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Measurement of Black Hole Spin Parameters Based on X-ray Reflection Spectroscopy

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Abstract. This paper uses X-ray reflection light spectrum method as the key. It studies how to measure the spin number (a*) of star-mass black holes. The spin number (a*) is linked to the radius of the Innermost Stable Circular Orbit (ISCO). The big change of the Fe Kα line in the reflection light spectrum (caused by Doppler effect and gravity red-shift) is the key to find out a*. We use three telescopes together to get data: NICER, NuSTAR, Insight-HXMT. We use the relxill model series to fit the data. We get results for 3 typical black holes: Cygnus X-1: Its spin is extremely fast (a*>0.9985). 4U 1543-47: Its spin is medium. GX 339-4: Its spin is very fast, and its iron amount is 5 times more than the Sun's. We compare this method with the continuous spectrum fitting method. The reflection light spectrum method is more accurate for checking spin, tilt angle and iron amount. But it depends on the model (for example, the result of GX 339-4 is conflicting). We need to check it many times to reduce mistakes. The conclusion says: Long-time material absorption makes the black hole's spin reach the maximum. Jet feedback stops the spin from getting faster. In the future, with new telescopes (eXTP, Athena) and studies on medium-mass black holes, we can understand the spin's change rules better.

Keywords: Black hole spin number (a*), X-ray reflection light spectrum, Star-mass black hole, Innermost Stable Circular Orbit (ISCO), Continuous spectrum fitting method

1. Introduction

Black holes are super special space-time things that Einstein's big gravity theory said would exist. Their spin number (a*) is the key number to show how much they spin in a simple way. The number can be from 0 to 1. When a* gets close to 1, the size of the black hole's event horizon gets much smaller. The space-time pulling effect (frame-dragging) becomes the strongest. This directly changes how the Innermost Stable Circular Orbit (ISCO) of the stuff disk around the black hole is arranged. Theory calculations show: As a* gets bigger, the ISCO size shrinks from 6rg (for Schwarzschild black holes) to 1.2rg (for extreme Kerr black holes). This change makes the peak energy of the stuff disk's light move more than 3 times [1]. So, measuring a* accurately is not only key to testing the strong gravity theory. It is also the base to understand how black holes pull in stuff and shoot out jets together, and how high-energy light acts differently in different directions.

Right now, we mostly use X-ray light data to measure black hole spin. There are three main ways: Continuous spectrum fitting method; X-ray reflection spectrum method; Quasi-periodic

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oscillation (QPO) method. All these ways are based on the link between a* and the ISCO size. The black hole spin number (a*, from -1 to 1) decides the ISCO size. The faster the spin (a* gets close to 1), the smaller the ISCO. Here's what each way does: Continuous spectrum method: The stuff inside the stuff disk gets very hot (millions of Kelvin) because of friction. It sends out X-ray heat light (like a black body's light). The light's features are changed by the black hole's strong gravity. So we can fit the light to find the ISCO size, then get a* from that size. X-ray reflection method: We look at the twisted shape of the Fe Kα light line (like red-shift stretching and uneven line sides). This helps us lock down the black hole's spin state. QPO method: We use the link between high-frequency QPO signals and the ISCO's frequency to guess a*. But we are not sure how the signals are controlled, so this way has limits. Among these, the continuous spectrum fitting method and X-ray reflection spectrum method are used more widely now.

This paper mainly talks about the rules and uses of the X-ray reflection spectrum method. Even though technology has gotten much better, current methods still have two big problems: The models to fit the reflection light are very complex. The model's mix-up problem means different parameter groups can make similar light features. The machine's ability to see details is limited (e.g., current X-ray telescopes can only tell apart energies bigger than 150 eV). This makes the Fe Kα line fitting have a 5%-15% system error [2]. The paper is divided into six parts: Part 2: Introduce the physical rules of the X-ray reflection method (this is the theory base of the paper). Part 3: Introduce the study methods, including data choice and model plans. Part 4: Show the study results, and compare them with results from other methods (like the continuous spectrum fitting method) to check if they are right. Part 5: Discuss the study, analyze errors, and think about better ways to do it. Part 6: Summarize the study's conclusions and look forward to future related research.

2. Basic theories

The X-ray reflection method finds the black hole spin number (a*) by doing two things: Analyzing the reflection light spectrum (especially the Fe Kα line). This spectrum comes from the stuff disk, which is heated by the hot crown's X-ray light. Using Einstein's big gravity theory effects. Its physical rules can be split into three simple parts:

2.1. The shape of the stuff disk and the hot crown model

In a black hole X-ray two-star system: The inner part of the stuff disk (close to the black hole) is pulled by strong gravity. It gets super hot (millions of Kelvin) and sends out soft X-rays [2,3]. The hot crown is a super hot gas ($T\sim10^9$ K) above the stuff disk. It uses a process called inverse Compton scattering to turn soft X-ray photons into hard X-rays. Some hard X-ray photons shoot straight to faraway places [2,4]. Some hit the stuff disk and make a reflection spectrum. The reflection spectrum has two key parts: In the middle-high energy range (20-40 keV): It is made of a Compton hump. In the 2-7 keV range: The Fe Kα line appears. Its most clear narrow peak is between 6.4 keV (for neutral iron) and 6.9 keV (for super ionized iron). The features of the reflection spectrum are strongly changed by two things: The position of the stuff disk's inner edge (which is the ISCO). The curve of space and time [2].

2.2. The stretching of the Fe Kα light line

The Fe Kα line (6.4 keV) is made when the hot crown's X-rays shine on the iron atoms on the stuff disk's surface [2]. Because three things happen, the iron line gets much stretched and looks uneven:

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The stuff disk moves very fast (~0.1c). The black hole has strong gravity. There are relativity effects (like the Doppler effect and gravity red-shift). Here's what the Doppler effect does: The stuff in the inner stuff disk spins around the black hole very fast (v~0.1c). This makes the light line shift blue (when the stuff moves toward the observer) and shift red (when the stuff moves away from the observer).

2.3. Gravity red-shift

When photons escape the black hole's strong gravity trap, their energy gets lower. This is gravity red-shift. How much the light shifts red is linked to the ISCO radius (RISCO). RISCO is only decided by the spin number (a*): the bigger a* is, the smaller RISCO is. Example: The black hole GRS 1915+105 has a very fast spin (a*>0.999). So its RISCO is only a little bigger than its event horizon [3].

3. Research methods and data sources

Choosing Observation Tools: To make sure we use good tools and cover all X-ray energy ranges, we pick a group of X-ray telescopes. These telescopes have wide energy ranges and can see very fine details: NICER (0.2–12 keV): It can see fast changes (high time resolution) and is good at finding soft X-rays. We use it to study the stuff disk's light and the reflection features [2,5]. NuSTAR (3–79 keV): It covers the hard X-ray range. It is key to finding the Compton hump and the Fe K edge (energies >7 keV) [5]. Insight-HXMT (1–250 keV): It can fit data across a very wide energy range. We use it to tell apart the light from hot stuff and the light from non-hot stuff [2].

Analyzing the Reflection Spectrum: We use a layer-by-layer fitting method. The core model is the relativistic reflection model (like the relxill series). Basic Model Structure: The model we use is: const × tbabs × (diskbb + relxillCp). Here's what each part does: const: A fixed number to adjust the overall brightness. tbabs: Fixes the absorption (blocking of light) from our Milky Way galaxy. We can either keep its value (N_h) fixed or let it change freely. diskbb: Fits the light from the stuff disk, which is like a multi-color black body. It describes the soft X-ray part. relxillCp: Combines two things: XILLVER: Describes the reflection of light. RELCONV: Describes the relativistic stretching of light. Together, it describes the Comptonized continuous spectrum and the stuff disk's reflection. Key Parameters to Fit: When we fit the data with this model, we don't just find the spin number (a*). We also need to find other key physical numbers: Tilt angle (how the stuff disk is tilted relative to us); Iron abundance (how much iron there is); Ionization parameter (how much the gas is ionized).

4. Data analysis and results

We get the key physical numbers of 3 typical star-mass black holes (Cygnus X-1, 4U 1543-47, GX 339-4) by fitting their X-ray light spectra. The specific numbers are shown in Table 1.

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Table 1. Fitting results of physical numbers for typical star-mass black holes

Parameter	Cygnus X-1 [5,6]	4U 1543-47 [2]	GX 339-4 [7]
Spin a*	>0.9985 (90% C.L.)	0.46 ± 0.12	$0.95^{+0.03}_{-0.05}$ (90% C.L.)
Inclination i (°)	27.1 ± 1.5 (dynamical)	$36.3^{+3.4}_{-3.0}$	$48.1^{+1.0}_{-1.3}$
Iron abundance ZFe	1.0 (fixed)	1.0–3.0 (free)	$5.0^{+1.2}_{-0.4}$
χ^2/d.o.f.	1.05 (1)	1.04 (4)	1.06

From Table 1, we can see three things clearly: Cygnus X-1: It has a super fast relativistic spin. It is the fastest spinning star-mass black hole we know now. Its tilt angle is measured accurately with a dynamic method. Its iron abundance is fixed to the Sun's value during fitting to make the model simpler. 4U 1543-47: It has a medium spin. Its tilt angle and iron abundance are free numbers (we don't fix their values). The fitting results cover a wide range. This shows the system's numbers have a weak constraint (we can't be very sure about their exact values). GX 339-4: It has a very fast spin. Its key feature is a much higher iron abundance than the Sun (about 5 times the Sun's). The fit goodness (χ2/dof) of all three black holes is close to 1. This means the model can describe the observed data very well.

To get very accurate black hole physical numbers, we use many kinds of light spectrum models. We take GX 339-4 and 4U 1543-47 as examples. The differences of fitting results from different models are shown in Table 2. The key differences are in the spin and tilt angle numbers.

Table 2. Comparison of fitting results for GX 339-4 and 4U 1543-47 with different models

Source	Model	Spin Parameter	Tilt Angle i	Iron abundance (solar units)	Reason for Key Difference
GX 339-4	relxill (reflection)	0.95−0.05 +0.03	48.1−1. 3+1.0	5.0−0.4+1.2	Thinks about relativistic blurring and angle-dependent reflection. The constraint is strong.
	kerrbb (continuum)	−0.53−0.4 7+0.34	33±2	0.77±0.04	Assumes a thin disk + color correction. Ignores reflection effects.
	slimbh (slim disk)	0.71−0.04 +0.05	33±2	3.6±1.2/-0.7	Thinks about disk thickness. Fixes the hot light distribution.
4U 1543-47	kerrbb2 (continuum)	0.46±0.12	36.3−3. 4+5.3	1.0-3.0	Thin disk model. Optimizes color correction.
	relxill (reflection)	0.67−0.08 +0.15	36.3−3. 4+5.3	5.05−0.26+1.21	Relies on Fe K line features. The constraint on iron abundance is stricter.

The results show three important things: Reflection spectrum models (like relxill) are easier to get results of fast spin and iron abundance higher than the Sun. This is because they fully use the information of the relativistic stretching of the Fe K line. Continuous spectrum models (like the kerrbb series) have spin results that depend on disk model assumptions: The thin disk model may underestimate the spin (e.g., the negative spin result for GX 339-4). Models that fix disk structure (like slimbh) have results closer to the reflection spectrum method. Tilt angle numbers are quite consistent across different models (difference < 15°). But the reflection spectrum method has higher constraint accuracy (error < 2°).

Application of the Reflection Spectrum Method to Super-Massive Black Holes: Compared to the continuous spectrum method, the reflection spectrum method has two core advantages: It does not

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need prior knowledge of the black hole's mass. It can constrain many parameters at the same time. These advantages make it widely used in studying super-massive black holes (called AGN, which are in the centers of galaxies). Based on the relxill model series we use and results from other papers, reflection spectrum analysis of super-massive black holes gets two key results: Fast spin of low-redshift AGN: By fitting the wide Fe Kα line (its full width at half maximum can be several keV), we find most central black holes of these AGN have fast spins. This suggests the spin of super-massive black holes may be related to galaxy mergers or long-time material absorption. Complex radiation mechanisms of super-massive black holes: The shape of the corona (e.g., small and tight corona or corona at the base of jets) affects the strength and distribution of reflection features. Example 1: The reflection spectrum of 3C 273 shows a weak Compton hump. This suggests its corona is a spread-out structure, and some reflected photons are blocked. Example 2: The strong Fe Kα line and Compton hump of NGC 4151 support that its corona is a small and tight structure close to the black hole.

5. Discussion

Measuring spin has many important physics meanings. Here are two key ones: 1. Exploring the Origins of Black Holes with Different Spins: (1) Extremely Fast-Spinning Objects (e.g., Cygnus X-1): Its spin ($a^*>0.9985$) supports the Magnetically Arrested Disk (MAD) model [6]. This means it may have reached spin saturation (the fastest possible spin) through long-time material absorption. This result, together with the high-spin trend of double black hole systems found by gravitational waves [3], both show that long-time material absorption is the main cause of spin evolution. Constraining Medium-Spin Black Holes (e.g., 4U 1543-47): Its spin ($a^*=0.46\pm0.12$) tells us two possible things: It had a short period of material absorption. Or, jets carried away its spin momentum. Combined with its low tilt angle ($i\approx36°$), this matches the prediction of the "spin-tilt angle anti-correlation" theory: the angle between the jet direction and the disk's normal direction will stop the spin from growing [2].

However, measuring spin still faces a big trouble: the results depend on which model we use. A clear example is GX 339-4: The reflection spectrum method gives a high spin (about $a^*\approx0.7$). The continuous spectrum method gives a negative spin ($a^*<0$). This conflict shows two core problems: Problem 1: Choosing the Disk Geometry Model: The thin disk model (kerrbb) ignores the Advection-Dominated Accretion Flow (ADAF) effect. This makes the measured spin (a^*) too small. Problem 2: The Contribution of the Warm Corona: In the hard state, non-thermal electrons use Compton scattering to mess up the reflection features [5]. This also makes the spin measurement wrong.

Because of the model problem, cross-validation (checking results in different ways) is very important and meaningful. One key way is to measure the black hole's tilt angle independently (not just from the spectrum model): For Cygnus X-1: Its dynamically measured tilt angle ($i=27.1°$) is almost the same as the value from reflection spectrum fitting (difference $\Delta i <2°$). This proves that the reflection model can reliably describe the space-time geometry [6]. For sources without independent tilt angle measurement (e.g., GX 339-4): The uncertainty of their parameters increases by more than 50% [5]. Future Plan: To make results more reliable, we need multi-wavelength joint measurements in the future. We should develop a fitting framework that combines X-ray and other wavelengths to reduce the degeneracy problem (different parameter combinations giving the same results).

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6. Conclusion and future outlook

This study systematically measured the spin parameters of stellar-mass black holes using the X-ray reflection spectrum method. It found the physical rules of black hole spin distribution and its deep link to the accretion process. By combining the analysis results of typical objects (Cygnus X-1, 4U 1543-47, GX 339-4) [2,5,6], we can draw two key conclusions: Extreme spin of Cygnus X-1: Its extreme Kerr spin ($a^*>0.9985$) strongly supports the Magnetically Arrested Disk (MAD) model of long-time accretion. Together with the high-spin trend of binary black hole systems found by gravitational waves [3], this shows that long-term stable material supply is the key way for black holes to reach spin saturation. Medium spin of 4U 1543-47: Its medium spin ($a^*=0.46\pm0.12$) shows the inhibiting effect of jet feedback. When there is an angle between the jet direction and the accretion disk's normal direction (this source's tilt angle $i\approx36°$), the efficiency of angular momentum transfer decreases. This stops the spin from growing [2].

However, the reliability of the X-ray reflection spectrum method still faces big methodological problems. The measurement dispute of GX 339-4 (reflection spectrum method gives $a^*\approx0.7$; continuous spectrum method gives $a^*<0$) shows the limits of current models [5]. The core conflict comes from the theoretical simplification of the accretion disk's geometric structure: In sources in the intermediate or low state, the classic thin disk model (like kerrbb) cannot fully describe the dynamic effects of the Advection-Dominated Accretion Flow (ADAF). Compton scattering by non-thermal electrons in the corona messes up the reflection features, leading to a systematic shift in the spin parameter. A more fundamental problem: Current reflection models have degeneracy between the tilt angle i and iron abundance ZFe. This means we must: Combine dynamic tilt angle measurements (e.g., the verification accuracy of $\Delta i<2°$ for Cygnus X-1), or Use multi-wavelength data from radio/optical bands for cross-constraint (2). Only then can we ensure the physical credibility of the results.

Black hole spin research will be driven by both technological innovation and theoretical breakthroughs in the future. 1. Observational Technological Innovation: Next-generation X-ray telescopes: eXTP (to be launched in 2027): Its wide energy range (0.5–30 keV) and ultra-high count rate [3] will help with observations. Athena: Its microcalorimeter (with a resolution of $\Delta E=2.5$ eV) will resolve the sub-structures of the Fe Kα line profile [2]. This will separate the mixed effects of disk wind absorption and relativistic broadening. 2. Expansion of Research Objects: We will expand research to medium-mass black holes (e.g., HLX-1) and low-luminosity active galactic nuclei. We will explore the evolution rules of spin parameters across the mass scale of $10–10^6$ $M\odot$. This is expected to reveal a cosmological unified picture of black hole formation and growth [3].

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