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# *Evolution Solutions of Friedmann Equations in Modified Gravity Theories*

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***Abstract.*** Modern cosmology faces a persistent crisis—the physical origin of the observed late-time cosmic acceleration remains unexplained. Although invoking "dark energy" via the cosmological constant offers a mathematical fix, it forces an unnatural fine-tuning of the vacuum energy density—down to $10^{-120}$ Planck units—alongside the nagging coincidence problem. Instead of inventing exotic fluids, this study investigates large-scale modifications to General Relativity, focusing specifically on $f(R)$ gravity theories where a generalized curvature function replaces the standard Ricci scalar $R$. Deriving the modified Friedmann equations reveals that these new geometric terms naturally provide the missing repulsive force, acting as "geometric dark energy" to drive the expansion. To rigorously evaluate this framework, this research maps the modified equations onto the cosmic apparent horizon using nonequilibrium thermodynamics, directly linking gravitational dynamics to entropy production. The analysis confirms that specific $f(R)$ models not only successfully reconstruct the expansion history but also strictly satisfy local solar system gravity tests, establishing a robust, purely gravitational solution to the dark energy dilemma.

***Keywords:*** modified gravity, friedmann equations, cosmic acceleration, geometric dark energy

## 1. Introduction

One of the outstanding issues of cosmology is the expansion of the universe accelerated later. This effect is described by the standard model of cosmic models namely, the ΛCDM model, which introduces a cosmological constant, Λ, into the right-hand side of the field equations of Einstein as has a source of vacuum energy. Nonetheless, the hypothetical problems of this dark force portion have caused a change of orientation. Recent arguments indicate a pull out in questioning the large-scale validity of General Relativity instead of pull-ins of extra exotic fluids. As an example, Yang presented the existing theoretical issues and observations data on this component [1]. These questions drive researchers to the investigation of the modified gravity as an alternative to the dark energy. These theories include $f(R)$ gravity that tries to cue the situation with geometric transformations. In this method the Ricci scalar R in the Einstein-Hilbert action is substituted by an arbitrary f into which terms of higher order derivative are added to the equations of gravity field.

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Previous studies by Nojiri and Odintsov showed that specific $f(R)$ functions can reconstruct the expansion history, where the geometric terms act as an effective dark energy [2]. In addition to pure fitting, a thermodynamic point of view was mentioned through the work of Tian and Booth. The Friedmann equations are in this case modified with the relation to energy dissipation and entropy production at the apparent horizon with gravity treated as emergent phenomenon connected to the thermodynamic flux. Other, such as $f(R,T)$ gravity, include a coupling between the Ricci scalar and the trace of the energy-momentum tensor T to generate acceleration [3].

This paper illustrates that the model of acceleration can be understood through a manipulation of the field equations without involving the dark energy. In section 2, the theoretical framework of $f(R)$ gravity is also given and the modified Friedmann equations obtained to determine the particular geometric terms. Section 3 examines the solutions to evolution, both dynamically and on nonequilibrium thermodynamics without contradiction to the local solar system gravity tests. The last part is the conclusion of the findings in comparison to the existing observations and the test of these changes that will be made in the future.

## 2. Modified gravity theory

The basis of the theoretical interpretation of explaining the acceleration of the cosmos without a dark energy fluid commences with an amendment to the Einstein-Hilbert action. In $f(R)$ gravity, the standard Ricci scalar $R$ is replaced by a generalized function $f(R)$ . The field equations are derived by varying this action with respect to the metric $g_{\mu\nu}$ , resulting in:

$$F(R)R_{\mu\nu} - \frac{1}{2}f(R)g_{\mu\nu} - (\nabla_\mu\nabla_\nu - g_{\mu\nu})F(R) = \kappa^2 T_{\mu\nu} \tag{1}$$

where $F(R) \equiv \partial f/\partial R$ is the derivative of the term with respect to R. These equations imply that gravity will not act in the same way when at cosmological dimensions as it does at the nearby environment, may propel the noted acceleration through geometric alterations [4].

Using Friedmann-Lemaire Robertson-Walker universe of the homogeneous and isotropic type, complex field equations can be simplified to modified Friedmann equations and the particular equations that describe the scale factor a(t) evolution are obtained. The first Friedmann equation in $f(R)$ gravity is expressed as:

$$3FH^2 = \kappa^2\rho_m + \frac{RF-f}{2} - 3H\dot{F} \tag{2}$$

This is to show that the rate of expansion H depends not just upon the matter density rho m but also upon geometrical quantities which include $F(R)$ and its time derivatives ( $\dot{F}, \ddot{F}$ ). Unlike General Relativity, where $F = 1,$ here $f$ is a dynamical variable that acts as an effective gravitational coupling.

The terms resulting as a consequence of the curvature function may be rearranged to show as an effective energy density $P_{MG}$ on the right-hand side of the Einstein equations. This "geometric dark energy" approach argues that cosmic acceleration is an inherent property of modified spacetime geometry rather than an exotic physical substance. Analysis of the effective Equation of State (EoS) parameter, defined as $w_{eff} = P_{eff}/\rho_{eff}$ , reveals how the geometry triggers the observed acceleration. When the specific form of $f(R) allows w_{eff}$ to achieve values less than $-1/3$ , the

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universe will begin to experience accelerated expansion, essentially resembling the action of a cosmological constant or quintessence field [5].

## 3. Solutions and implications for cosmic acceleration

This chapter has a certain sequence to find observable acceleration in the cosmos through a modified gravitational action. Initial, the change is made to the Friedmann equations to a moving system by means of the dimensionless density parameters $\Omega_i$ . This move assists in determining in which conditions the effective geometric fluid contains the negative pressure ( $w_{eff} < -1/3$ ) necessary to accelerate is in the existence of the effective geometric fluid. This is followed by a reconstruction method to solve the actual functions of the expansion history so as to be used in determining the actual functions of the observational Hubble. This demonstrates that a significant geometric change could imitate the dark energy without a cosmological constant -Lambda. To be physically-viable, these solutions are sifted using the Dolgov Kawasaki criterion of stability ( $f''(R) > 0$ ) and local screening, including the Chameleon effect. These filters ensure that the model is consistent with local Solar System tests of gravity [6]. Lastly by taking into account the laws of thermodynamics at the cosmic apparent horizon, the Friedmann equations are formulated as equations of state of a nonequilibrium system. According to this approach, cosmic acceleration is attributed as due to entropy maximization in a modified geometry.

### 3.1. Reconstruction of f(R) models from expansion history and stability analysis

The reconstruction program is one of the foundations of $f(R)$ weighted gravity research because it seeks to answer the inverse problem: which kind of the gravitational action $f(R)$ would have created a given history of cosmic expansion? In the standard model of cosmology, the history of expansion in models based on the dark energy is dictated by an eternal dark energy density but in modified gravity, the objective is to replace this fixed dark energy density with the dynamical geometric term. It is possible to put the field equations in a form of a differential equation of $f(R)$ by expressing the modified Friedmann equations as a product of the number N = ln (a) of e-folding. This kind of mathematical structure permits the researcher to insert observational evidence of Supernovae Ia or Cosmic Microwave Background (CMB) to find the solution of the necessary curvature function. As demonstrated in Figure 1 where different observational data give the empirical boundaries of the expansion history.

But reconstruction is not just a curve-fitting game and the solutions obtained should be examined in the context of dynamical stability and physical viability. To be a candidate of the real universe, it must pass a number of important "viability conditions on a model. The first, is that the condition $F(R) = \partial f/\partial R > 0$ has to exceed zero, otherwise there must be no anti-gravity condition that causes the theory to degenerate into an anti-gravity state. Second the second derivative of $\partial^2 f/\partial R^2 > 0$ is a condition required to prevent tachyonic instabilities, sometimes known as Dolgov Kawasaki instabilities or ghosts. Failure to do so would lead to a progressive increase in the curvature $R$ on scales of very short time on which it would be impossible to have stable stars and galaxies.

In addition, the effective reconstruction of $f(R)$ is expected to consider the cosmological changeover of various epochs. The universe had gone through a radiation dominated period, followed by an epoch of prolonged matter dominated ( $a \propto t^{2/3}$ ) before entering the current accelerated expansion phase. A viable $f(R)$ model would have to have a fixed point in phase

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space, which is a "stable matter-dominated point" and in which the structure forms. The geometric terms in the modified Friedmann equation (including those involving the $(RF - f)/2$ term) should automatically start to play a leading role as the matter density, the density of the universe itself, this allows the universe to transition smoothly into an accelerated phase without the fine-tuning problems typically associated with the cosmological constant $\Lambda$. By analyzing these evolution solutions, it can be confirmed that specific models, such as the Hu-Sawicki or Starobinsky models [7], successfully mimic the $\Lambda$ CDM expansion at late times while offering distinct, testable predictions for the growth of cosmic structure.

The testing of these models has now entered the multi-messenger astronomy era as clear in recent articles dealing with the journey beyond Einstein. Observation of gravitational wave events, like GW170817, have placed very tight limits on the speed of gravitational propagation ( $c_T^2 \approx c^2$ ), effectively ruling out a large class of modified gravity theories that predict a variable speed of gravity. This observational 'filter' ensures that only $f(R)$ models and scalar-tensor theories that maintain the speed of light for gravitational waves remain viable candidates.

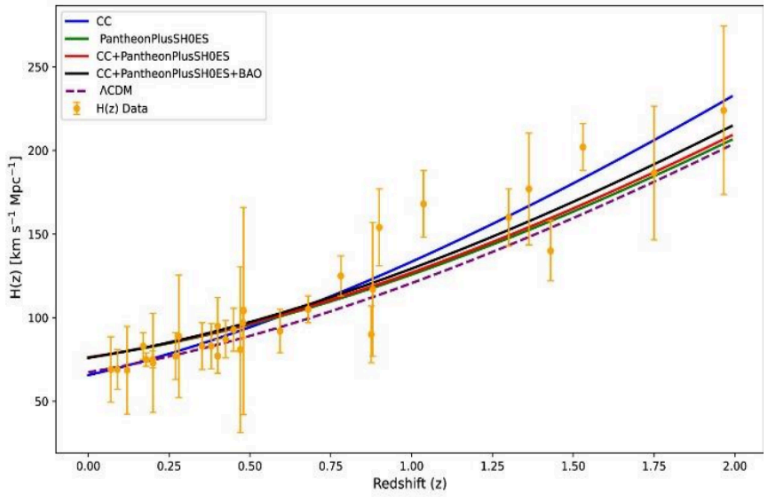



Figure 1. Comparison of the Hubble parameter $H(z)$ evolution between various observational datasets (including Cosmic Chronometers, Pantheon+, and BAO) and the theoretical predictions of theΛCDM model [8]

### 3.2. Screening mechanisms and compatibility with local gravity tests

A significant hurdle for any modified gravity theory is the "Solar System Constraint.". While gravity is desired to deviate from General Relativity (GR) on the scale of billions of light-years to explain cosmic acceleration, it is known from high-precision experiments (such as Cassini tracking) that GR works perfectly within the solar system. This creates a scale-dependent paradox: $f(R)$ gravity must be "strong" in deep space but "weak" near massive bodies. To resolve this, viable $f(R)$ theories must incorporate a "screening mechanism," the most prominent of which is the Chameleon Effect.

The Chameleon mechanism operates on the principle that the effective mass of the new scalar degree of freedom (the "scalaron") depends on the local matter density. In high-density environments like the Earth or the Sun, the scalaron becomes extremely massive. According to the Yukawa potential, a massive force carrier has a very short range, meaning the "fifth force" generated by the $f(R)$ modification is suppressed or "screened" to a level that is undetectable by current

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experiments. In contrast, in the near-vacuum of intergalactic space, the density is extremely low, the scalaron becomes nearly massless, allowing it to drive the accelerated expansion observed.

Another mechanism often discussed is the Vainshtein mechanism, which relies on non-linear derivative couplings to screen gravity near massive sources. For $f(R)$ models to remain competitive with the $\Lambda$CDM model, they must demonstrate that these screening effects are efficient enough to satisfy the parameterized post-Newtonian (PPN) bounds. This requirement significantly narrows the field of acceptable $f(R)$ functions. For instance, the parameter $f$ (the value of the derivative of $f(R)$ today) is often constrained to be less than $10^{-5}$ or $10^{-6}$ to stay within experimental limits. This interplay between large-scale acceleration and small-scale suppression is what makes $f(R)$ gravity a robust and sophisticated alternative to dark energy, as it offers a unified geometric explanation for diverse gravitational phenomena across vastly different scales.

### 3.3. The nonequilibrium thermodynamic origin of friedmann equations

A rather theoretically intriguing advancement in contemporary cosmology has been the discovery that gravity is not necessarily a primary force, but the emergent thermodynamic feature. This is equivalent to the original point of view, introduced by Ted Jacobson in the case of General Relativity, which was generalized to modified gravity theories. Here the Friedmann equations are modified as not only field equations but also as thermodynamic identities on the cosmic apparent horizon. Thermodynamics actually yields the expansion dynamics of the universe by subjecting the first law of thermodynamics $T\,dS = \delta Q$ —to the horizon, one can actually derive the dynamics of the universe's expansion.

There is however a serious complication presented by $f(R)$ gravity; it is associated with a nonequilibrium thermodynamic state. In the General Relativity, the area is proportional to the horizon entropy ( $S = A/4G$ ), which is a measure of equilibrium. The entropy in the $f(R)$ gravity is generalized to the general curvature dependent coupling:

$$S = \frac{AF(R)}{4G} \tag{3}$$

Due to the dynamical nature of the term $F(R)$ which varies in the course of the evolution of the universe, the entropy of the horizon does not obey the area law. This internal entropy production or $d_iS$ , which is a characteristic of nonequilibrium systems is a result of this evolution. This term of entropy production is the same mathematical term as the additional geometrical terms in the modified Friedmann equations, which form geometric dark energy.

This thermodynamic relation also gives $f(R)$ gravity a better physical rationale. It implies that the acceleration of the universe at the end of time is not due to some unknown energy fluid, but rather a macroscopic effect of the microstates at the base of the universe settling to the maximum entropy state. The rate of changing of the effective gravitational coupling F H is directly proportional to the energy dissipation of the changing $H$ . A non-locally uniform picture is obtained by using the universe as a thermodynamic system, in which all of the geometric structure of spacetime, the generation of entropy, and the cosmic acceleration are causally interrelated. Not only does this framework have the merit of building on the theoretical foundation of modified gravity, but also makes it consistent with the larger investigation of understanding the holographic nature of the universe and the quantum origin of the spacetime.

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In order to further differentiate these geometric changes over the traditional dark energy, future high-precision surveys will be of the deciding factor. According to Wang and Xu [9], the Euclid satellite, the Large Synoptic Survey Telescope (LSST) and the gravitational wave detectors of the future are aimed at determining the growth of structure and the rate of expansion at the rates never seen before. Such observations will play a key role in the thermodynamic development of the phenomenon of gravity and finding out whether the observed cosmic acceleration is indeed a feature of a modified geometry of spacetime or a to-be-no-longer-known energy component [10, 11].

## 4. Conclusion

This paper demonstrates that the effect of $f(R)$ gravity can serve as a geometric substitute to dark energy and cause the late-time acceleration of the cosmic flow. Through determining and solving the derived and modified Friedmann equations, it can be seen that the non-linear effects in the geometrical sector have the ability to yield a cosmological constant effect of some quantity ae termed the cosmological constant ae. This method is geometric and not fine-tuned since it solves the coincidence problems and the vacuum energy problem of the standard 6CDM model. The analysis of phase-space indicates that a smooth transition between a matter dominated period and an accelerated expansion phase can be achieved in particular $f(R)$ models. Regarding the underlying physics, the mapping of such modified field equations on cosmic apparent horizon is related to nonequilibrium thermodynamics. Through this mapping, field equations are explained as thermodynamic identities. Direct relation with production of entropy and dissipation of energy is the evolution of the Ricci scalar and effective gravitational coupling. The consistency reveals that $f(R)$ right consistency behaves as the gravity is an emergent phenomenon, and not merely a curve-fitting procedure.

This thermodynamic perspective is also associated with quantum gravity and the spacetime origin. Quantum modifications of the Friedmann equations, that happen to be significant at the Planck curvature scale indicate that quantum effects were dominant in the early universe. These corrections can aid in the avoidance of big bang singularity and a way of offering inflation mechanism. This finding aligns with the nonequilibrium thermodynamic interpretation, and one may conclude that cosmic acceleration is an emergent phenomenon concerning the fundamental physics. Whereas great scale cosmic acceleration can be attributed to $f(R)$ gravity, it has to a satisfying degree local gravity tests. Non-linear screening mechanisms like Chameleon effect are used to fulfill this requirement. The high precision of a survey such as Euclid and LSST will be necessary in the future to determine the difference between the modified gravity and dynamical dark energy. The data will assist in the breaking of the parameter degeneracy between the two models. The rate of expansion of large structures can be used to tell whether the acceleration is produced by a new venue of energy or by distorted gravity. What is more, it is now known that early dark energy in the framework of $f(R)$ may be useful to both address the Hubble tension and reconcile the discrepancies between early and late-universe measurements.

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