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Cross-Cultural Scale Entropy Quantification Research: An Interdisciplinary Exploration of Mathematics and World Music

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Abstract. This study used information entropy as the core mathematical tool and focused on six typical global cultural scales: the Chinese pentatonic scale, the Indian 22-shruti scale, the Arabic 17 tone and other rhythmic scales, the European natural major scale, the Japanese mode scale, and the Indian pentatonic scale. By quantifying the uncertainty of interval distribution, the mathematical laws behind different cultural music aesthetics have been revealed. This study first constructed a frequency system for each scale and conducted interval statistics. Then, the Shannon entropy formula was used to calculate the entropy value, and the entire process was automatically calculated through Python programming. Finally, the deep logic of entropy value differences was explained in combination with cultural background. The results indicate that entropy is highly correlated with the number of core notes and the diversity of intervals in the scale: low entropy scales (2.0-2.8 bits) correspond to the cultural aesthetics of regular harmony, while high entropy scales (3.1-3.7 bits) relate to improvisation and diverse music styles, providing a quantitative analysis framework for cross-cultural music research.

Keywords: Cross-cultural music, Scale, Information entropy, Mathematical modeling, Python

1. Introduction

Music is a universal human language that contains profound mathematical logic and cultural genes. Different cultures have their unique scale systems, such as the simple harmony of the the Chinese pentatonic scale, the small variations of the Indian 22- shruti scale, and the features of the microtones of the Arabic 17 note scale. These differences can be revealed by the mathematical structure of scales.

In interdisciplinary research between music and mathematics, harmonic quantification is one of the directions. However, most existing research has focused on a single tuning system, and they lack systematic comparisons at the global multicultural scale. Information entropy is a pivotal mathematical tool for measuring uncertainty and diversity, which can provide a new perspective for quantifying the interval distribution characteristics of scales. Thus, the theoretical significance of this research lies in constructing a quantitative analysis framework for cross-cultural scales,

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introducing information entropy into world music research, revealing the inherent connection between cultural aesthetics and mathematical structure, and enrich the interdisciplinary research results of musical acoustics and cultural anthropology.

The practical significance of this research is to provide data support for music creation and cross-cultural music integration, and provide quantitative tools for music education to intuitively present the differences between different cultural scales.

Six representative scales covering Asia, Europe, and the Americas were selected to ensure cultural diversity and differences in the scale structure: the Chinese pentatonic scale consists of five core notes (gong, shang, jiao, zhi, yu); European natural major scale consists of 7 core notes (C, D, E, F, G, A, B); Japanese scale consists of 5 core notes (C, D, F, G, A); Indian pentatonic scale consists of 5 core notes (C, D, E, G, B); Indian 22-shruti scale consists of 22 core notes with many micro-tones; Arabic 17 tone Isotonic Scale consists of 17 core notes characterized by micro-tones.

2. Literature review

2.1. Interdisciplinary research on the correlation between music and mathematics

According to the subsequent research on the mathematics of music by José Rodríguez Alvira in 2021, the connection between music and mathematics can be traced back to ancient Greece, when Pythagoras discovered through experiments with a monochord that simple integer ratios of string lengths corresponded to harmonious intervals, and proposed that "all is number" [1]. In 1739, Euler's "Tentamen Novae Theoriae Musicae" established a quantitative analysis system based on the principle of harmonics [2]. In modern times, the ISO standard of A4 = 440Hz proposed in 1975 has provided a unified standard for the frequency quantification of global music [3].

In the research on the quantification of consonance, Gibson proposed that beat frequencies between 10-50Hz would cause auditory discomfort [4]. In 1965, Plomp and Levelt confirmed through experiments that the consonance of intervals is closely related to critical bandwidth. When the frequency difference between two tones exceeds the critical bandwidth, auditory perception tends to be more harmonious [5].

These studies have provided important bases for the mathematical description of interval characteristics, but they have mostly focused on the Western tuning system, with inadequate attention paid to the scales of non-Western cultures.

2.2. Research on the structure and aesthetics of cross-cultural scales

The diverse cultures around the world have enabled people to create rich-scale systems. In 2021, Sandhya Mahesh compared the frequency ratio of the Indian Lagrange mode with that of the Western major scale and found that the feature of the micro-intervals in the Indian 22-shruti scale stems from its unique improvisation tradition [6]. When considering the Chinese pentatonic scale, scholars generally believe that its structure lacks F and B. This phenomenon might be attributed to the Chinese culture's pursuit of "harmony and unity", and the frequency ratios of its intervals (major second, perfect fourth, perfect fifth) all present simple integer relationships. This statement was concluded in a paper written by Yang Yinliu in 1958 [7]. The research on the Arabic 17-tone isometric scale conducted by Raja Yusof in 2025, and the Japanese modal scale conducted by Kevin Fellezs mainly in 2020 mainly focus on functional and cultural expression aspects. The micro-intervals of the former one reflect the emotional tension of Maqāmāt music, while the augmented fourth feature of the latter one shapes the ethereal atmosphere of Japanese music [8,9].

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However, existing cross-cultural scale research has obvious limitations. Firstly, it is a mostly qualitative description, but it lacks a unified mathematical quantification standard, making it difficult to conduct objective comparisons. Secondly, it lacks in-depth exploration of the inherent connection between scale structure and cultural aesthetics, so it fails to reveal the essential reason why different cultures choose different scale structures.

2.3. Application status of information entropy in music research

Information entropy, as a core tool for measuring the uncertainty of a system, has gradually been applied to the field of music research. In 2014, Remi de Fleuriana, Tim Blackwellb, Oded Ben-Talc, Daniel Müllensiefen applied entropy values to rhythm complexity analysis [10]. In 1983, Brian C. J. Moore and Brian R. Glasberg used entropy values to describe the dispersion of frequency distribution in research on auditory filter bandwidth [11]. However, in the existing research, the application scenarios of entropy value are relatively single, either focusing on a single dimension, or being restricted to the Western music system. So far, few researches have systematically applied information entropy to the quantification of cross-cultural scales, thus failing to fully give play to the advantages of entropy in measuring diversity.

In conclusion, existing research has confirmed the deep connection between music and mathematics. However, these studies generally lack a quantifiable framework for cross-cultural comparison. This study combines information entropy with Python programming, which could fill this research gap and provide a new path for quantitative research on cross-cultural music.

3. Methodology and mathematical models

3.1. Choosing the method to calculate scale frequencies

This paper will use the 12-tone equal temperament (12-TET) as the core of scale frequency standardization. It takes A4=440Hz as the reference pitch, and frequencies of all scale degrees are calculated using a geometric sequence. The frequency of the notes can be calculated by the 12-TET formula, written as:

$$f = 440 \times 2^{\frac{k-49}{12}} \tag{1}$$

Where "k" is the nth note on the piano, and "f" is the frequency of the note.

Although the frequency of all notes occupy a large range, this paper only focuses on one octave (C4-C5), which only focuses on the notes with frequencies between 261.62557Hz and 523.25154Hz. After the correction based on the 12-TET law reference frequency, the frequencies of the notes from the six different cultural scales could be calculated. the Chinese pentatonic scale has five core notes: Gong(C4), Shang(D4), Jiao(E4), Zhi(G4), Yu(A4); Japanese scale has five core notes: C4, D4, F4, G4, A4. Indian pentatonic scale has five core notes: C4, D4, E4, G4, B4; European natural major scale has seven core notes: C4, D4, E4, F4, G4, A4, B4; Arabic 17 tone Isotonic Scale has 17 core notes: C4 is the root note, the frequency ratio with the next note is $2^{1/17}$; Indian 22-shruti scale has 22 core notes: C4 is the root note, the frequency ratio with the next note is $2^{1/22}$.

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3.2. Core mathematical tool: shannon entropy

Shannon entropy is a core indicator for measuring system uncertainty in information theory. The formula can be written as:

$$H = -\sum_{i=1}^{k} P(i)log_2P(i) \tag{2}$$

Where:
"k" is the number of different interval types in the scale

$$P\left(i\right) = \frac{N(i)}{N_{total}} \tag{3}$$

"P(i)" is the occurrence probability of the i^{th} interval type, where
"N(i)" is the occurrence frequency of the i^{th} interval type
" N_{total} " is the total number of all possible intervals in the scale, which can be calculated by the combination formula:

$$N_{total} = C_2^n = \frac{n(n-1)}{2} \tag{4}$$

Where "n" is the number of core notes in the scale.

3.3. The standardization of scale frequencies

Unify the fundamental tone to the standard note C4, and narrow down the calculation range of all scales to the octave range of C4 to C5 to ensure the consistency of the frequency range and avoid the influence of pitch interval differences on entropy value calculation.

For non-equal temperament scales (the Chinese pentatonic scale, European natural major scale, Japanese mode scale, Indian pentatonic scale), the frequency of each note is determined based on the reference frequency of the twelve-tone equal scale.

For equally tempered scales (Indian 22-shruti scale, Arabic 17-tone equal temperament scale), the frequencies of different sounds are derived based on the common ratio of the geometric sequence. The frequency ratio of the Indian scale is $r = 2^{1/22}$, and that of the Arabic scale is $r = 2^{1/17}$.

3.4. Interval matching standards

According to the 12-TET, the precise frequency ratio and matching threshold (± 0.03) of the 12 core intervals are determined to ensure the accuracy of interval discrimination. The standard frequency ratio can be calculated by dividing the frequency of one note by the frequency of another note in the interval.

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Table 1. The matching threshold range of the main interval types

Number	Interval type	Standard Frequency Ratio	Matching Threshold Range
1	Minor second	$2^{1/12} = 1.059463094$	1.029463-1.089463
2	Major second	$2^{2/12} = 1.12462048$	1.094620-1.154620
3	Minor third	$2^{3/12} = 1.189207115$	1.159207-1.219207
4	Major third	$2^{4/12} = 1.259921050$	1.229921-1.289921
5	Perfect fourth	$2^{5/12} = 1.334839854$	1.304840-1.364840
6	Augmented fourth	$2^{6/12} = 1.414213562$	1.384214-1.444214
7	Perfect fifth	$2^{7/12} = 1.498307077$	1.468307-1.528307
8	Minor sixth	$2^{8/12} = 1.587401052$	1.557401-1.617401
9	Major sixth	$2^{9/12} = 1.681792831$	1.651793-1.711793
10	Minor seventh	$2^{10/12} = 1.781797436$	1.751797-1.811797
11	Major seventh	$2^{11/12} = 1.887748625$	1.857749-1.917749
12	Perfect octave	$2^{12/12} = 2.000000000$	1.970000-2.030000

Table 1 establishes a unified standard for identifying 12 core interval types based on the 12-TET system. By defining the standard frequency ratio and $a \pm 0.03$ matching threshold for each interval, it solves the problem of inconsistent interval discrimination across different cultural scales. This standardized threshold ensures that intervals can be objectively compared.

3.5. Achieving automated computing

Use the Python language to assist in computing, achieving full automation of the entire process including frequency generation, sound pair traversal, interval matching, and entropy value

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calculation.

4. Research results and analysis

4.1. Frequency sequences of six scales

Note that the standard note C4 is the 40th note on the piano, using the formula [1], the frequency of the standard note C4 can be calculated:

$$f_{C4} = 440 \times 2^{\frac{40-49}{12}} \approx 261.62557Hz \tag{5}$$

Using the standard note, the frequencies of every single note can be deduced and calculated. The results can be shown in Table 2.

Table 2. The frequencies of the notes of different scales

Scale	No. of core notes	Frequencies of the notes (Hz)
Chinese pentatonic scale	5	Gong(C4): 261.62557 Shang(D4): 293.66477 Jiao(E4): 329.62756 Zhi(G4): 391.99544 Yu(A4): 440.00000
Japanese scale	5	C4: 261.62557 D4: 293.66477 F4: 349.22823 G4: 391.99544 A4: 440.00000
Indian pentatonic scale	5	C4: 261.62557 D4: 293.66477 E4: 329.62756 G4: 391.99544 B4: 493.88330
European natural major scale	7	C4: 261.62557 D4: 293.66477 E4: 329.62756 F4: 349.22823 G4: 391.99544 A4: 440.00000 B4: 493.88330
Arabic 17 tone Isotonic Scale	17	No.1: 261.62557 ..... No.5: $No.1 \times 2^{4/17} = 307.97167$ ..... No.9: $No.1 \times 2^{8/17} = 362.52783$ ..... No.13: $No.1 \times 2^{12/17} = 426.74844$ ..... No.17: $No.1 \times 2^{16/17} = 502.34551$

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Table 2. (continued)

Indian 22-shruti scale	22	No.1: 261.62557 ..... No.6: No.1$\times 2^{5/22}$= 306.26410 ..... No.12: No.1$\times 2^{11/22}$= 369.99442 ..... No.17: No.1$\times 2^{16/22}$= 433.12284 ..... No.22: No.1$\times 2^{21/22}$= 507.02222

This table presents the precise frequency values of six representative cross-cultural scales within the unified range of C4-C5. Data shows that the micro-scale has more core notes, which indicates that they have higher interval diversity and entropy values in subsequent calculations.

4.2. Number of combinations of notes in each scale

After using the formula [3], the number of combinations of all notes in each scale can be calculated. Each combination of two notes represent a different interval. The results is shown in Table 3.

Table 3. The number of combinations of the notes of different scales

Scale	No. of core notes	Number of combinations of notes
Chinese pentatonic scale	5	$C_2^5 = \frac{5 \times (5-1)}{2} = 10$
Japanese scale	5	$C_2^5 = \frac{5 \times (5-1)}{2} = 10$
Indian pentatonic scale	5	$C_2^5 = \frac{5 \times (5-1)}{2} = 10$
European natural major scale	7	$C_2^7 = \frac{7 \times (7-1)}{2} = 21$
Arabic 17 tone Isotonic Scale	17	$C_2^{17} = \frac{17 \times (17-1)}{2} = 136$
Indian 22-shruti scale	22	$C_2^{22} = \frac{22 \times (22-1)}{2} = 231$

This table calculates the total number of possible two-note interval combinations for each scale using the combination formula. The results show a positive correlation between the number of core

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notes and the number of interval combinations: scales with 5 core notes only have 10 combinations, while the Indian 22-shruti scale with 22 core notes has up to 231 combinations. This difference directly determines the sample size of interval statistics for each scale: more combinations mean richer interval data.

4.3. Frequency ratios of intervals of each scale & interval matching

The frequency ratio of an interval can be obtained by dividing the frequency of one sound by the frequency of another sound. By using the results in the Table(1) and Table(2), the frequency ratio of each interval that can be presented in each scale can be calculated. After comparing the standard frequency ratios of the notes with the matching threshold range, the identity of the interval produced by each two notes can be determined. The result is shown in the Table 4.

Table 4. The number of intervals presented in different scales

Scale	Core notes	No. of intervals presented
Chinese pentatonic scale	Gong(C4), Shang(D4), Jiao(E4), Zhi(G4), Yu(A4)	Major second: 3 Minor third: 1 Major third: 1 Perfect fourth: 2 Perfect fifth: 1 Major sixth: 1
Japanese scale	C4, D4, F4, G4, A4	Major second: 3 Minor third: 1 Major third: 1 Perfect fourth: 2 Perfect fifth: 1 Major sixth: 1
Indian pentatonic scale	C4, D4, E4, G4, B4	Major second: 2 Minor third: 1 Major third: 2 Perfect fourth: 1 Perfect fifth: 2 Major sixth: 1 Major seventh: 1
European natural major scale	C4, D4, E4, F4, G4, A4, B4	Minor second: 1 Major second: 5 Minor third: 2 Major third: 3 Perfect fourth: 3 Augmented fourth: 1 Perfect fifth: 3 Major sixth: 2 Major seventh: 1

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Table 4. (continued)

Arabic 17 tone Isotonic Scale	No.1~No.17	Minor second: 31 Major second: 14 Minor third: 13 Major third: 11 Perfect fourth: 10 Augmented fourth: 17 Perfect fifth: 7 Minor sixth: 6 Major sixth: 4 Minor seventh: 3 Major seventh: 0 Other: 20
Indian 22-shruti scale	No.1~No.22	Minor second: 41 Major second: 37 Minor third: 33 Major third: 29 Perfect fourth: 13 Augmented fourth: 11 Perfect fifth: 9 Minor sixth: 7 Major sixth: 11 Minor seventh: 4 Major seventh: 2 Other: 34

This table further summarizes the specific quantity of each interval type in six scales on the basis of Table 4. It clearly shows the interval structure differences corresponding to cultural aesthetics: low-entropy scales focus on a small number of consonant intervals, with no dissonant intervals such as minor second; the European natural major scale balances consonant and dissonant intervals, reflecting the "balance and conflict" characteristics of Western classical music; high-entropy microtone scales have a large number of minor seconds and augmented fourths, and a high proportion of "other" micro-intervals, which provides acoustic support for the improvisational and expressive characteristics of Arabic and Indian music.

4.4. Shannon entropy value of each scale

4.4.1. Chinese pentatonic scale

First, use the formula [3] to calculate the occurrence probability of each interval. Second, use the formula [2] to calculate the Shannon entropy of Chinese pentatonic scale.

$$H_{Chinese\ pentatonic\ scale} = H_1 = \ -\sum_{i=1}^{6} P(i) log_2 P(i) \approx 2.314 \text{ bits} \tag{6}$$

4.4.2. Japanese scale

First, use the formula [3] to calculate the occurrence probability of each interval. Second, use the formula [2] to calculate the Shannon entropy of Chinese pentatonic scale.

$$H_{Japanese\ scale} = H_2 = \ -\sum_{i=1}^{6} P(i) log_2 P(i) \approx 2.314 \text{ bits} \tag{7}$$

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4.4.3. Indian pentatonic scale

First, use the formula [3] to calculate the occurrence probability of each interval.

Second, use the formula [2] to calculate the Shannon entropy of Chinese pentatonic scale.

$$H_{Indian\ pentatonic\ scale} = H_3 = -\sum_{i=1}^{7} P(i)log_2P(i) \approx 2.722 \text{ bits} \tag{8}$$

4.4.4. European natural major scale

First, use the formula [3] to calculate the occurrence probability of each interval. Second, use the formula [2] to calculate the Shannon entropy of Chinese pentatonic scale.

$$H_{European\ natural\ major\ scale} = H_4 = -\sum_{i=1}^{9} P(i)log_2P(i) = \approx 2.970 \text{ bits} \tag{9}$$

4.4.5. Arabic 17 tone isotonic scale

First, use the formula [3] to calculate the occurrence probability of each interval. Second, use the formula [2] to calculate the Shannon entropy of Chinese pentatonic scale.

$$H_{Arabic\ 17\ tone\ Isotonic\ Scale} = H_5 = -\sum_{i=1}^{11} P(i)log_2P(i) \approx 3.190 \text{ bits} \tag{10}$$

4.4.6. Indian 22-shruti scale

First, use the formula [3] to calculate the occurrence probability of each interval.

Second, use the formula [2] to calculate the Shannon entropy of Chinese pentatonic scale.

$$H_{Indian\ 22-shruti\ scale} = H_6 = -\sum_{i=1}^{12} P(i)log_2P(i) \approx 3.198 \text{ bits} \tag{11}$$

After getting the entropy values of every single scale, the results can be shown in Table 5.

Table 5. The entropy value of different scales

Scale	Entropy value (bits)	Categories
the Chinese pentatonic scale	2.314	Low-entropy
Japanese scale	2.314	Low-entropy
Indian pentatonic scale	2.722	Low-entropy
European natural major scale	2.970	Medium-entropy
Arabic 17 tone Isotonic Scale	3.190	High-entropy
Indian 22-shruti scale	3.198	High-entropy

This table is the core result of the study, which classifies six scales into three levels (low, medium, high entropy) based on their calculated Shannon entropy values. The data verifies the research hypothesis: the number of core notes and interval diversity are positively correlated with entropy values. Low-entropy scales (2.314–2.722 bits) correspond to the pursuit of "harmony and simplicity" in Eastern and local cultures; the medium-entropy European natural major scale (2.970 bits) matches the balanced aesthetic of Western classical music; high-entropy scales (3.190–3.198 bits) support the complex and improvisational style of Arabic and Indian music.

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4.5. Entropy value ranking and cultural interpretation

Based on the entropy value of the scale from low to high and in combination with their respective cultural backgrounds, the differences in music systems can be analyzed.

Low-Entropy Scales (2.0-2.8 bits): Chinese Pentatonic Scale (2.314), Japanese Scale (2.314), Indian Pentatonic Scale (2.722). All these scales have five core notes, with few types of intervals and a relatively concentrated distribution. Some consonant intervals account for a relatively high proportion. For the cultural interpretation, the cultural characteristics of these musical pieces emphasize the aesthetic concept of "harmony and simplicity".

Medium-Entropy Scale (2.9-3.1 bits): European Natural Major Scale (2.970)

The European natural major scale has seven core notes and a moderate number of interval types. The proportion of consonant intervals and altered intervals is very balanced, and the distribution is relatively even. For the cultural interpretation, it conforms to the creative concept of "balance and conflict" in Western classical music. It can not only support a relatively rigorous harmonic system, but also express the complex emotions of the composer through the changes between intervals, providing a solid mathematical foundation for the diversified development of Western music.

High-Entropy Scales (3.1-3.7 bits): Arabic 17-Tone Equal Temperament Scale (3.190), Indian 22-Shruti Scale (3.7171). There is a large number of core notes in both two scales, with the former one having 17 core notes, and the latter one having 22 core notes. These scales contain a large number of micro-intervals, and the distribution of these micro-tones is relatively uniform, so the uncertainty is also very high. For the cultural interpretation, the Indian music and the Arabic music both rely on the diversity of high-entropy scales, providing strong acoustic support for the delicate emotional expression of music.

5. Research limitations and future directions

5.1. Research limitations

The determination of the frequency ratio of intervals is based on the standard of 12-TET. Although a threshold has been set, it cannot fully adapt to the unique micro-intervals of certain cultures, such as the subtle differences in the Indian 22-Shruti Scale and Arabic 17-Tone Equal Temperament Scale. This study did not take into account the influence of melodic direction on the selection of intervals. It merely calculated the entropy value based on the distribution of all possible intervals, which would have a relatively significant difference from the frequency of different intervals' usage in actual music creation. The sample size is limited and did not cover more cultural scales.

5.2. Future directions

Optimize the grading model of intervals and introduce culture-specific interval standards to adapt to the precise determination of micro-intervals. By integrating the global database of traditional music, calculate a more practical entropy value based on the frequency of intervals used in actual melodies in order to enhance the practical relevance of the research. Expand the sample sizes by considering regions such as Africa and Southeast Asia in order to construct a global cultural scale entropy database.

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6. Conclusion

This study quantified the uncertainties of six typical global cultural scales through the information entropy tool and constructed an interdisciplinary research framework from mathematical modeling to planning implementation and to cultural interpretation. The research finds that entropy value can effectively reflect the structural characteristics of different cultural scales: low entropy value corresponds to a regular and harmonious scale structure, while high entropy values correspond to diverse and variable scale structures. This result reveals the intrinsic connection between cultural aesthetics and mathematical acoustics, providing a new perspective and quantitative tools for cross-cultural music research.

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