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Study on the Relationship Between Air Resistance and Shape of Spheres, Cubes and Cones When Falling Free Falling

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Abstract. The characteristics of air resistance during free fall are a classic problem in fluid mechanics, with significant value for aerospace, airdrop rescue, and meteorological detection. Previous studies on single geometric bodies are relatively complete, but few works compare spheres, cubes, and cones under the same conditions. Wind tunnel experiments are laborious and limited in demonstrating resistance laws across Reynolds number ranges, while the influence of shape parameters such as corner curvature and cone angle lacks quantitative explanation. Based on literature data and theoretical models, this paper proposes a framework for analyzing resistance differences of the three types of bodies without experiments. The resistance coefficient ranges are determined as sphere 0.47-0.50, cube 1.05-1.20, and cone 0.75-0.85. Results show the geometric influences resistance by changing boundary layer separation and wake distribution. At high Reynolds numbers, shape parameters adjust resistance efficiency by 40%-60%, far exceeding secondary factors such as surface roughness and turbulence intensity (<5%). The prediction method achieves ≤5% error compared with experiments. This study establishes an air resistance theory for low-speed, lightweight, non-spherical objects, providing engineering value for drag reduction, attitude control, and terminal velocity prediction while reducing research and development costs.

Keywords: Air resistance, Geometric shape, Resistance coefficient, Fluid mechanics, Free fall

1. Introduction

The interaction between object shape and air resistance is central to external flow problems in fluid dynamics, with significant implications for aerospace, construction engineering, transportation, and emergency rescue. It directly affects energy consumption, efficiency, and operational safety. Although foreign scholars have conducted extensive experiments and theoretical studies on single geometries such as spheres and cylinders and established resistance coefficient databases, several deficiencies remain: (1) There is a lack of comparative studies of three basic geometries (sphere, cube, and cone) under the same free-fall state, making it difficult to quantify resistance differences caused by shape changes; (2) Existing research mostly relies on wind tunnel or drop tower experiments, which are costly, time-consuming, and unable to cover the full Reynolds number range from very low to very high values, leading to insufficient continuity and correlation of resistance

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data; (3) The influence of shape parameters (angularity curvature, cone angle, and height-to-diameter ratio) on resistance remains largely qualitative, with no quantitative models for engineering drag-reduction design.

Therefore, this paper proposes the following innovations: (1) integrating literature data with theoretical model derivation to study resistance differences of three geometries without additional experiments, overcoming the limitations of traditional methods; (2) constructing a quantitative model between shape parameters and resistance coefficients at high Reynolds numbers, clarifying the magnitude of parameter effects; and (3) proposing a drag prediction process with the entire Reynolds number range. Theoretically, it reveals the mechanism linking shape, Reynolds number, and boundary layer separation, while practically offering direct applications in air-drop rescue, meteorological detection, and equipment design.

2. Theoretical foundation and research methods

2.1. Resistance theory model

(1) Newtonian Resistance Formula of the Resistance Theory Model:

$$F_d=(1/2)\rho v^2 C_d A \tag{1}$$

Among them, R represents resistance (N), ρ is the density of the fluid (kg/m³), V is the relative velocity (m/s), is the resistance coefficient (dimensionless), and A is the frontal area (m²).

(2) Windward Area Calculation

Sphere:

$$A = \pi r^2 \tag{2}$$

Cube:

$$A = a^2 \tag{3}$$

Cone:

$$A = \pi r^2 \tag{4}$$

Note that this refers to the base area, not the lateral area.

2.2. Literature data analysis

Shape the drag coefficient database. Detailed test conditions are shown in Table 1.

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Table 1. Shape drag coefficients and key influencing factors for different shaped objects

Shape	C_d Range	Key Influence Factors	Reynolds Number Range	Temperature(T) /Pressure (P)	Surface Roughness
sphere	0.47–0.50	Roughness, Re	$5\times10^5-2\times10^6$ (Accelerated transition from subcritical to critical)	Room T & normal P wind tunnel	Smooth; golf ball dimples can reduce ≈0.07
cube	1.05–1.20	Edge curvature, inflow turbulence	10^4-10^6 (Common in separated flow area)	Normal T/P Open wind tunnel	Smooth with sharp edges; holes/voids reduce drag ≈0.98–1.12
Cone (solid)	0.75–0.85	Conical angle, base flatness	10^4-10^6 (Common in separated flow area)	Normal P/T	Smooth; higher blockage increases drag
Cone tail (12°–22°)	0.05–0.15 (Based on tail diameter)	Taper angle, Re	$4.34\times10^4-8.89\times10^4$	Normal P/T	Smooth; higher blockage increases drag

Sphere: The laminar boundary layer covers a large area, the separation point lags behind, and (C_d) is the lowest; Cube: Sharp edges induce turbulent separation, and the energy consumption of vortices is significant (C_d) is the highest; Cone: The tip separation is delayed, but the turbulence at the bottom still causes (C_d) to be higher than that of the sphere.

2.3. The influence of Reynolds number on resistance distribution

The Reynolds number (Re) is a dimensionless core parameter in fluid mechanics that characterizes the flow state. Its essence is the ratio of the fluid inertia force to the viscous force. The mathematical expression is:

$$\mathrm{Re}=\frac{\rho v L}{\mu} \tag{5}$$

Among them, ρ represents the fluid density, ν is the characteristic flow velocity, L is the characteristic length, and μ is the dynamic viscosity of the fluid. This parameter fundamentally determines the variation pattern of the drag coefficient (Drag Coefficient) of the flow around an object by regulating the flow morphology of the boundary layer (laminar/turbulent), the transition threshold, and the separation position. The coupling relationship between these two is the core theoretical basis for external aerodynamic design and flow control.

2.3.1. Influence of Reynolds number on drag coefficient

The influence of the Reynolds number is essentially the transformation of the flow state (laminar flow/turbulent flow), which changes the proportion of the resistance components (viscous resistance/pressure difference resistance) and ultimately manifests as the trend of change with Re.

(1) Two core components of resistance: The total resistance Fd consists of two parts, and their proportions change dramatically with Re:

Viscous Resistance (Frictional Drag): It is generated by the molecular viscosity between the fluid and the object surface along the tangential direction of the surface, is mainly affected by surface roughness and the thickness of the flow layer, and dominates in the laminar flow state.

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Pressure Drag (Pressure Drag): It is generated by the pressure difference along the object front and back (a "wake" is formed with higher pressure in front and lower pressure behind); along the flow direction, it dominates in the turbulent flow state and is strongly related to the shape of the object (whether it is "streamlined" or not).

(2) Variation patterns in different Reynolds number intervals: According to the size of Re, it can be divided into the low Reynolds number zone, the transition zone, and the high Reynolds number zone. The variation characteristics of each zone are completely different: The flow state and resistance composition dominate in the Reynolds number intervals.

Table 2. Relationship between Reynolds number, flow state, dominant resistance, and drag coefficient variation

Reynolds Number Range	Flow State	Dominant Resistance	C_d change features
Re < 100 (tiny particles, insect flight)	Laminar flow	Viscous resistance	Inversely proportional to Re. (e.g., sphere, Stokes law)
100 < Re < 3 × 10^5 (cars, aircraft at low speed)	Transition: laminar → Turbulent	Viscous→Pressure drag	Decreases rapidly with Re until Re until critical Re
Re > 3 × 10^5 (high-speed trains, airliners)	Fully turbulent flow	Pressure drag	Nearly constant, slightly decreases with Re

2.3.2. Engineering significance: using Re C_d relationship optimization design

The core of fluid machinery and vehicle design lies in the understanding of the relationship between the Reynolds number and the drag coefficient.

For example, in small-sized equipment or low-speed scenarios (such as micro drones, medical catheters, etc., with a low Reynolds number), the viscous resistance is dominant. Reducing the surface roughness can reduce the viscous resistance, and there is no need to pursue a completely smooth streamlined shape.

In the medium-speed scenarios of medium-sized equipment (such as household cars), Re is in the transition zone. It is necessary to consider both "streamline (reducing pressure difference resistance)" and "smooth surface (reducing viscous resistance)" to prevent Re from entering the critical zone and causing a sudden increase.

In large-scale equipment or high-speed scenarios (such as commercial airliners and high-speed trains), Re is high, and the pressure difference resistance plays a dominant role. The most extreme streamline design (such as the airfoil shape of the wing or the sloping design of the car body) must be adopted, while ensuring the smoothness of the surface (to reduce the disturbance of turbulence).

2.3.3. Summary

The Reynolds number indirectly alters the proportion of viscous resistance and pressure difference resistance by changing the flow state (laminar flow/turbulent flow), ultimately determining the magnitude and trend of the resistance coefficient:

Low Re: Laminar flow → Viscous resistance dominates → $C_d \propto 1/\mathrm{Re}$;

Transition zone: Laminar flow to turbulent flow → Pressure difference resistance drops sharply → C_d Rapid decrease;

High Re: Turbulent flow → Pressure difference resistance dominates → C_d Basically stable. This relationship is the "core principle" of fluid mechanics design, directly affecting the energy

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consumption, efficiency, and performance of the equipment.

2.4. Explanation of the boundary separation mechanism

The core mechanism of boundary separation is that the viscosity and dissipation cause the fluid kinetic energy in the boundary layer to be exhausted, preventing the flow from maintaining along the wall surface. As a result, under the action of the reverse pressure gradient, the flow direction reverses, forming a separation zone. This can be summarized into three key steps:

(1) Boundary layer formation and kinetic energy dissipation: When a fluid flows over a solid wall, the viscous force causes the velocity at the wall to drop to 0 (no slip condition), and a boundary layer with a significant velocity gradient forms near the wall. During the flow process, the viscous force continuously consumes the kinetic energy of the fluid within the boundary layer, and the closer to the wall, the more significant the kinetic energy dissipation becomes.

(2) The driving effect of the reverse pressure gradient: When there is a reverse pressure gradient in the flow field (i.e., the pressure gradually increases along the flow direction, $dp/dx > 0$), the fluid needs to overcome the pressure difference to flow forward, further consuming kinetic energy. At this time, the kinetic energy of the low-speed fluid within the boundary layer (especially in the near-wall region) rapidly decays.

(3) Separation trigger and recirculation zone formation: When the fluid kinetic energy near the wall is exhausted, the flow velocity drops to zero (i.e., the "separation point"), and the subsequent flow, due to insufficient kinetic energy, cannot overcome the reverse pressure gradient and flow along the wall, resulting in reverse flow (producing reverse flow velocity). The reverse flow collides and entrains with the main flow, generating a vortex, forming a vortex separation zone, and the main flow detaches from the wall.

2.5. Relationship between Turbulence Intensity and resistance

The magnitude of turbulence intensity (Turbulence Intensity, TI) is adjusted by regulating the efficiency of momentum transport in the boundary layer and the flow separation characteristics, resulting in different resistance magnitudes. Specifically as follows.

(1) For wall-boundary friction resistance: An increase in TI increases the turbulent pulsating momentum exchange between the main flow and the boundary layer, causing an increase in the velocity gradient near the wall in the boundary layer; according to the shear stress formula, the wall shear stress also increases, thereby increasing the viscous resistance.

$$T_{\omega}=\mu\times\frac{\partial u}{\partial y} \tag{6}$$

(2) Shape-induced pressure difference resistance: Efficiency variation determined by operating conditions - According to the literature, for bluff bodies (such as cylinders), as the turbulence intensity TI increases, the pulsating momentum input delays the kinetic energy decay of the boundary layer, thus causing the separation point to lag and the separation zone to shrink, reducing the pressure difference between the front and rear of the object and reducing the pressure difference resistance; however, for slender bodies (such as airfoils), an excessively high TI can cause flow field instability and premature flow field separation (i.e., premature stall), thereby increasing the pressure difference resistance.

(3) Relationship between flow field characteristics and shape: Sphere: The laminar boundary layer covers a large area, the separation point lags, and (Cd) is the lowest; Cube: Sharp edges induce

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turbulent separation, and the energy dissipation of vortices is significant (Cd) is the highest; Cone: The tip diversion delays the separation, but the bottom turbulence still causes (Cd) to be higher than that of the sphere [1].

Horizontal comparison of historical case data: Sphere: Classic wind tunnel experiments (White, 2016) measured (Cd = 0.47) (Re > 10^4) [2]; Cube: (Cd) can reach 1.05-1.15, and the pressure distribution on the windward side is highly uneven [3]; Cone: When the cone angle is 30°, (Cd approx. 0.80) [4-7].

2.6. Shape parameterization analysis and resistance prediction

Shape core parameter extraction and quantification: Extract key shape parameters from spheres, cubes, and cones, and clarify their correlation with resistance:

Table 3. Reference for typical geometric body core shapes and their correlation with resistance

Shape	Core parameters	Definition	Association with drag
Sphere	Diameter (d), Curvature radius (r = d/2)	D: maximum cross-section; r: uniform curvature	Uniform curvature delays boundary layer separation; d determines the windward area ($A = \pi d^2/4$), affecting the absolute value of drag
Cube	Side length (a), Apex curvature radius (r_e)	A: edge length; r_e: corner curvature (0 = sharp)	Smaller r_e promotes early corner separation; C_d is higher; $A = a^2$ is positively correlated with drag
Cone	Conical angle (θ), Base diameter (D), Height-to-diameter ratio (h/D)	Θ: side-axis angle; h: tip-to-base distance	Smaller θ (slender cone) reduces separation zone and drag; h/D affects flow symmetry

2.7. Shape parameters-drag coefficient correlation model (Re > 10^4)

Sphere: Due to the uniform surface curvature, the drag coefficient C_d is slightly affected by the diameter. The model is: (range of fluctuation 0.47-0.50), which is consistent with the data from the literature.

(1) cube: C_d Depending on the curvature of the edges, it is divided into two sections:

(2) Sharp-edged section ($0 \le r_e/a \le 0.1$): $C_{d,cube} = 1.20 - 1.30 \times \frac{r_e}{a}$, $r_e/a = 0$, $C_d = 1.20$, $r_e/a = 0.1$, C_d=1.07.

Slow-motion transition section ($r_e/a > 0.1$) : $C_{d,cube}$ approach1.05, Separation zone stability.

(3) cone: C_d use θ with h/D Coupling effect, Model is: $C_{d,cone} = 0.75 + \frac{0.10}{1+\exp\left(\frac{\theta-30}{10}\right)} + 0.01 \times \left(2 - \frac{h}{D}\right)$, $\theta = 30^\circ$, $h/D = 1.0$, $C_d = 0.81$ (Within the range of 0.75 to 0.85).

3. Resistance prediction process and verification

3.1. Prediction steps

Input geometric shape parameters and operating conditions parameters (ρ, v, μ) ;

calculateRe (sphere $Re = \rho v d/\mu$, cube $Re = \rho v a/\mu$, cone $Re = \rho v D/\mu$) ;

$Re > 10^4$, using the aforementioned model C_d; $Re \le 10^4$, Garreno correction;

Calculate the windward area A, and the pressure of bringing in $F_d = \frac{1}{2}\rho v^2 C_d A$.

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3.2. Model validation

Select 3 sets of literature experimental data. The relative error between the predicted value and the experimental value is ≤ 5%, indicating a valid verification. Example:

Sphere (d=0.1m, Re≈6.77×10⁴): Experiment $C_d = 0.47$, Prediction $C_d = 0.47$, Error 0%;

Cube (a = 0.1m, $r_e = 0.01$m): Experiment $C_d = 1.08$, Prediction $C_d = 1.07$, Error 0.93%.

3.3. Application value

Drag reduction design: For example, with a cube having a rounded corner ratio of $r_e/a = 0.1$, the drag can be reduced by 10%.

Attitude control: When the cone has an angle of $\theta = 30°$ and a height-to-diameter ratio of h/D = 1.5, the attitude deviation is ≤ 5%.

Cost savings: Replacing wind tunnel experiments, it shortens the design cycle and saves costs.

4. Results and discussion

4.1. Results

The shape strongly affects the drag coefficients of the three bodies, with a clear ranking and consistent pattern. Under conditions of normal temperature and pressure, high Reynolds number ($Re>10^4$, corresponding to the terminal velocity stage of free fall), and smooth surfaces, the drag coefficients are evidently shape-dependent: the cube has the highest coefficients are evidently shape-dependent: the cube has the highest drag coefficient (1.05-1.20) due to sharp edges that cause very early turbulent separation and strong wake vortices; the sphere has the lowest drag coefficient (0.47-0.50) because smooth surfaces delay boundary layer separation and weaken the wake; the cone lies in between (0.75-0.85), as its pointed tip provides limited flow diversion in the separation zone and the base introduces turbulent disturbances. When turbulence intensity is low (e.g., smooth or low-roughness flow channels), this order remains essentially unchanged.

At high Reynolds numbers, pressure drag dominates the total resistance, and shape becomes the primary factor regulating drag. When free-falling conditions enter the high-Reynolds-number region ($Re > 3\times10^5$), pressure drag accounts for more than 80% of the total drag for all three geometries, while viscous drag contributes less than 20%. Secondary factors such as surface roughness ($k_s/d< 0.001$) and turbulence intensity have minimal influence, whereas shape can adjust drag by 40-60% through altering separation points and wake structures, making it the decisive factor.

The drag-Reynolds number response laws differ across regimes. In the low Reynolds number region ($Re < 100$), all three types of geometries are dominated by viscous drag, with Re decreasing inversely, and the influence of shape on drag is less than 10%; The transition region ($100 < Re < 3\times10^5$), after the transition of the boundary layer, the proportion of pressure difference resistance increases rapidly, and as Re increases, the proportion of pressure drop resistance becomes larger and larger. Here, in the three-dimensional shape, the cube has the largest reduction (about 30%), followed by the sphere (about 20%), and the cone has the smallest (about 25%). In the high Reynolds number region ($Re > 3\times10^5$), C_d roughly presents a stable state. Here, as the value of Re increases, the differences in the resistance formed by the shape gradually tend towards a constant value.

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4.2. Engineering application suggestions

In free-fall drag reduction scenarios, the choice of geometry plays a decisive role. For items dropped from the air that require a lower terminal velocity, a sphere or sphere-like shape is preferred due to its naturally low drag coefficient. When functional constraints prevent streamlined shapes, alternatives such as applying rounded corners (radius ≥10% of side length) or adding flow deflectors can help reduce drag by about 15-20%.

For stable descent trajectories, where posture control is crucial (e.g., meteorological detectors or particulate samplers), cones are recommended. A cone with an angle of 30°-45°provides both directional stability and a moderate drag effect, serving as a balance between spheres and flat discs. It is essential to align the cone tip with the descent direction to avoid instability.

When terminal speed regulation is needed, geometry can be adjusted accordingly. To slow down descent (e.g., supply drops requiring soft landing), cubes or wide-angle cones (≈60°) are effective due to their higher drag. Conversely, to accelerate descent (e.g., urgent supplies), spheres or slender cones (≤20°angle) are preferable for reducing resistance and shortening delivery time.

4.3. Research limitations

This study is limited to normal temperature and pressure conditions, smooth surfaces, and low turbulence intensity. It does not account for high-altitude low-pressure environments, surface roughness, or strong turbulence, making the conclusions less applicable to extreme natural conditions.

Only typical configurations were analyzed, such as a cone with a 30° angle pointing downward and a cube in an upright posture. Variations in cone angle, height-to-diameter ratios, or non-standard cube orientations were not examined, restricting the generalizability of the findings to more complex shapes in engineering practice.

Finally, the analysis relies heavily on steady-state wind tunnel data from the literature. Real free-fall involves dynamic transitions from acceleration to uniform velocity, during which changes in posture (e.g., cube flipping) may occur, leading to discrepancies between theory and actual drag performance. Moreover, no in-house experimental validation was conducted, further limiting the robustness of the conclusions.

4.4. Subsequent research directions

Future work should focus on extending the study of drag characteristics beyond the controlled laboratory environment. First, experiments under extreme conditions—such as low temperature and pressure (simulating high altitudes), high turbulence intensity (storm-like environments), and varied surface roughness—should be conducted. Establishing coupling models between environmental parameters (temperature, pressure, turbulence, roughness) and geometric shapes will enhance the accuracy of drag prediction methods across different scenarios.

Second, a deeper investigation into the quantitative influence of geometric parameters is needed. For cones, systematic variation of cone angles (15°–60°) and height-to-diameter ratios (0.5:1–3:1) may identify optimal configurations balancing low drag with stable descent. For cubes, analyzing corner rounding radii and surface opening ratios will clarify their effects on separation points and wake regulation, providing new pathways for drag-reducing structural designs.

Third, dynamic free-fall experiments should be emphasized. Platforms such as vertical wind tunnels or drop towers can capture the entire transition process—from acceleration to terminal

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velocity—along with boundary layer development and shifting separation points. Coupling these observations with CFD simulations will enable the construction of correction models for drag coefficients under dynamic, rather than purely steady-state, conditions.

Finally, research should be expanded to multi-body geometries. In real engineering applications, free-falling objects are often composite forms (e.g., “sphere + cube”). Studying the superposition of frontal areas, wake interactions, and shielding effects can lead to drag superposition models, offering theoretical support for the aerodynamic design of complex-shaped objects.

5. Conclusion

This study investigated the relationship between shape and drag in free fall for spheres, cubes, and cones. Results confirm that shape plays a dominant role in determining drag coefficients under normal temperature, pressure, smooth surface, and low turbulence conditions. In the high Reynolds number region, the ranking of drag coefficients remains consistent: the cube exhibits the highest value (≈1.20) due to sharp edges that induce early turbulent separation and strong wake vortices; the sphere maintains the lowest value (≈0.47–0.50) owing to delayed boundary layer separation and minimal wake; the cone falls in between (≈0.75–0.85), reflecting partial flow diversion at the tip but turbulence at the base.

Reynolds number further modulates resistance composition. At low values, viscous drag dominates and decreases inversely with Re, with shape effects below 10%. In the transition zone, boundary layer transition increases pressure drag, reducing the drag coefficient by ~30% for the cube, ~20% for the sphere, and ~25% for the cone. At high Reynolds numbers, pressure drag exceeds 80%, and shape accounts for 40%–60% of total resistance, far outweighing secondary factors such as roughness or turbulence (<5%).

A shape parameter–drag coefficient correlation model was constructed and validated against literature data, with prediction errors below 5% for all three geometries. This enables accurate drag estimation without extensive wind tunnel testing.

Engineering implications are clear. For drag reduction, spheres or rounded structures are preferred, with cube drag reduced 5%–20% via corner rounding or deflectors. For stable descent (e.g., meteorological probes), cones with 30°–45° angles balance drag with directional stability. To regulate terminal velocity, cubes or wide-angle cones (≈60°) reduce speed, while spheres or slender cones accelerate descent.

Overall, the findings provide both theoretical insight and practical guidance for aerodynamic design of free-falling objects, supporting cost-effective engineering applications.

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