantify the "lumpiness" of the universe. Below is the two-point correlation function:

$$dP = n[1 + \xi(r)]dV$$

Applying the Fourier transform to ξ(r), we get the matter power spectrum:

$$\left\langle \delta(k)\delta^{*}\left(K'\right)\right\rangle = (2\pi)^{3}\delta_{D}\left(k - k'\right)P\left(k\right)$$

Instead of describing clustering in terms of distances between galaxies, the matter power spectrum uses spatial frequencies, represented by the wavenumber k. P(k) is often the more useful function because it provides a more straightforward way to quantify and visualize observations. P(k) tells us how much structure exists on scale k ~ 1/Λ.

2.2. Cold vs warm vs mixed DM: the big picture / predictions

Each DM model exhibits their own characteristics, and leads to drastically different outcomes.

Cold Dark Matter (CDM): CDM particles are heavy and became non-relativistic very early in the history of the universe. They can cluster on all scales, so the power spectrum should reflect high power levels at all scales. CDM is the baseline of the standard ΛCDM model of cosmology.

Warm Dark Matter (WDM): WDM particles are lighter and move faster, typically found at around 1 to 2 keV. They moved at relativistic speeds longer in the early universe and could "free-stream", meaning they were going for longer distances before slowing down. This free-streaming eliminates fluctuations below a certain scale, as represented by a cutoff in k in the power spectrum, corresponding to small physical scales.

Mixed Dark Matter (MDM): In this model, the universe contains both cold and warm components. The suppression of small-scale power will be weaker than the pure WDM model, but it would still be noticeable. The suppression of small-scale power is weaker than in pure WDM but still noticeable. The specific strength and scale of the suppression will depend on the ratio between WDM and CDM.

3. Methodology

3.1. Boltzmann solver (CLASS)

To compute the matter power spectrum, I used CLASS (Cosmic Linear Anisotropy Solving System), a state-of-the-art Boltzmann solver. CLASS solves the Einstein–Boltzmann system, a system of equations which combines Einstein's field equations with the Boltzmann equation for particle distributions to describe the evolution of matter and energy in the history of the universe.

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For this project, I used the Python wrapper Classy, which allows CLASS to be run directly with Python code. This made creating different models with different parameters particularly straightforward, and the linear matter power spectrum P(k,z) can be directly plotted using Matplotlib.

3.2. Baseline parameters

```
lcdm = {
    'h':          0.6736,
    'omega_b':    0.02237,
    'omega_cdm':  0.1200,
    'A_s':        2.1e-9,
    'n_s':        0.9649,
    'tau_reio':   0.0544,
    'N_ur':       3.044,
    'output':     'mPk',
    'P_k_max_1/Mpc': 100.0,
    'z_pk':       '0,1,3',
    'k_per_decade_for_pk': 50
}
```

Figure 1. Outputs

To start, I used the Planck 2018 best-fit ΛCDM parameters, which are currently the most precise measurements of cosmological parameters from the cosmic microwave background. The key parameters included values for the Hubble constant h, baryon density Ω_b, cold dark matter density Ω_cdm, scalar amplitude A_s, scalar spectral index n_s, and optical depth τ_reio.

The outputs requested from CLASS were the linear matter power spectrum up to k = 100 Mpc^{-1}, with sampling of 50 points per decade in k. I computed the spectra at three redshifts: z = 0 (today), z = 1 (earlier in the history of the universe), and z = 3 (even earlier in history) (Figure 1).

3.3. Model details

To explore different dark matter scenarios, I modified the baseline ΛCDM parameters in CLASS to create two additional models:

ΛCDM (baseline): Standard cold dark matter with no modifications. All parameters taken directly from Planck 2018.

Pure WDM: All CDM was replaced with a thermal relic of mass 2 keV. In CLASS, this was implemented by setting the CDM density to zero and adding a non-cold dark matter (NCDM) species with the correct mass, temperature ratio, and energy density.

Mixed DM: A model with 80% CDM and 20% WDM, where the warm component had a particle mass of 3 keV. This was implemented by splitting the dark matter density between the CDM and NCDM components.

These modifications were done using Python functions that adjusted the CLASS input dictionaries before computation. The models were chosen with the goal of identifying clear differences: pure WDM for strong suppression, and MDM for an in-between effect.

3.4. Computation loop & output

For each of the three redshifts (z = 0, 1, 3), I computed:

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The linear matter power spectra P(k, z) for all three models.

The ratios $P_{model}(k, z) / P_{\Lambda CDM}(k, z)$

The ratios are particularly useful because they emphasize the differences in the curve shapes caused by free-streaming suppression.

In total, this produced six plots: three showing the power spectra themselves and three showing the suppression ratios relative to ΛCDM.

3.5. Reproducibility

To ensure reproducibility, the software environment and versions are listed below:

Python: 3.13

CLASS: 3.3.2

NumPy: 2.3.3

Matplotlib: 3.10.6

Python script: (will include GitHub link once I upload it)

4. Results

4.1. P(k, z) at z = 0 (today)

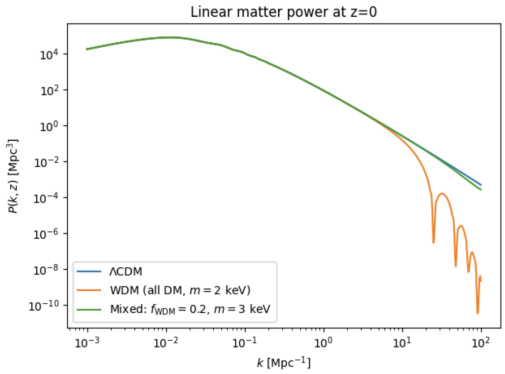

Figure 2. Linear matter power at z=0

At the present day, all three dark matter models behave similarly on large scales (low k), but show clear differences on small scales (high k). For $k < 10$ Mpc^{-1}, the curves for ΛCDM, WDM, and MDM overlap almost perfectly. This is very much expected, since free-streaming effects are negligible on large scales (Figure 2).

Differences start to show at a higher k. The WDM model shows a clear and strong decline. Its power spectrum drops drastically compared to ΛCDM once k exceeds about 10 Mpc^{-1}. The mixed dark matter model also shows mild suppression, with intermediate levels of intensity between the CDM and pure WDM curves. The behavior of these curves aligns with the hypothesis: free-streaming eliminates small-scale fluctuations for WDM, and partially reduces small-scale fluctuations for MDM.

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4.2. Suppression ratios at z = 0

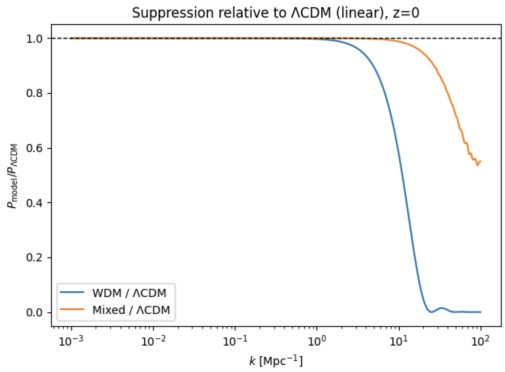

Figure 3. Suppression relative to ΛCDM (linear), z=0

The scale-dependent suppression becomes clearer when looking at the ratios of each model to ΛCDM. At low k, the ratios are close to one, showing no suppression. At high k (k > 1), however, the ratios fall quickly below one (Figure 3).

Pure WDM drops to around 0.5 by k ~ 10 Mpc^{-1}, and approaches 0.0 as k approaches $10^2 Mpc^{-1}$.

Mixed DM drops more mildly, reaching around 0.5 by k = $10^2 Mpc^{-1}$.

These plots demonstrate that the differences between models are not just in its values, but in the shape of the spectrum, with small scales most affected.

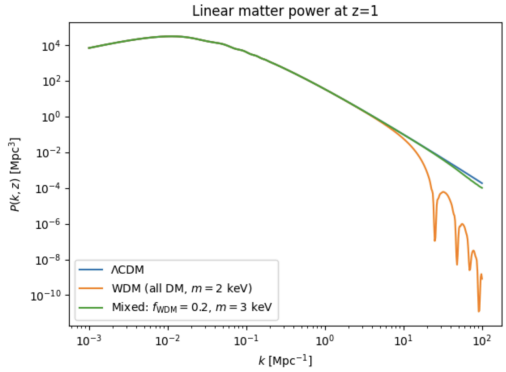

Figure 4. Linear matter power at z=1

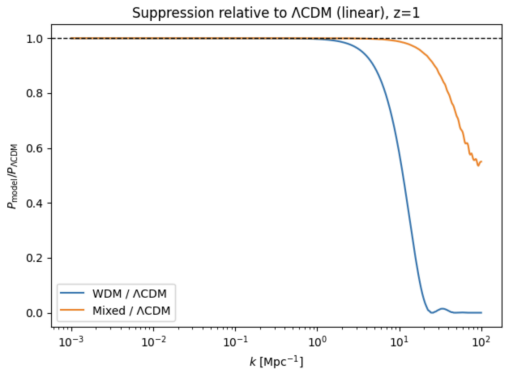

Figure 5. Suppression relative to ΛCDM (linear), z=1

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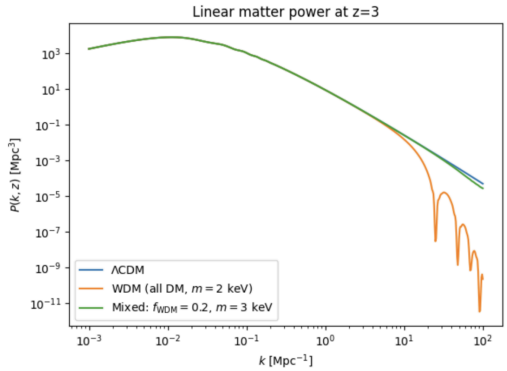

Figure 6. Linear matter power at z=3

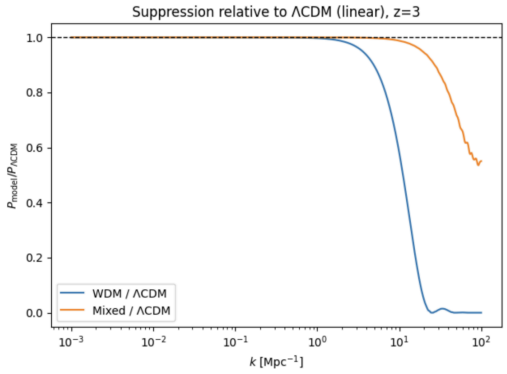

Figure 7. Suppression relative to ΛCDM (linear), z=3

4.3. Evolution with redshift (z = 1 and z = 3)

The same patterns are visible at earlier times. At z = 1, the spectra and ratios look very similar to those at z = 0, with WDM showing strong signs of suppression and MDM showing partial suppression. At z = 3, suppression is already present, the shapes of the curves almost identical to those at z = 0.

This tells us something significant: the small-scale suppression caused by free-streaming is already present in the linear matter power spectrum at high redshift. In other words, the defining characteristics of dark matter particles with different properties appear early in the history of the universe and remain visible as the universe expands and evolves (Figure 4-7).

4.4. Summary of findings

Overall, the simulations confirm the hypothesis:

On large scales (low k), all models overlap, since free-streaming has no effect there.

On small scales (high k), WDM suppresses structure strongly, while MDM produces an intermediate level of suppression.

The suppression is already set by high redshift and persists unchanged to today.

These results highlight how the matter power spectrum can help visualize the physical differences between dark matter models in a clean and straightforward way based on cosmic scales.

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5. Interpretation

5.1. Physical significance

The simulation results reflect the basic physics of how different kinds of dark matter particles move and cluster. On large scales (low k), all models behave the same, since free-streaming is irrelevant there. On small scales (high k), however, WDM and MDM show suppressed power due to the warm component's ability to travel and erase fluctuations before slowing down. The sharp decline in the WDM spectrum and the same pattern milder in MDM are clear evidence of this effect. In short, the shape of the power spectrum directly reflects the properties of dark matter.

5.2. Parameter significance / implications

The strength of suppression depends on the mass and number of warm particles. Smaller particle masses correspond to larger free-streaming lengths, which erase more structure. A higher ratio of WDM in mixed models also leads to deeper suppression. In the simulation, a 2 keV WDM relic showed a strong cutoff, while a model with 20% of 3 keV WDM showed the same patterns, just less drastic. Altering the initial parameters would make the scale and position of the cutoff shift in predictable ways. This means the power spectrum could, in principle, be used to infer particle mass and amount from data.

5.3. Observational handles

The characteristics seen here can be tested against actual observations. For example:

High-redshift galaxies: The number of small galaxies that form early on depends on the amount of small-scale power.

Satellite galaxies: In the Milky Way, the abundance of dwarf satellites reflects the level of clustering on small scales.

Future surveys such as Euclid, LSST, and JWST will improve these measurements and could either detect or place strong limits on WDM- or MDM-like models. This makes the matter power spectrum a bridge between particle physics and observations of the universe.

5.4. Strengths & limitations

A strength of this study is that it provides a clean, controlled comparison of dark matter models using a reliable tool (CLASS). Additionally, comparing ratios between ΛCDM and other models reduces potential uncertainties with calibration.

Limitations include:

Only linear scales were considered, so nonlinear growth and galaxy formation were not included.

Only thermal relic WDM was modeled, not other possible candidates.

Only one specific ratio was tested in the MDM case.

Even so, the results are meaningful because they highlight the qualitative differences between models.

5.5. Future work

Future studies could extend this work in these directions:

- Explore a wider range of warm particle masses and mixed dark matter ratios.

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• Include nonlinear computations.

These steps would bring the analysis closer to what professional cosmologists do when constraining dark matter properties.

6. Conclusion

This project aims to investigate how different types of dark matter affect the linear matter power spectrum. Using CLASS simulations, I compared three models: standard ΛCDM, pure warm dark matter, and a mixed dark matter scenario.

The results show that differences in dark matter particle properties directly affect the formation and clustering of cosmic structures and lead to different characteristics of the matter power spectrum. On large scales, all models exhibit similar behavior, since free-streaming has no effect on large scales. On small scales, however, the warm dark matter model shows strong suppression, while mixed dark matter also produces mild suppression with levels in between WDM and CDM. These patterns are always established by high redshift ($z \sim 3$) and remain visible to the present day. Free streaming is the key mechanism that links the properties of different dark matter particles to the formation of cosmic structures.

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