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Application of Residue Theorem in Rational Integral Calculation

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Abstract. The infinite improper integrals of rational functions are often taught in mathematics and often used in the calculation of engineering, but traditional calculation methods in the real variable are often large in size, and non-universal. As the primary tool, Residue Theorem of the complex analysis will be used in this paper to explore the analysis of infinite improper integral of rational functions. It introduces some important concepts like Isolated Singularities, Simple Poles and Simple Residues, takes the usual contour of the upper half plane, uses Maximum-Length inequality to demonstrate convergence and simplifies the parities of functions, and uses the simplified formula to compute the residue of a function at simple singularities to systematically solve four classes of integrals. The results indicate that Residue Theorem is excellent in convenient and universality and it is developed the universal and reusable structure of the solution. In this study, there are some possible fields of applications in educating of Math and engineering calculation. It can be extended to other more complicated improper integrals and special function integrals in the future.

Keywords: Residue Theorem, Mathematical Analysis, Integral Equation

1. Introduction

The Residue Theorem is a key tool for computing infinite improper integrals of rational functions, and is often employed in the engineering calculation process and taught in mathematics classes. The current literature is mostly targeted at the solution of individual-type integrals or verification of the individual cases, without any systematic classification of typical forms of integrals and a single and reusable scheme of solutions. While most of the conventional methods use the whole Laurent's series expansion to obtain the residues, these residues are inefficiently computed and there is not much literature available to present a standard operating procedure in their computation [1].

The paper will focus on Residue Theorem as well as its use in evaluating infinite improper integrals of rational functions. This systematically lists the main theories and categorizes the research of the typical integrals of this type, and constructs a system of solutions of such types of integrals that can be reused. This paper explains the basic concepts (isolated singularities, simple poles, Laurent expansion and residues) and clarifies the essence of the central implication of Residue Theorem (between contour integrals and residues of function singularities) both in theory and practice. However, to avoid complicated expansion of the whole Laurent series and to increase the effectiveness of the calculations, a simple formula for the calculation of residue at simple poles

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of rational functions is introduced [2] to create a simple overall theoretical supporting system of applied research.

The paper lists in the practical application section four common types of improper integrals for which the problem is seeking the solution. Each of the types of integrals are systematically solved and verified using the Residue Theorem, on the standard upper half-plane semicircular contour, the Maximum-Length (ML) inequality to verify the convergence of arc integrals and the parity of the functions, to simplify the calculation [3]. The approach employed in this paper has proven to have obvious convenience and universality advantages to the old real integration methods and the present disjointed literature. In conclusion, the paper is a regular solution process for infinite improper integral of rational function in a type easily popularized by the sorting in theory and case checking approaches. The conclusions of the study have the undeniable theoretical value and practical significance, and can be directly used in the practical application of engineering calculus teaching.

2. Basic theories of the Residue Theorem

2.1. Basic concepts: isolated singularities and residues

In this paper the author begins by taking a complex function, $f(z)$. The function at a given parameter z_0 is not analytic, but is smooth in its vicinity. There may be three types of isolated singularities. This work deals only with poles — certain simple poles. The author considers a rational function analyzed for which there are only a few poles leaving [4].

If $\mathrm{f}(\mathrm{z})$ is continuous in some regions $\mathrm{r}_1 < |\mathrm{z} - \mathrm{z}_0| < \mathrm{r}_2$, one can transfer the function into its Laurent expansion:

$$\sum_{n=-\infty}^{\infty} a_n (z - z_0)^n = \sum_{n=0}^{\infty} a_n (z - z_0)^n + \sum_{n=1}^{\infty} a_{-n} (z - z_0)^{-n} = f(z) \tag{1}$$

In practice, the first and second terms are the regular part and principal part, respectively.

There are two important parts above, principal part and regular part. The consists of all negative terms of power in Eq. (1) is called the principal part. The regular part is indicated by the group of the constant and all positive terms. The main part is devoted to the classification of the types of the isolated singularity. When the principal is finite, z_0 is a pole. In contrast, if the principal is infinite, z_0 is an essential singularity. If the principal only has one term, and the corresponding power is exactly -1, and there are no terms with negative exponents smaller than -1 that come out, then the singularity z_0 is noted a first-order pole (or simple pole) of the $f(z)$, given as: $\frac{\mathrm{a}_{-1}}{\mathrm{z}-\mathrm{z}_0}$.

The coefficient of the term is a_{-1} and one calls it residue. In this way, one has the residue as the pre-factor of $(\mathrm{z} - \mathrm{z}_0)^{-1}$ term in the Laurent expansion.

2.2. Statement of the Residue Theorem

The calculation of contour integrals of complex functions around closed curves is done with the Residue Theorem. Assuming $f(z)$ is analytic in a simply-connected region Ω, there are at least finitely many isolated singularities a_k ; and assume that a_k is a positively-oriented closed curve in Ω, which includes the isolated singularities:

$$\oint f(z)dz = 2\pi i \sum_{a_k \in int(r)} Res(f, a_k) \tag{2}$$

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As discussed in Section 2.1, the residue a_k is the coefficient of the term $(z-a_k)^{-1}$ in the Laurent series expansion of $f(z)$ in a neighbourhood of a_k, i.e., the coefficient a_{-1} as in Section 2.1. This is a very crucial relationship. It relates the residue theorem to the previous fundamental ideas, e.g., isolated singularities and Laurent series and results in a more coherent theory as compared to stating the theorem in isolation. Since the residue of the Laurent series is the same as this coefficient in this series, one can relate it at contour integrals which is one of the secrets to the validity of the residue theorem.

Thus, the line of reasoning behind the Residue Theorem is to make the calculation of contour integrals of complex functions a bit easier. Also, instead of calculating integrals directly one only has to calculate the residues of all the singularities within the contour, add them up and multiply by $2\pi i$ to get the integral.

In this paper, simple poles are only taken into account as defined in Section 2.1. The term of lowest degree in the principal of the Laurent expansion at some such pole is the term of (z- a_k)$^{-1}$. and there are no powers of $z-a$ with negative exponents of lower degree. All the functions used in this paper are rational functions $f(z) = P(z)/Q(z)$ with $P(z)$ and $Q(z)$ polynomials. To such rational function, when a_k is a simple pole, i.e., when a_k is a simple zero of the denominator $Q(z)$, a simple formula can be used to find the residue: the residue equals to $P(a_k)/Q'(a_k)$, and also equals to derivative of $Q(a_k)$. The calculation is much easier since this formula does not involve the expansion to the full Laurent series, as done in Section 2.1. It is also the basic technique of calculation with which people use it to compute integrals in the next section.

3. Application of the Residue Theorem

3.1. High-degree roots of unity rational functions

The author now gives an integral to evaluate, namely $I = \int_0^{+\infty} \frac{x^2}{x^8+1}\,dx$.

To investigate the improper real integral, the author continues the integrand into the complex plane and choose some suitable closed contour of integration. The complex rational function is given by the complex rational function $f(z) = \frac{z^2}{z^8+1}$ [5].

The function c is meromorphic, analytic in $\mathbb{C}$ and has simple poles at the roots of the equation $z^8=-1$. In light of the residue theorem, the integral of $f(z)$ around the contour C is equal to:

$$\oint_C f(z)dz = 2\pi i \sum_{z_k \in Int(C)} Res[f, z_k] \tag{3}$$

On the interval of $f(z)$, the contour integral can be decomposed into real-axis and semicircular parts:

$$2\pi i \sum_{\substack{poles\ z_k \\ Imz_k > 0}} Res[f, z_k] = \int_{-R}^{R} f(x)dx + \int_{C_R} f(z)dz \tag{4}$$

To computes the corresponding residues at simple poles, the solution is to identify all isolated singularities. The poles of $f(z)$ are the roots to $z^8 = -1 = e^{\pi i}$. The 8th roots of -1 are given by $z_k = e^{i\frac{(2k+1)\pi}{8}}$, $k = 0,1,\ldots,7$. Only poles with positive imaginary part lie inside the contour C for sufficiently large R. But only $z_0 = e^{i\pi/8}, z_1 = e^{i3\pi/8}, z_2 = e^{i5\pi/8}, z_3 = e^{i7\pi/8}$ conform to the requirements [6].

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For a rational function $f(z)=\frac{P(z)}{Q(z)}$, where P(z) and Q(z) are analytic at z_k and Q(z_k)=0, $Q'(z_k)\neq 0$, the residue at a simple pole is $\mathrm{Res}[f,z_k]=\frac{P(z_k)}{Q'(z_k)}$. Here $(z)=z^2$, $Q(z)=z^8+1$, $Q'(z)=8z^7$. Thus $\mathrm{Res}[f,z_k]=\frac{z_k^2}{8z_k^7}=\frac{1}{8z_k^5}$. The integral over the upper semicircular arc in the contour integral, leaving only the integral along the real axis, which is then related to the desired improper integral. On the arc C_R with $|z|=R\to\infty$: $|f(z)|=\left|\frac{z^2}{z^8+1}\right|\leq\frac{|z|^2}{||z|^8-1|}=\frac{R^2}{R^8-1}\sim\frac{1}{R^6}$. The length of the semicircular arc C_R is πR. Using the ML inequality,

$$|\int_{C_R} f(z)dz|\leq|f(z)|\cdot length(C_R)\leq\frac{\pi R}{R^8-1}\cdot R^2=\frac{\pi R^3}{R^8-1}. \tag{5}$$

The right-hand side goes to 0 as R goes to $+\infty$. Therefore $\lim_{R\to\infty}\int_{C_R}f(z)dz=0$. If $f(x)$, $f(x)$ is even on $\mathbb{R}$. So, it follows that: $\int_{-R}^{R}f(x)dx=2\int_0^R\frac{x^2}{x^8+1}dx$ The above results can now be put into the contour-integral identity: $\lim_{R\to\infty}\left(2\int_0^R\frac{x^2}{x^8+1}dx+\int_{C_R}f(z)dz\right)=2\pi i\sum_{k=0}^{3}\mathrm{Res}[f,z_k]$ [7].

The integral around the semi-circular arc C_R approaches zero as R approaches ∞. Therefore, one needs to vanish arc and denote $I=\int_0^{+\infty}\frac{x^2}{x^8+1}dx$, giving $I=\pi i\sum_{k=0}^{3}\mathrm{Res}[f,z_k]$. Since $\mathrm{Res}[f,z_k]=\frac{1}{8z_k^5}$, $z_k=e^{i(2k+1)\pi/8}$, $I=\frac{\pi i}{8}\sum_{k=0}^{3}\frac{1}{z_k^5}$. Using $z_k^8=-1$, answer of $\frac{1}{z_k}$ is $-z_k^7$, so $\frac{1}{z_k^5}=-z_k^3$.

The sum can be expressed in terms of roots of unity and can be evaluated by using the geometric-series formula or by the sine and cosine formulas for the eighth roots of negative one. After direct computations, the imaginary parts merge and the final result is: $I=\frac{\pi}{4\sqrt{2}}$.

3.2. Odd-function numerator rational functions

Now the author defines I as the integral from $-\infty$ to $+\infty$ of $\frac{x}{x^4+4x^2+3}$ with respect to x: $I=\int_{-\infty}^{+\infty}\frac{x}{x^4+4x^2+3}dx$ In this case, the author is concerned with the analysis of an improper integral of a rational expression which satisfies the same conditions as contour integration and analysis in Section 3.1, namely, the numerator is odd-function and the denominator even-function. All this computation is built around a theoretical basis, determined in Chapter 2, properties of simple poles, the residue computation rule of rational functions, which was presented in Section 2.1, the residue theorem (2), the contour decomposition expression (3), and the ML inequality of integral estimation (4).

To calculate the value of the target real improper integral, the author expands the real integrand into the complex plane where the most important step in the expansion is to use contour integration to calculate real integrals. This extension will help construct a meromorphic complex function which has the same properties of calculus of residue. The complex rational function that corresponds is defined as: f(z)=xx4+4x2+3.The function f(z) is analytic at all points with the exception of the simple poles that are the zeros of the equation $z^4+4z^2+3=0$. Factoring the denominator gives $z^4+4z^2+3=(z^2+1)(z^2+3)$, so the poles of $f(z)$ are the solutions to $z^2=-1$ and $z^2=-3$. Namely, $z=i$, $z=-i$, $z=\sqrt{3}i$, and $z=-\sqrt{3}i$.

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The contour is the usual contour in upper half plane (UHP) $z = R$, $Im\, z \geq 0$ taken in positive (counterclockwise) direction and −R, R is the straight line along real axis from −R to R. This leads to closed contour: $C = C_R \cup [-R, R]$. If R is chosen large enough then this contour will only enclose the poles of $f(z)$ in the upper half plane: $z_1 = i$ and $z_2 = \sqrt{3}i$.

Thus, the fundamental relation of contour integration is fulfilled by the residue-theorem integral of $f(z)$ around C. The contour integral can be decomposed into (real-axis and semicircular) components like Eq. (3). This decomposition breaks this original integral into two parts, one of which is the integral along the real axis and is proportional to the target improper integral, and the other is the integral along the semicircular arc which can be estimated and eliminated in the limit process [8].

To get the residues at the enclosed simple poles, the author uses the residue formula of rational functions directly in the Section 2.1. In the case of the rational function $\mathrm{f}(\mathrm{z}) = \frac{\mathrm{P}(\mathrm{z})}{\mathrm{Q}(\mathrm{z})}$ having a pole at $\mathrm{z_k}$, the residue is $\mathrm{Res}[\mathrm{f}, \mathrm{z_k}]$. Here, $\mathrm{P}(\mathrm{z}) = \mathrm{z}$, $\mathrm{Q}(\mathrm{z}) = \mathrm{z}^4 + 4\mathrm{z}^2 + 3$, and the 1st derivative of the denominator is $\mathrm{Q}'(\mathrm{z}) = 4\mathrm{z}^3 + 8\mathrm{z}$. Replacing the upper half-plane poles into the formula provides the value of the residue directly. The integral over the upper semicircular arc $\mathrm{C_R}$ is estimated by the ML inequality (4). On the arc $\mathrm{C_R}$ with $|\mathrm{z}| = \mathrm{R} \to \infty$, the modulus of $\mathrm{f}(\mathrm{z})$ decays rapidly, and the product of the maximum modulus and the arc length approaches zero as $\mathrm{R} \to +\infty$. Therefore, the integral along the semicircular arc vanishes in the limit, leaving only the integral along the real axis to be considered.

The integrand $\mathrm{f}(\mathrm{x}) = \frac{\mathrm{x}}{\mathrm{x}^4+4\mathrm{x}^2+3}$ is an odd function of R. In the symmetric case of an odd function, that is, with the symmetric interval of $(-\infty, +\infty)$, the positive and negative terms are canceled. Adding the arc-integral limit value, the symmetry of odd functions and the computed residues, the result of the improper integral is algebraically simplified to the final result: $\mathrm{I} = 0$.

3.3. Homogeneous mixed-denominator rational functions

Now the author defines I as the integral from $-\infty$ to $+\infty$ of $\frac{1}{\mathrm{x}^4+2\mathrm{x}^2+2}$ with respect to x: $\mathrm{I} = \int_{-\infty}^{+\infty} \frac{1}{\mathrm{x}^4+2\mathrm{x}^2+2}\,\mathrm{dx}$. The section is dedicated to the analysis of an improper integral of a rational function with a homogeneous mixed denominator, the contour integration scheme and analysis steps are similar to Section 3.1 and Section 3.2. It is all on the theoretical basis introduced in Chapter 2, properties of simple poles, the expression of calculation of residues of rational functions (2.1), the contour decomposition expression (3) and the ML inequality of estimate of integral (4) [9].

The integrand can then be extended to the complex plane and a complex closed contour of integration formed, in order to compute the improper real integral. It is an extension precisely needed to use contour integration to solve real integrals and it helps in constructing a meromorphic complex function, whose analytic properties can be used. The associated complex rational expression can be defined as: $\mathrm{f}(\mathrm{z}) = \frac{1}{\mathrm{z}^4+2\mathrm{z}^2+2}$. The function has simple poles at the solutions z of the equation $\mathrm{z}^4+2\mathrm{z}^2+2=0$ and is analytic at all other places. The four simple poles of $f(z)$ are given by the solutions of the quartic equation as $\mathrm{f}(\mathrm{z})$, $z = -1 + i$, $z = -1 - i$, $z = 1 + i$ and $z = 1 - i$.

In the upper half-plane (UHP), the poles of $f(z)$ are $\mathrm{z_1}$=−1+i, $\mathrm{z_2}$=1+i and on a sufficiently large radius R, one has: CR, is the semi-circular contour: z=R, Imz≥0, oriented in the counterclockwise direction; −R, R is the straight line segment along the real axis: −R to R; and the closed contour is

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written as $C=C_R \cup [-R, R]$. Therefore, the residue of the closed contour integral of $f(z)$ by the residue theorem (2) is the core relation of contour integration.

The contour integral can be split up into the real-axis and semicircular part as shown in (3). This kind of decomposition breaks up the original integral into two independent parts: the integral over the real axis, which is directly proportional to the improper integral to be evaluated; and the integral over the semicircular arc, which can be estimated and neglected in the limiting process.

In order to compute the residues of the enclosed simple poles, the formula of residues of rational functions in Section 2.1 is used directly. Residue at a simple pole: If $f(z) = \frac{P(z)}{Q(z)}$, where $P(z)$ and $Q(z)$ are analytic at z_k and $Q(z_k) = 0$, $Q'(z_k) \neq 0$, then the residue of f at z_k is $\mathrm{Res}[f, z_k] = \frac{P(z_k)}{Q'(z_k)}$. In this case, P(z)=1, $Q(z) = z^4 + 2z^2 + 2$ and first derivative of the denominator is $Q'(z) = 4z^3 + 4z$. The residues can be obtained from the upper half plane poles simply by substitution in the formula.

The integral over the upper semicircular arc C_R is estimated by the ML inequality (4). On arc C_R with $|z| = R \to \infty$: $|f(z)| = \left|\frac{1}{z^4+2z^2+2}\right| \leq \frac{1}{R^4-2R^2-2} \sim \frac{1}{R^4}$ The length of semicircular arc C_R is πR. By the ML inequality (4), the product of the maximum modulus and the arc length tends to zero as $R \to +\infty$. Therefore $\lim_{R\to\infty} \int_{C_R} f(z)dz = 0$[10].

In the case of $f(x)$, $f(-x) = f(x)$, therefore, $f(x)$ is even on $\mathbb{R}$. This implies that the integral of the whole real line can be expressed as two times integral of 0 to $+\infty$. The limit result of the arc integral and the symmetry of the even functions are combined with the calculated residues, and all the terms are replaced in the contour integral identity and simplified stepwise. Once the similar terms are lumped together and the denominator is rationalized, the value of the improper integral is then obtained: $I = \frac{\pi\sqrt{2}}{2}$.

3.4. Even symmetric non-unity-root rational functions

Now the author defines I as the integral from $-\infty$ to $+\infty$ of $\frac{1}{x^4+6x^2+8}$ with respect to x : $I = \int_{-\infty}^{+\infty} \frac{1}{x^4+6x^2+8} dx$. This section is dedicated to the discussion of an improper integral of a non-unity-root even symmetric rational function, which uses the same contour integration scheme and analysis steps, as in Section 3.1 and Section 3.2. The entire calculation is built up on the theoretical basis that was decided in Chapter 2 that encompasses properties of simple poles, the rule of computing the residue of rational functions that is introduced in Section 2.1, the residue theorem (2), the expression of contour decomposition (3) and the ML inequality to estimate the worth of integrals (4). Unlike high-degree roots of unity rational function in the previous section, all poles of the integrand in this section are not on the unit circle, the characteristic of the non-unity-root rational function in this section.

To find the improper real integral, one can extend the function to be integrated to the complex numbers and form a suitable closed contour of integration. The extension is the step needed to compose contours to compute real integrals, and helps in building a meromorphic complex function with known analytic properties.

The complex rational function of this is: $f(z) = 1/\left(z^4 + 6z^2 + 8\right)$. Hence, the basic relation for contour integration is given by the residue theorem (2) of the integral of fz along the closed contour $\mathbb{C}$, for which the simple poles correspond to the zeros of the equation $z^4+6z^2+8=0$ and f is analytic elsewhere. Factoring the denominator shows that $z^4+6z^2+8=(z^2+2)(z^2+4)$, so the four simple

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poles of $f(z)$ are z= $\sqrt{2}$ i, z=− $\sqrt{2}$ i, z= $\sqrt{2}$ i, and v , with moduli of $\sqrt{2}$ and 2, respectively, which are both not equal to 1, and which are in complete keeping with the non-unity-root definition of this section.

The semicircular contour is chosen as the upper half-plane (UHP), which is the semi-circle z=R, Imz≥0, oriented counter-clockwise, together with the real axis segment −R, R, which is closed by the contour C=C_R ∪ −R, R. This contour will only surround poles of $f(z)$ that are in the upper half plane: z_1= $\sqrt{2}$ i and z_2=- 2 i if R is large enough. Thus, by the residue theorem Eq. (2), the contour integral of $f(z)$ around C is the sum of residues of integrand at the poles. Hence, according to the residue theorem Eq. (2), the following contour integration core relation is true around the closed contour C. The contour integral may be broken up into real-axis and semicircular components as in (3). Contour integral can be broken down into real-axis and semicircular components illustrated in (3). The breakup of the original integral into two independent terms one an integral along the real axis which is directly proportional to the target improper integral and the other an integral over the semicircular arc that is estimable and can be dropped in the limit procedure.

In order to evaluate the residues at the enclosed simple poles, the formula used to calculate the residues of rational functions in Section 2.1 is used straight away. For the residue at a simple pole z_k of a rational function $\mathrm{f}(\mathrm{z}) = \frac{\mathrm{P}(\mathrm{z})}{\mathrm{Q}(\mathrm{z})}$, it is found that $\mathrm{Res}[\mathrm{f}, \mathrm{z_k}] = \frac{\mathrm{P}(\mathrm{z_k})}{\mathrm{Q}'(\mathrm{z_k})}$. Here $\mathrm{P}(\mathrm{z}) = 1$, $\mathrm{Q}(\mathrm{z}) = \mathrm{z}^4 + 6\mathrm{z}^2 + 8$, with the first derivative of the denominator Q′(z)=4z3+12z. The residue values can be obtained directly as $\mathrm{Res}\left[\mathrm{f}, \sqrt{2}\mathrm{i}\right] = \frac{1}{8\sqrt{2}\mathrm{i}}$, $\mathrm{Res}[\mathrm{f}, 2\mathrm{i}] = -\frac{1}{16\mathrm{i}}$.

The integral over the upper semicircular arc $\mathrm{C_R}$ is estimated by the ML inequality (4). On the arc $\mathrm{C_R}$ with $|\mathrm{z}| = \mathrm{R} \to \infty$, the modulus $|\mathrm{f}(\mathrm{z})|$ decays quickly toward zero. The length of the semicircular arc $\mathrm{C_R}$ is $\pi\mathrm{R}$. By the ML inequality (4), the product of the maximum modulus and the arc length goes to zero as $\mathrm{R} \to +\infty$. Therefore $\lim_{\mathrm{R}\to\infty} \int_{\mathrm{C_R}} \mathrm{f}(\mathrm{z})\mathrm{dz} = 0$.

In the case of $\mathrm{f}(\mathrm{x})$, $\mathrm{f}(-\mathrm{x}) = \mathrm{f}(\mathrm{x})$, so $\mathrm{f}(\mathrm{x})$ is even on $\mathbb{R}$, which coincides with the even symmetric property in the section title. The result is that the integral of the whole real line can be expressed as an integral of 0 to $+\infty$ twice. The limit result of the arc integral, the symmetry of the even function, and the residues obtained are combined, and all the terms are replaced into the contour integral identity and simplified step by step. Upon addition of similar words and rationalization of the denominator, the end result of the improper integral is found: $\mathrm{I} = \frac{\pi\left(2-\sqrt{2}\right)}{8}$.

4. Conclusion

The basic theories of Residue Theorem are systematically discussed and its application in the evaluation of infinite improper integrals of rational functions is investigated, which are widely used in teaching mathematics and engineering computation. Various basic concepts like isolated singularity, simple pole, Laurent expansion and residue are well defined and a simplified formula to calculate the residue of a rational function at the simple pole is derived. The formula can prove handy when it comes to avoiding the tedious full Laurent series expansion, as well as to facilitate faster computation.

The four popular types of infinite improper integral of rational function and their solutions have been completely and standardized on the conventional upper half-plane semicircular contour, as well as the arc integral convergence and parity of function simplification methods. Computing has proved that the Residue Theorem is better than the more usual real and complex valued integral forms in

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several aspects. The framework provides helpful and easy-to-understand hypothetical guidance and operation examples to teach mathematics in college, curriculum practice and other engineering calculation situations. Further research is required to expand the scope of application of the Residue Theorem and improve the elasticity of the solution system for various problems involving integrals by expanding the proposed method to more complex improper integrals, such as trigonometric integrals, integrals with higher poles and special function integrals.

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