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Velocity Field, Pressure Distribution, and Aerodynamic Forces on Airfoils via Conformal Mapping

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Abstract. The method to analyse the incompressible potential flow over airfoils is Conformal mapping, using the specific capabilities of the Joukowski transformation, because this method can convert complicated airfoil profiles into a simple circular cylinder within the complex plane. This property enables us to make direct computation of velocity fields, pressure distributions, and the aerodynamic lift. Importantly, a physically bounded solution requires the accurate application of Kutta condition at the trailing edge, a feature central to this framework. We confirm that thw model robustly reproduces the expected linear dependencies of lift coefficient on both angle of attack and camber according to the validation against thin airfoil theory. A noticeable finding is that the overestimation of lift is the modest but consistent. This difference is because of the finite thickness effects inherent in the Joukowski transformation, which are systematically ignored in classical thin airfoils estimations. The analysis of the obtained data definitely points out the upper-surface suction peak as the final source of lift, proved by the smoothness characteristics of irrotational streamline visualizations. The model is inherently constrained by its fundamental inviscid assumption though its computational elegance. This theoretical limitation basically prevents the capture of boundary layers, turbulence, or viscosity, leading to the avoidless zero-drag prediction(d' Alembert's paradox) and an inflated calculation of lift at high angles. Hence, while conformal mapping retains essential value as a highly efficient tool for elementally design, its transition to high-fidelity, estimative real-world analysis entirely upon future efforts to integrate feasible viscous correction models.

Keywords: Conformal mapping, Airfoil, Potential flow, Kutta condition, Lift coefficient

1. Introduction

Incompressible flow analysis [1, 2] can be used to simulate the negligible fluid behavior of density changes. It is the basis of aerodynamics [3] and fluid dynamics, affecting aircraft design, turbine efficiency and fluid dynamics systems [4, 5], such as analyzing the air flow around vehicles such as cars and airplanes, or the water flow around ships with usually low Mach numbers [6]. In addition, it is also suitable for hydraulic systems and pipe networks to optimize liquid flow in pipeline and pump design. Internal flow, such as the efficiency analysis of coolant cycle or centrifugal pump in analog electronic equipment, and external flow, such as aerodynamic profile of aircraft wings or wind load calculation of skyscrapers, are all its application fields. Although numerical methods such

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as CFD (computational fluid dynamics) dominate in modern applications, analytical methods, especially angle-retenting mapping, still have mathematical elegance and computational efficiency in ideal flow.

This paper comprehensively studies the application of angle preservation mapping technology in solving the problem of incompressible winding current in the wing type. The preservation angle mapping (formerly known as "contention angle mapping") is a powerful mathematical tool for simplifying the solution of fluid dynamics problems and can transform complex geometric forms into simple geometric forms. This article first explains its basic principle and its connection with aerodynamics, focusing on the Joukowski transformation [7] - this special angle-maintening mapping can map the flow around the cylinder to the flow around the wing, and make the velocity potential and flow function of the wing different from that of the cylinder wing. On this basis, the velocity field and pressure of the wing type are calculated in detail. The correctness of the analytical solution was verified by numerical simulation and compared with the experimental data in the literature. The results show that the angle preservation mapping method can accurately and quickly solve the incompressible flow under the geometric shape of the wing, which is an important tool for aerodynamic analysis and design. The influence of parameters such as angle of attack and wing shape on the flow characteristics was further analyzed, and it was found that this method was universal when dealing with different wing geometry and flow conditions. Finally, the role of angle preservation mapping in solving complex gas dynamics problems is summarized, and suggestions are put forward to deepen the research of angle preservation mapping, expand the application field of angle preservation mapping, and improve the accuracy of angle preservation mapping.

2. Methodology: application of conformal mapping

This section elaborates on the specific process of solving the analytical solution of the potential flow around a specified airfoil using conformal mapping.

2.1. The mathematical foundation

The analysis work starts from the basic principle of complex potential flow [7, 8]. For incompressible irrotational two-dimensional flows, their velocity fields can be described by the complex function W(z), where the expression of z is x + iy.

The complex is an analytic function [9], which means that it satisfies the Cauchy-Riemann equation.The real part of W(z) is the velocity potential, $\Phi(x,y)$, and the imaginary part is the stream function, $\Psi(x,y)$. The velocity components, u and v, are given by the complex velocity, which is the derivative of the complex potential with respect to z:

$$V_c = u - iv = \frac{dW}{dz}$$

Both the velocity potential and the stream function are harmonic functions, satisfying Laplace's equation:

$$\nabla^2\Phi = 0 and \nabla^2\Psi = 0$$

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The stream function is constant along streamlines, and the body of the airfoil itself is a streamline. The primary objective is to find a suitable complex potential that satisfies the boundary conditions of the flow: the velocity must be zero at infinity (for a uniform flow) and the normal component of velocity must be zero on the surface of the airfoil.

2.2. The conformal mapping strategy

The core idea of conformal mapping is to transform a complex geometry (the airfoil shape in the physical z-plane) into a simpler geometry (a circle in the transformed ζ-plane) where the flow problem is readily solvable [5, 8]. The transformation is defined by a mapping function,

$$z = f(\zeta)$$

A key property of a conformal map is that it preserves angles, which implies that the relationship between the complex potential in the z-plane, W(z), and the complex potential in the ζ-plane , W(ζ), is straightforward. The complex velocity in the z-plane is related to the complex velocity in the ζ-plane by the chain rule:

$$\frac{dW}{dz} = \frac{dW}{d\zeta}\ \frac{d\zeta}{dz} = \frac{dW/d\zeta}{dz/d\zeta}$$

This relationship allows us to solve the problem in the simpler ζ-plane and then map the solution back to the physical z-plane.

2.3. Step-by-step procedure

Step A: Define the Airfoil Geometry and Choose the Mapping Function

The first step is to determine the specific wing type to be analyzed. For this paper, the Joukowsk wing type of a family will be used as a major example due to the elegance of Joukowski's transformation [7]. The Joukowski transformation is given by the formula:

$$z = \zeta + \frac{c^2}{\zeta}$$

Where z is a point on the physical plane (wing type), ζ is a point on the mapping plane (circle), and c is a real constant. This transformation maps a circle with a radius of R, and the center of the circle is

$$\zeta_0 = -\epsilon + i\eta$$

The wing type from the ζ plane to the z plane. The parameters c, R, ε and η can be adjusted to control the thickness, radiance and angle of attack of the wing shape.

Step B: Define the process in the mapping plane (ζ)

Uniform flow around the circle is a classic solved problem in fluid mechanics [1, 7, 10]. The complex potential for a uniform flow with velocity U_∞ at an angle of attack α around a circle of radius R centered at the origin in the ζ-plane, with a circulation Γ, is given by:

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$$W(\zeta) = U_\infty e^{-i\alpha}\zeta + \frac{U_\infty R^2 e^{i\alpha}}{\zeta} + \frac{i\Gamma}{2\pi}\ln(\zeta)$$

The first term represents the uniform flow, the second term is the doublet (representing the flow blocking the circle), and the third term is the vortex (responsible for lift). The center of the circle is shifted to ζ_0 for a more general case, and the complex potential is adjusted accordingly. For the Joukowski transformation, the circle is not centered at the origin, so the complex potential for the flow around a displaced circle is used.

Step C: Apply the Kutta Condition

A crucial aspect of obtaining a realistic solution for lifting airfoils is the application of the Kutta condition [1, 10]. This physical principle states that for a sharp trailing edge, the flow must leave the trailing edge smoothly. Mathematically, this means the velocity at the trailing edge must be finite. In the transformed ζ-plane, the trailing edge of the airfoil corresponds to the point where the circle passes through the point

$$\zeta = c.$$

The Joukowski transformation has a derivative of zero at this point,

$$dz/d\zeta = 1 - c^2/\zeta^2 = 0$$

$$for \zeta = \pm c$$

This would lead to an infinite velocity if the velocity in the ζ-plane were non-zero. The Kutta condition, therefore, requires that the complex velocity in the ζ-plane at the trailing edge point must be zero. This condition uniquely determines the value of the circulation, Γ. By setting the complex velocity $\frac{dW}{dz}$ to zero at the trailing edge point, we can solve for the circulation Γ.

Step D: Map the Solution Back to the z-Plane

Once the complex potential W(ζ) is determined with the correct circulation, we can find the complex velocity in the physical z-plane using the chain rule relation derived in step 2.2.

$$V_c\left(z\right) = \frac{dW}{dz} = \frac{dW/d\zeta}{dz/d\zeta}$$

This expression gives the velocity at any point in the flow field around the airfoil.

Step E: Calculate Physical Quantities

With the velocity field determined, we can calculate key aerodynamic quantities.

* Pressure Distribution: Using Bernoulli's equation, the pressure P at any point can be calculated from the velocity magnitude $|V_c|$:

$$P - P_\infty = \frac{1}{2}\rho\left(U_\infty^2 - |V_c|^2\right.$$

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Where P_∞ and U_∞ are the pressure and velocity at infinity.

Lievature and drag: The lift (L) and drag (D) forces can be calculated by using the Blasius theorem to integrate the pressure on the surface of the wing type (circle on the ζ plane). The Blasius theorem provides a simple way to calculate the force acting on an object:

$$F_x - iF_y = \frac{i\rho}{2} \oint_c \left(\frac{dW}{dz}\right)^2 dz$$

By applying this theorem to the circular flow on the ζ plane and transforming its integral, we can derive the lift and drag. For potential currents, the resistance is always zero (Da Lambert's paradox), and the lift is not zero, given by the Kutta-Joukowski theorem [1, 4, 10]:

$$L = \rho U_\infty \Gamma$$

This method provides a complete, step-by-step framework, using shape-preserved mapping to find the solution to analyze the fluid flow around the wing type, allowing the calculation of pressure distribution and lift.

3. Results and discussions

The application of angle preservation mapping to analyze the incompressible potential current around the Joukowski wing type has produced profound results, verified the method, and highlighted its theoretical advantages and practical limitations. To ensure the reliability of the approach, the method was first validated against established theoretical benchmarks. For a symmetric Joukowski airfoil, defined by parameters such as c =1 , a circle center at

$$\zeta_0 = -0.1 + 0i \text{ , } andradius\ R \approx 1.1$$

the lift coefficient C_L was computed for angles of attack $\alpha = 0^\circ$, 5° and 10° , resulting in values of 0.0, 0.597, and 1.190, respectively. These figures demonstrate a close alignment with thin airfoil theory [1, 10], which predicts $C_L = 2\pi \sin(\alpha)$, yielding 0.0, 0.548, and 1.091 for the same angles. The modest overprediction in the computed values can be attributed to the airfoil's finite thickness (chord length approximately 4.033), which augments lift beyond the idealized thin airfoil approximation. Extending this to a cambered airfoil, with the circle center shifted to

$$\zeta_0 = -0.1 + 0.05i \quad \text{and} \quad R \approx 1.104 \text{ ,}$$

produced a C_L of 0.312 at $\alpha = 0^\circ$, illustrating the inherent positive lift from camber and a negative shift in the zero-lift angle, consistent with classical aerodynamic principles. Such validations affirm the method's efficacy for inviscid flows, though they underscore the need to consider real-world complexities where viscous effects introduce deviations.

Examination of the pressure coefficient C_p distributions further elucidates the physical dynamics of lift generation under varying flow conditions. For the symmetric airfoil at $\alpha = 5^\circ$, the C_p on the upper surface exhibits a pronounced suction peak near the leading edge, with values such as -1.89 at $x \approx -1.95$, contrasting sharply with the lower surface's near-stagnation value of 0.76. This disparity drives the net upward force essential for lift. As the flow progresses toward the trailing edge, the upper surface C_p transitions from negative (e.g., -0.97 at $x \approx$ -1.30) to near-positive (0.14 at $x \approx 1.88$), while the lower surface maintains a more subdued variation,

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decreasing to approximately 0.19. At an elevated $\alpha = 10^\circ$, the suction intensifies and shifts forward, amplifying the pressure gradient and, consequently, the lift. In the cambered case, even at zero incidence, an asymmetric C_p distribution persists, with more negative values on the convex upper surface, explaining the baseline lift. These patterns emphasize the critical role of the Kutta condition in enforcing finite velocities at the trailing edge, averting unphysical infinite speeds and ensuring realistic flow behavior. However, they also reveal the method's idealization, as real incompressible flows around airfoils involve boundary layer interactions that can alter pressure profiles, particularly under adverse gradients leading to separation.

The lift coefficient calculations reinforce the linear trends estimated by the Kutta-Joukowski theorem [1, 4, 10], where $C_L = 2\Gamma/(U \bullet \text{chord})$, with circulation Γ determined by the Kutta condition. For the symmetric airfoil, C_L rises linearly with α—from 0.0 at 0° to 1.190 at 10°—with a slope of approximately 6.82 per radian, surpassing the thin airfoil value of $2\pi \approx 6.28$ due to thickness-induced enhancements. The cambered variant displays an upward offset, achieving 0.312 at 0°, 0.908 at 5°, and 1.497 at 10°, reflecting camber's equivalence to an effective angle of attack. This linearity holds well for small angles, aligning with the theorem's assertion that lift derives solely from circulation in potential flow. Yet, in more complicated conditions, such as those involving higher angles or non-ideal fluids, this model falters, as it predicts indefinite linear growth without accounting for stall phenomena.

Visualizations of streamlines, obtained from the stream function ψ (the imaginary part of the complex potential), provide a qualitative description of the flow patterns, underscoring the assumptions of irrotational and incompressible behavior. In the symmetric airfoil at low α, streamlines approach uniformly, diverge smoothly around the profile, and reconverge downstream, with stagnation points at the leading and trailing edges for zero incidence. At $\alpha = 5^\circ$, the forward stagnation shifts to the lower surface, and streamlines curve more acutely over the upper side, indicating accelerated flow and suction regions. For the cambered airfoil, asymmetry is evident even at zero angle, with streamlines adhering more closely to the upper convex surface, correlating with non-zero circulation. These illustrations confirm the method's ability to capture additional smooth flow, but they ignore sticky complexity, such as separation of bubbles or trails, which dominate the actual wing type performance under turbulent or high Reynolds conditions.

Although this study has achieved the above results, the analysis shows that the non-viscous flow assumption has inherent limitations - this assumption ignores viscosity, boundary layer and turbulence, which are precisely the core influencing factors of the complex conditions of incompressible flow around the actual airfoil. For example, the model sustains linear C_L growth beyond $\alpha = 10^\circ$, ignoring stall onset typically at 10°-15° due to adverse pressure gradients causing separation.The d'Alembert paradox [4] further appeared, predicting zero resistance contradicts actual viscosity and induced resistance. There are also challenges when mapping complex geometric figures, among which complex transformations increase the computational requirements, especially near singularities, such as the back edge. When juxtaposition with computational fluid dynamics (CFD) or experimental data, the method underestimates the resistance at large angles and overestimates the lift, making it an ideal tool for preliminary, low-angle analysis, rather than a comprehensive simulation of the complexity of the real world.

This research goes beyond the scope of conventional applications. In this work, we explore how mapping parameters affect airfoil performance, which lets us analyze more sophisticated flow conditions. Moving from symmetric (ε = -0.1, η = 0) to cambered (η = 0.05) constructions does two things: it ensures known results and also shows how small adjustments in η can create

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important zero-incidence lift (e.g., $C_L = 0.312$). This provides useful parametric data for optimizing designs, take an example, in cruise flight.

The method can also implement adjustable lift without changing the wing's shape, simply by applying different boundary conditions (like a new angle of attack). However, this clearly demonstrates a trade-off. While higher angles might improve efficiency, they also increase the risk of flow separation because of viscous effects (which are present in real-world flow). This parameter research uses the standard Joukowski analysis. It helps connect the simple, classical airfoil theory with the more complicated realities of the actual flow environment.

4. Conclusion

Modeling incompressible flow over airfoils is achieved by using conformal mapping, leveraging the specialized Joukowski transform [7]. The main methodology, based on the complex potential theory, mandates strict adherence to the Kutta condition to yield a physically meaningful result. Indeed, our computations efficaciously reproduce precise lift coefficients, streamline patterns, and pressure distributions across both symmetric and cambered profiles, a finding that powerfully validates the method's efficacy and alignment with classical aerodynamic principles for ideal flow assessment.

Such theoretical elegance, however, comes at the cost of high-fidelity to the physical world. The inviscid prerequisite necessarily excludes key phenomena—viscosity, turbulence, and boundary layer effects—which immediately results in the well-known difference of zero-drag prediction (d'Alembert's paradox [4]). Furthermore, this same simplification magnifies lift at increased angles of attack. Moreover, computationally intensive transformations surface when applying the method to complex geometries, particularly around the trailing edge singularities. Closing this high-fidelity gap necessitates targeted research. The crucial next step involves integrating viscous correction models to move beyond purely potential flow. Meanwhile, optimizing the transformation algorithms for demanding constructions and broadening the scope of parametric investigations are crucial if conformal mapping is to fulfill its potential as a basic aerodynamic design tool, providing a reference basis for actual fluid dynamics.

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