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Input-to-State Stability of Nonlinear Systems with Time Delays and Delayed Impulses under a Flexible Event-Triggered Mechanism

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Abstract. This paper studies the input-to-state stability (ISS) of a class of nonlinear time-delay systems affected by delayed pulse disturbances. To address the challenges arising from the coexistence of delays, pulse effects, and external inputs, a novel event-triggered impulsive control (ETIC) strategy is proposed to ensure the maintenance of ISS. Adjusting the coefficients of the triggering mechanism provides greater flexibility, and the triggering threshold can dynamically vary along the sequence of events. Under the condition of cumulative exponential decay, a generalized Razumikhin-type ISS criterion is established to characterize the intrinsic coupling relationship among triggering parameters, pulse gains, and pulse delays. Furthermore, for systems with impulsive dynamics and delays, constructive design conditions for the triggering mechanism and pulse gains are derived, and the feasibility of the proposed method is verified through numerical simulations.

Keywords: Event-triggered impulsive control (ETIC), input-to-state stability (ISS), time delay, Zeno behavior, nonlinear impulsive systems, Time-delay pulse.

1. Introduction

Research on input-to-state stability (ISS) of nonlinear systems based on event-triggered impulsive control (ETIC) has become relatively mature. ETIC has drawn increasing attention because it can reduce unnecessary control update [1,2], but in existing frameworks, the key factor of time delay is often simplified. Recently, some studies have begun to explore the impact of delays on the stability of event-triggered impulsive systems. For example, [3] investigated case of state delays, while [4]further considered dual delays. Chen et al [5] studied large-scale nonlinear systems with complex structures, delays, and uncertain measurement channels. However, most of these studies adopt uniform triggering parameters, which may limit the flexibility of the triggering mechanism and pose challenges in handling more complex delay scenarios. Based on these considerations, Unlike the method in literature [3] that uses fixed parameters in the triggering rule, this paper proposes a novel ETIC strategy with non-uniform triggering parameters, where the triggering thresholds can be dynamically adjusted based on the occurrence of events. Specifically, the exponential weights α_k and β_k change with k. By introducing a more flexible design of triggering thresholds, we establish ISS stability results for nonlinear systems with dual delays.

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This paper proposes an ETIC strategy with adjustable coefficients for nonlinear impulsive systems involving time delays. On this basis, time-delay impulses are introduced. The proposed scheme ensures system operation without Zeno phenomena and provides additional flexibility for adjusting the triggering frequency and disturbance robustness. Moreover, delays exceeding the inter-pulse interval are allowed, thus alleviating the conservative assumptions commonly seen in literature [6].

2. Preliminaries

Notation: $\varsigma_0 < \varsigma_1 < \varsigma_2 < \cdots$ be the impulsive times with $\lim_{k\to\infty} \varsigma_k = \infty$,denoted by $\{\varsigma_k\}$ For $\mathscr{X} \subset R$ and $Y \subset R^n$, define $\mathbb{PC}(\mathscr{X} \to Y) := \{\phi : \mathscr{X} \to Y | \phi \text{ is piecewise right-continuous}\}$ .A continuous function $\gamma : R_+ \to R_+$ is said to belong to class K if it is strictly increasing and $\gamma(0) = 0$, in addition, $\gamma(s) \to \infty$ as $s \to \infty$, then γ is said to belong to class K_∞ .A function $\delta : R^+ \times R^+ \to R^+$ is said to belong to class KL ,for each fixed $t \geq 0$, $\delta(\cdot, t)$ is of class, and for $\mathscr{K}$ each fixed $r \geq 0$, $\delta(r, t) \to 0$ as $t \to 0$. For $\tau > 0$, let $PC_\tau := PC([-\tau, 0], R^n)$ be the space endowed with the norm $\|\phi\|_\tau := \sup_{\kappa\in[-\tau,0]} |\phi(\kappa)|, \phi \in PC_\tau$.

Consider the following impulsive delay system:

$$\begin{cases} (t) = f(t, y_t, \vartheta(t)), t \neq \varsigma_k, t \geq \varsigma_0 \\ \Delta y(\varsigma_k) = g_k(t, y(\varsigma_k^- - \tau), \vartheta(\varsigma_k^-)), t = \varsigma_k, k \in N_+ \\ y(t) = \xi(t), \varsigma_0 - \tau \leq t \leq \varsigma_0 \end{cases} \tag{1}$$

where $y \in R^n$ is the system state, and $\dot{y}$ is right-hand derivative of y . $\vartheta(t) \in R^m$ is a disturbance input. $\xi \in PC_\tau$,and $\|\xi(t)\| = \sup_{\varsigma_0-\tau\leq\kappa\leq\varsigma_0} |\xi(\kappa)|$, $y_t(\kappa) = y(t + \kappa) \quad \kappa \in [-\tau, 0]$ .Assuming that f and g meet the necessary regularity requirements, the existence of solutions over the specified time intervals is guaranteed [5] .The solution of system is assumed to be right-continuous, i.e., $y(t^+) = y(t)$ for all $t \geq \varsigma_0$.

Definition1 For a given sequence $\{\varsigma_k\}$, system is said to be input-to-state stable (ISS) if there exist functions $\beta \in KL$ and $\phi \in K_\infty$ such that for any input $\vartheta \in PC([\varsigma_0, \infty], R^m)$ and any initial condition $\xi \in PC([-\tau, 0], R^n)$, the solution $y(t, \varsigma_0, \xi)$ exists globally and satisfies

$$|y(t)| \leq \beta\left(\|\xi\|_\tau, t - \varsigma_0\right) + \sup_{\varsigma_0\leq\kappa\leq t} \phi\left(|\vartheta(\kappa)|\right), \forall t \geq \varsigma_0.$$

Definition2 Given a locally Lipschitz function $V : R^n \times R_+$,the upper right-hand Dini derivative of V along the solution of system is defined by

$$D^+V\left(y\left(t\right)\right) = \limsup_{h\to0} \frac{1}{h}\left[V\left(y\left(t\right) + hf\left(y\left(t\right)\right)\right) - V\left(y\left(t\right)\right)\right]$$

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3. Results

In this section, criteria are derived for system to rule out Zeno behavior and to establish its ISS performance within the ETIC setting with delayed impulses. We begin with the following event-triggering mechanism (ETM):

$$\varsigma_k = min\{\varsigma_k^*, \tau_k\}, \varsigma_k^* = inf \\ \{t \geq \varsigma_{k-1} : \upsilon(t) \geq exp(s_k)\upsilon(\varsigma_{k-1}) + exp(z_k)\chi(\|\varpi\|_{[\varsigma_0,t]}) + \gamma exp(-\eta(t-\varsigma_0))\upsilon_0\} \tag{2}$$

where $\chi \in K_\infty$, γ and η are positive constants, Define $\upsilon(t) := V(y(t))$, and $\upsilon_0 = \sup_{\kappa\in[-\tau,0]} \upsilon(t+\kappa)$. For the sequences τ_k, s_k ,and z_k in R_+ , assume that $\inf_{k\in\mathbb{N}_+} \{\tau_k\} > 0$.

Lemma1 Assume that there exists locally Lipschitz function $V : R^n \times R_+, \chi \in K_\infty$ and constants $s_k, z_k \in R_+$, $\zeta_1, \zeta_2, \iota, \tau_k > 0$ such that for all $t \in R_+$, $l \in Z^+$,Then, under the ETM, system exhibits no Zeno behavior.

$$(i) D^+V(t, x(t)) \leq \zeta_1 V(t, x(t)) + \zeta_2 V(t-\tau, x(t-\tau)) + \chi(\|\varpi(\varsigma)\|), \text{ for all } \kappa \in [-\tau, 0];$$

$$(ii) V\left(\varsigma_k, g_k\left(\varsigma_k^- - \tau, x\left(\varsigma_k^- - \tau\right)\right)\right) \leq e^{-\iota} V\left(\varsigma_k^- - \tau, x\left(\varsigma_k^- - \tau\right)\right) + \chi\left(\left\|\varpi\left(\varsigma_k^- - \tau\right)\right\|\right), \ k \in N_+.$$

(iii) The following cumulative inequality holds for any $k \geq 2$ and $0 \leq l \leq k-2$:

$$\sum_{i=0}^{l} s_{k-i} + z_{k-l} + \zeta l\tau - l\iota \leq -\zeta_4 l + \iota$$

Proof: From the ETM, for $t \in [\varsigma_{k-1}, \varsigma_k)$, the following three scenarios are considered to exclude Zeno behavior:

Case 1: The impulsive time sequence $\{\varsigma_k\}$ consists entirely of forced impulsive moments τ , i.e., $\varsigma_k = \tau_k$. According to the definition of forced triggering, it holds that $\varsigma_k - \varsigma_{k-1} = \tau_k > 0$ for all $k \in N$. Obviously, Zeno behavior is excluded in this case.

Case 2: The impulsive time sequence $\{\varsigma_k\}$ consists entirely of event-triggered moments $\overset{*}{\varsigma_k}$, i.e., $\varsigma_k = \overset{*}{\varsigma_k}$. From the definition of $\overset{*}{\varsigma_k}$, we have:

$$\overset{*}{\varsigma_k} = inf\{t \geq \varsigma_{k-1} : exp(\zeta(t-\varsigma_0))\upsilon(t) \geq H_{k-1}\}$$

Where $H_{k-1} = exp(s_k + \zeta(t-\varsigma_0))\upsilon(\varsigma_{k-1}) + exp(z_k + \zeta(t-\varsigma_0))\chi\left(\|\varpi\|_{[\varsigma_0,t]}\right) + \gamma\upsilon_0$.

Let

$$\widetilde{\upsilon}(t) = \begin{cases} exp(\zeta(t-\varsigma_0))\upsilon(t), t \in [\varsigma_0, \infty) \\ \upsilon(t), t \in [\varsigma_0 - \tau, \varsigma_0) \end{cases}$$

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Then, the ETM is transformed into:

$$\varsigma_k = min\left\{\overset{*}{\varsigma_k}, \tau_k\right\},$$

$$\varsigma_k^* = inf\{t \geq \varsigma_{k-1} : (t) - \mathrm{H}_{k-1}(t) \geq 0\} \tag{3}$$

Where $\widetilde{H}_{k-1}(t) = exp(s_k + \zeta(t-\varsigma_{k-1}))\widetilde{\upsilon}(\varsigma_{k-1}) + \gamma\upsilon_0 + exp(z_k + \zeta(t-\varsigma_0))\chi\left(\| \varpi \|_{[\varsigma_0,t]}\right)$ .From condition (ii). First, we show that the conclusion holds on the initial interval. obviously, $\upsilon(t) \leq \upsilon_0$, for $t \in [\varsigma_0 - \tau, \varsigma_0]$ .From the continuous dynamics of system on the interval $t \in [\varsigma_0, \varsigma_1)$, it follows that

$$\widetilde{\upsilon}(t) = exp(s_1 + \zeta(t-\varsigma_0))\upsilon_0 + exp(z_1 + \zeta(t-\varsigma_0))\chi\left(\| \varpi \|_{[\varsigma_0,t]}\right) + \gamma\upsilon_0$$

Therefore, the inequality holds on the interval $[\varsigma_0, \varsigma_1)$ .Next, by condition (iii), for $t \in [\varsigma_1, \varsigma_2)$, we obtain

$$\widetilde{\upsilon}\left(\varsigma_2^-\right) \leq exp(s_2 + \zeta(t-\varsigma_1))\widetilde{\upsilon}(\varsigma_1) + exp(z_2 + \zeta(t-\varsigma_0))\chi\left(\| \varpi \|_{[\varsigma_0,t]}\right) + \gamma\upsilon_0$$

$$\leq exp(s_2 + \zeta(t-\varsigma_1+\tau) - \iota)\upsilon\left(\varsigma_1^- - \tau\right) + exp(s_2 + \zeta(t-\varsigma_0))\chi\left(\left\|\varpi\left(\varsigma_1^- - \tau\right)\right\|\right)$$

$$+exp(z_2 + \zeta(t-\varsigma_0))\chi\left(\| \varpi \|_{[\varsigma_0,t]}\right) + \gamma\upsilon_0$$

$$\leq exp(s_1 + s_2 + \zeta(t-\varsigma_0) - \iota)\upsilon_0 + \gamma exp(s_2 + \zeta(t-\varsigma_1+\tau) - \iota)\upsilon_0 + exp(s_2 + z_1 + \zeta(t-\varsigma_0))\chi\left(\| \varpi \|_{[\varsigma_0,t]}\right)$$

$$+exp(s_2 + \zeta(t-\varsigma_0))\chi\left(\left\|\varpi\left(\varsigma_1^- - \tau\right)\right\|\right) + exp(z_2 + \zeta(t-\varsigma_0))\chi\left(\| \varpi \|_{[\varsigma_0,t]}\right) + \gamma\upsilon_0 \leq G_k$$

When $\varsigma_0 \leq \varsigma_1 - \tau \leq \varsigma_1$, the conclusion holds. When $\varsigma_0 - \tau \leq \varsigma_1 - \tau \leq \varsigma_0$,we have,

$$\widetilde{\upsilon}(t) \leq exp(s_2 + \zeta(t-\varsigma_1) - \iota)\left(exp(s_1)\upsilon_0 + \gamma\upsilon_0 + exp(z_1)\chi\left(\| \varpi \|_{[\varsigma_0,t]}\right)\right)$$

$$+exp(s_2 + \zeta(t-\varsigma_1))\chi\left(\left\|\varpi\left(\varsigma_1^- - \tau\right)\right\|\right) + exp(z_2 + \zeta(t-\varsigma_0))\chi\left(\| \varpi \|_{[\varsigma_0,t]}\right) + \gamma\upsilon_0 \leq G_k$$

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Then, the conclusion holds for $\varsigma_1 - \tau \in [\varsigma_0, \varsigma_1]$ .Similarly, assume that for some $k \geq 2$, the conclusion holds for $t \in [\varsigma_{k-1}, \varsigma_k)$, that is,

$$\widetilde{\upsilon}(t) \leq exp\left(\sum_{i=0}^{k-1} s_{k-i} + \zeta(t - \varsigma_0 + (k-1)\tau) - (k-1)\iota\right)\upsilon_0 + \gamma\sum_{l=0}^{k-1} exp(\sum_{i=0}^{l-1} s_{k-i} + \zeta(t - \varsigma_{k-l} + l\tau) - l\iota)\upsilon_0$$

$$+\sum_{l=0}^{k-1} exp\left(\sum_{i=0}^{l} s_{k-i} + z_{k-l} + \zeta(t - \varsigma_0 + l\tau) - l\iota\right)\chi\left(\|\varpi\|_{[\varsigma_0,t]}\right)$$

$$+\sum_{l=0}^{k-2} exp\left(\sum_{i=0}^{l} s_{k-i} + \zeta(t - \varsigma_0 + l\tau) - l\iota\right)\chi\left(\left\|\varpi\left(\varsigma_{k-l-1}^{-} - \tau\right)\right\|\right)$$

Then, by substituting $\upsilon(t) = exp(-\zeta(t - \varsigma_0))\widetilde{\upsilon}(t)$ and applying condition , we have:

$$\upsilon(t) \leq exp\left(\sum_{i=0}^{k-1}(s_{k-i} + (k-1)\tau) - (k-1)\iota\right)\upsilon_0 + \gamma\sum_{l=0}^{k-1} exp\left(\sum_{i=0}^{l-1} s_{k-i} + \zeta l\tau - l\iota\right)\upsilon_0$$

$$\begin{aligned}&+\sum_{l=0}^{k-1} exp\left(\sum_{i=0}^{l} s_{k-i} + z_{k-l} + \zeta l\tau - l\iota\right)\chi\left(\|\varpi\|_{[\varsigma_0,t]}\right) + \\ &\sum_{l=0}^{k-2} exp\left(\sum_{i=0}^{l} s_{k-i} + \zeta l\tau - l\iota\right)\chi\left(\left\|\varpi\left(\varsigma_{k-l-1}^{-} - \tau\right)\right\|\right) \\ &= M_k\end{aligned}$$

Considering condition (iii), one can obtain

$$\begin{aligned}&\upsilon\left(t\right) \leq exp(-\zeta_3\left(k-1\right) + \delta)\upsilon_0 + 2\sum_{l=0}^{k-2} exp(-\zeta_4 l + \delta)\chi\left(\|\varpi\|_{[\varsigma_0,t]}\right) + \gamma \\ &\sum_{l=0}^{k-1} exp(-\zeta_4 l + \delta - \eta(\varsigma_{k-l-1} - \varsigma_0))\upsilon_0 \\ &\leq exp(\delta)(exp(-\zeta_4(k-1)) + \tfrac{\gamma}{1-exp(-\zeta_4)})\upsilon_0 + \tfrac{2exp(\delta)}{1-exp(-\zeta_4)}\chi(\|\varpi\|_{[\varsigma_0,t]})\end{aligned} \tag{4}$$

Thus, the following relation is satisfied for $t \in [\varsigma_{k-1}, \varsigma_k)$:

$$\upsilon\left(t + \kappa\right) \leq exp(\delta)\left(exp(-\zeta_4\left(k-1\right)\right) + \frac{\gamma}{1 - exp(-\zeta_4)}\right)\upsilon_0 + \frac{2exp(\delta)}{1 - exp(-\zeta)}\chi\left(\|\varpi\|_{[\varsigma_0,t]}\right)$$

With the aid of condition (i), we can estimate the upper bound of the derivative for $t \in [\varsigma_{k-1}, \varsigma_k)$ as:

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$D^{+}\upsilon\left(t\right)\leq\zeta_1\upsilon\left(t\right)+\zeta_2\left[exp(\delta)\left(exp(-\zeta_4\left(k-1\right)\right)+\frac{\gamma}{1-exp(-\zeta_4)}\right)\upsilon_0+\frac{2exp(\delta)}{1-exp(-\zeta)}\chi\left(\left\|\varpi\right\|_{[\varsigma_0,t]}\right)\right]$
$\chi\left(\left\|\varpi\right\|_{[\varsigma_0,t]}\right)$

Then, integrating the above inequality in $[\varsigma_{k-1},\varsigma_k)$, $\chi:=\underset{\kappa\in[\varsigma_0,\varsigma_k]}{sup}\chi\left(\left\|\varpi\right\|_{[\varsigma_0,t]}\right)$,one obtains

$$\upsilon(t)\leq exp(\zeta_1(t-\varsigma_{k-1}))\upsilon(\varsigma_{k-1})$$

$$+\int_{\varsigma_{k-1}}^{t}exp(\zeta_1\left(t-\kappa\right))\left[\zeta_2\left(exp(\delta)\left(exp(-\zeta_4\left(k-1\right)\right)+\frac{\gamma}{1-exp(-\zeta_4)}\right)\upsilon_0+\frac{2exp(\delta)}{1-exp(-\zeta)}\chi+\chi\right]d\kappa$$

$$\leq exp(\zeta_1\left(t-\varsigma_{k-1}\right))\upsilon\left(\varsigma_{k-1}\right)+\frac{exp(\zeta_1(t-\varsigma_{k-1})-1)}{\zeta_1}\chi$$

$$+\frac{\zeta_2}{\zeta_1}\left[exp(\delta)\left(exp(-\zeta_4(k-1))\right)+\frac{\gamma}{1-exp(-\zeta_4)}\right)\upsilon_0+\frac{2exp(\delta)}{1-exp(-\zeta)}\chi\left(\left\|\varpi\right\|_{[\varsigma_0,t]}\right)\right]\left(exp(\zeta_1\left(t-\varsigma_{k-1}\right)\right)-1\right)$$

Let $t=\varsigma_k^-$, together with $\upsilon\left(\varsigma_k^-\right)=exp\left(s_k\right)\upsilon\left(\varsigma_{k-1}\right)+exp\left(z_k\right)\chi\left(\left\|\varpi\right\|_{[\varsigma_0,t]}\right)+\gamma exp\left(-\eta\left(\varsigma_{k-1}-\varsigma_0\right)\right)\upsilon_0$

Implies that $exp(s_k)\upsilon\left(\varsigma_{k-1}\right)+exp\left(z_k\right)\chi+\gamma exp\left(-\eta\left(\varsigma_{k-1}-\varsigma_0\right)\right)\upsilon_0$

$$\leq exp(\zeta_1\left(\varsigma_k-\varsigma_{k-1}\right)\right)$$
$$\left[\upsilon\left(\varsigma_{k-1}\right)+\frac{\zeta_2}{\zeta_1}\left(exp(\delta)\left(exp(-\zeta_4\left(k-1\right)\right)+\frac{\gamma}{1-exp(-\zeta_4)}\right)\upsilon_0+\frac{2exp(\delta)}{1-exp(-\zeta_4)}\chi\right)-\frac{\chi}{\zeta_1}\right]$$

$$-\frac{\zeta_2}{\zeta_1}\left(exp\left(\delta\right)\left(exp\left(-\zeta_4\left(k-1\right)\right)+\frac{\gamma}{1-exp\left(-\zeta_4\right)}\right)\upsilon_0+\frac{2exp(\delta)}{1-exp(-\zeta_4)}\chi\right)-\frac{\chi}{\zeta_1}$$

If the Zeno behavior appears, then there exists a constant $t_r>0$ such that $\underset{k\to\infty}{lim}\varsigma_k=t_r$. It follows from the above estimate that

$$0\leq\frac{1}{\zeta_1}ln\left(\frac{exp(s_k)\upsilon(\varsigma_{k-1})+exp(z_k)\chi+\gamma exp(-\eta(\varsigma_{k-1}-\varsigma_0))\upsilon_0+J}{\upsilon(\varsigma_{k-1})+J}\right)$$

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$$\leq \varsigma_k - \varsigma_{k-1},\ \text{and}\ \varsigma_k - \varsigma_{k-1} \to 0\ \text{as}\ k \to \infty$$

Where

$$J = \frac{\zeta_2}{\zeta_1}\left(exp\left(\delta\right)\left(-\zeta_4\left(k-1\right) + \frac{\gamma}{1-exp\left(-\zeta_4\right)}\right)\upsilon_0 + \frac{2exp(\delta)}{1-exp(-\zeta_4)}\chi\right) + \frac{\chi}{\zeta_1}$$

This results in a contradiction. Accordingly, the conditions given in Theorem1 prevent Zeno behavior, and the proof is complete.

Case 3: The impulsive time sequence $\{\varsigma_k\}$ is determined by both forced and event-triggered mechanisms, i.e., $t_k = min\left\{\tau_k, \varsigma_k^*\right\}$. Since both Case 1 and Case 2 exclude the existence of an accumulation point for $\{\varsigma_k\}$, Zeno behavior is also excluded in this mixed scenario.

Theorem1 Under the conditions in Lemma1, assume that there exist function $\alpha_1, \alpha_2 \in K_\infty$ such that

$$(iv)\ \alpha_1\left(|y|\right) \leq V(t,y) \leq \alpha_2(|y|),\ t \geq \varsigma_0 - \tau, y \in R^n.$$

Then, system(1) coupled with ETM is ISS.

Proof: From the way the ETM is designed, it is clear that the impulse signal will occur infinitely often. To exclude Zeno behavior, it is sufficient to prove that the inter-impulse intervals are uniformly bounded away from zero, i.e., there exists a constant $\rho > 0$ such that $\varsigma_{k+1} - \varsigma_k \geq \rho$ holds for all $k \geq 0$ .Then, according to, and by a reasoning similar to that in Lemma1, the paper readily obtain that, for each $t \in [\varsigma_{k-1}, \varsigma_k)$, the following result holds:

$$\upsilon\left(t\right) \leq exp(-\zeta_3\left(k-1\right) + \delta)\upsilon_0 + + \frac{2exp\left(\delta\right)}{1-exp\left(-\zeta_4\right)}\chi\left(\left\|\varpi\right\|_{[\varsigma_0,t]}\right) + \gamma$$
$$\sum_{l=0}^{k-1} exp(-\zeta_4 l + \delta - \eta(\varsigma_{k-l-1} - \varsigma_0))\upsilon_0$$

$$\forall \varepsilon > 0, \exists K_1 = max\{-\tfrac{1}{\zeta_4}\, ln\tfrac{\varepsilon}{2}\left(1-exp(-\zeta_4)\right), \tfrac{1}{-\eta\rho}\, ln\tfrac{\varepsilon}{2}\left(1-exp(-\eta\rho)\right)\}, \forall k > K_1 \quad ,$$

$$\beta\sum_{l=0}^{k-1} exp(-\zeta_4 l - \eta(t_{k-l-1} - t_0))V_0 \leq \sum_{l=0}^{K_1} exp(-\zeta_4 l) + \sum_{l=K_1+1}^{k} exp(-\eta(k-l-1)\rho) \leq$$
$$\sum_{l=0}^{K_1} exp(-\zeta_4 l)$$

$$+\sum_{l=K_1+1}^{\infty} exp(-\eta(k-l-1)\rho) \leq \frac{exp(-\zeta_4 K_1)}{1-exp(-\zeta_4)} + \frac{exp(-\eta\rho(K_1+1))}{1-exp(-\eta\rho)} < \frac{\varepsilon}{2} + \frac{\varepsilon}{2} < \varepsilon$$

From condition(iii), one has

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$$\|y(t)\| \le \alpha_1^{-1} V(y(t)) \le \alpha_1^{-1} exp(\delta) \left(\frac{exp(-\zeta_4 K_1)}{1-exp(-\zeta_4)} + \frac{exp(-\alpha\rho(K_1+1))}{1-exp(-\alpha\rho)} \right) \alpha_2(\|\xi\|_\tau)$$

$$+\alpha_1^{-1} \left(\frac{2exp(\delta)}{1-exp(-\zeta_4)} \chi\left(\| \varpi \|_{[\varsigma_0,t]} \right) \right), \forall t \in \left[\varsigma_{k-1}, \varsigma_k \right)$$

Hence, system is ISS under the event-triggering mechanism , thereby completing the proof.

Remark Condition (i) describes the continuous-time dynamics of the system, including both delay-independent and delay-dependent components. condition (ii) describes the state transition at impulsive instants with delayed effects, reflecting the delayed impulsive action and providing a contraction mechanism that supports stability at discrete moments. condition (iii) imposes a cumulative decay condition on the combined influence of continuous dynamics and impulsive effects, ensuring that the overall evolution is dominated by an exponential decay rate. condition (iv) guarantees that the Lyapunov function is positive definite and radially unbounded. Compared with the event-triggering mechanism using fixed constant parameters in [3], the proposed scheme allows more flexible parameter design and broadens its applicability.

4. Numerical example

Consider a class of nonlinear time-delay systems with the following dynamics [7]:

$$\dot{y}(t) = 1.3|cost| \frac{y^3(t)}{1+y^2(t)} - 0.6|sint| y(t) + 0.5 costy(t-\tau) + \varpi(t), t \neq \varsigma_k$$

$$y(\varsigma_k^-) = exp(-0.7) y(\varsigma_k - \tau_{imp}) + \varpi(\varsigma_k), t = \varsigma_k \tag{5}$$

where $\tau > 0$ denotes the state delay and $\tau_{imp} > 0$, represents the impulsive delay. And $\varpi(t)$ is an external disturbance. The scalar $exp(-0.7) \in (0,1)$ is the impulsive strength. In this example, the disturbance is chosen as $\varpi(t) = 0.8 sin(1.2t)$ .To regulate the impulsive instants, an ETM is adopted. consider the candidate ISS-Lyapunov function $\upsilon(t) = |y(t)|$. For each interval $[\varsigma_k, \varsigma_{k+1})$.

Its upper-right Dini derivative along the system trajectories satisfies

$$D^+\upsilon(t) \le (1.3|cost| + 0.6|sint|)|y(t)| + 0.5|cost||y(t-\tau)| + |\varpi(t)|$$

$$\le 1.3\upsilon(t) + 0.5\upsilon(t-\tau) + |\varpi(t)|$$

where the dynamic threshold function is given by
$\Omega_k(t) = 1.3|y(\varsigma_{k-1})| + 3exp(-0.08(t-\varsigma_0))V_0 + 3|\varpi|_{[\varsigma_0,t]}$ with $V_0 = 0.8$.

If the above condition is satisfied, an event-triggered impulse is generated. Otherwise, a forced impulse will be achieved after a prescribed maximum interval $\tau > 0$ .For numerical verification, the

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system parameters are selected as $\tau = 6, \tau_{imp} = 0.4, \iota = 0.7, \gamma = 0.25, \varsigma = 0.08$. The time-varying parameters in triggering condition are defined as $s_k = 4 \times 10^{-6} + 1 \times 10^{-7}k, z_k = 3 \times 10^{-6} + 1 \times 10^{-7}k$ .The initial condition is chosen as $y(t) = 0.8, \quad t \in [-\tau, 0]$.

Figure 1 shows the state trajectory of the system, from which it can be seen that it is divergent. Figure 2 illustrates the state trajectory and the corresponding impulsive triggering instants of the system under the proposed event-triggered mechanism. Black triangles denote event-triggered impulses, while red circles represent forced-triggered impulses. It can be observed that the system state remains bounded under the external disturbance, and the triggering instants are well separated, which verifies the effectiveness of the designed mechanism and confirms the absence of Zeno behavior.

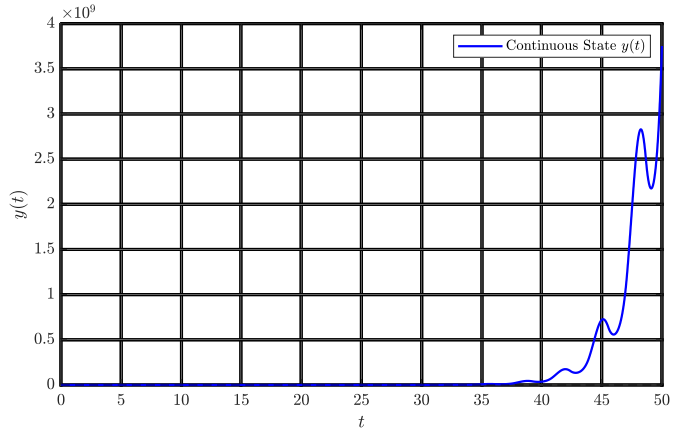


Figure 1. The state trajectories of system without impulsive control

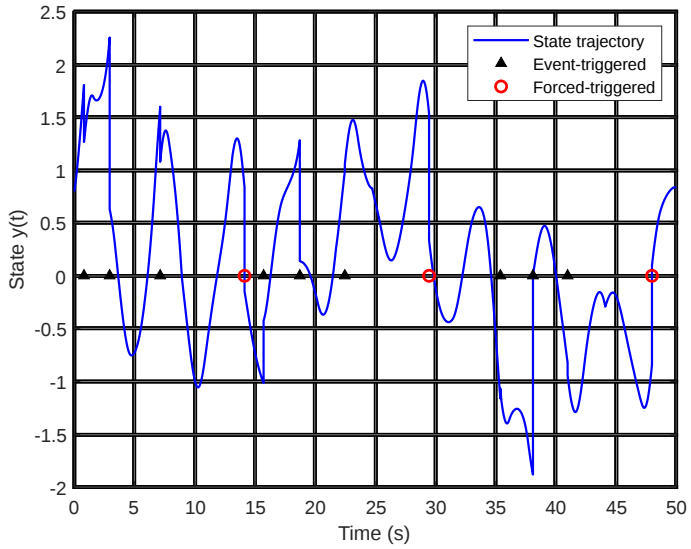


Figure 2. Simulation of system with impulsive control

5. Conclusion

Based on Lyapunov stability theory, this paper designs a more flexible event-triggered mechanism to address the impact of delays in discrete systems on nonlinear time-delay systems. Under the influence of dual delays, it ensures ISS of the system, while also proving that it does not generate the Zeno

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phenomenon. Finally, numerical simulations are conducted to demonstrate the effectiveness and advantages of the proposed results. Considering the wide application of the theory in real life and in emerging development industries, such as large-model robotics, future work will consider applying this to multi-agent systems and solving practical problems under this new ETM condition.

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