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A Review of the Theoretical Foundation and Research Progress of Gravitational Lensing Effect

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Abstract. Gravitational Lensing (GL) has long been studied as one phenomenon supporting Einstein's General Relativity (GR)—multiple studies focusing on the deflection angle, time delay of Gravitational Lensing, as well as its applications in cosmological tests and the verification of General Relativity have been publishe This paper serves as a review summarizing previous studies on the deflection angle, lens equation, time delay, and different types of lensing effects.This paper also analyzes previous studies on the applications of GL. It can be concluded that gravitational lensing is a phenomenon where light bends around massive objects due to spacetime curvature, with its deflection angle in general relativity being twice that predicted by Newtonian mechanics, that it has three main types—strong, weak, and microlensing—and is widely used for testing general relativity, measuring the Hubble constant, and detecting dark matter, and that the impact of plasma on lensing is limited, being significant only at low frequencies and under high plasma inhomogeneity.

Keywords: Gravitational Lensing, General Relativity, Dark Matter, Plasma, Massive Celestial Bodies

1. Introduction

The establishment of general relativity in the early twentieth century paved the way for the further development of modern physics, as it reveals that space-time is not absolute; instead, matter tells space-time how to curve, and space-time tells matter how to move. Simultaneously, light always travels through the shortest distance in space: the geodesic line. Ultimately, these evidences predict a new phenomenon—as a distant light source, a massive celestial body and an observer are on the same line, the image captured by the observer would be divided into several pieces or becomes an arc or a ring-like structure, the famous Einstein Ring—and this phenomenon is called Gravitational Lensing(GL) [1, 2]. With the advancement of research on GL-related topics, the applications of GL in determining the Hubble Constant, detecting dark matter, and some of its limitations have been revealed. Nevertheless, a general review and conclusion of previous research is rare. Therefore, this research paper aims to provide a general review of existing research papers and generate constructional and referential guidance for future development based on the foundation of GL.

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This paper is structured as follows: the main body will first discuss the foundation of GL within the framework of general relativity; the second part will introduce the current research on the applications of GL; the final part will focus on the shortcomings of the practical application of GL. The study facilitates readers to build a comprehensive understanding of current research progress and flaws in GL, providing suggestions on later developments based on analysis of existing research.

2. Origin of Gravitational Lensing (GL) under the frame of General Relativity (GR)

When Special Relativity was first proposed in the 1910s, a universal truth was revealed: Newtonian classical mechanics' absolute space-time theory is no longer applicable when the speed discussed approaches the speed of light c. Subsequently, with the introduction of Minkowski geometry, General Relativity was successfully established, revealing that space-time is a relative concept and that it can interact with matter. Therefore, Euclid geometry was no longer a suitable tool for describing space. Alternatively, the metric was applied to GR as an auto-suitable "ruler" which could modify itself to fit the space.

In Newtonian classical physics photon, the particle that constructs light would be affected by gravitational force when assuming its mass to be larger than zero. However, GR reveals that gravitational force reflects the influence of matter on space-time, demonstrating that light bends in space because it always travels along the shortest path in space, which is called a geodesic. Furthermore, light is influenced by gravitational force regardless of its mass; instead, its behavior depends on the influence of matter on space-time. Thus, if a distant light source like a star, a relatively massive celestial object such as a cluster of galaxies and an observer coincidentally share one same line, the light that initially couldn't be captured by the observer now becomes deflected and captured by the observer. This is known as Gravitational Lensing (GL).

In the Newtonian Mechanics Frame, as the speed of light surpasses the escape velocity for one certain celestial object, the trajectory of the light would be hyperbolic, and its eccentricity could be represented as [1]:

$$\varepsilon=\sqrt{1+\frac{2EL^2}{G^2m^3M^2}} \tag{1}$$

Here, G refers to the Gravitational Constant, m is the mass of a photon, M is the mass of the celestial object. E refers to the total mechanical energy of the photon, and L is the angular momentum of the photon.

By substituting E and L with their corresponding expressions, the expression for eccentricity can be written as [1]:

$$\varepsilon=\frac{r_p v_t^2}{GM}-1 \tag{2}$$

Here, r_p refers to the impact parameter of the photon to the celestial body, and v_t refers to the tangential velocity of the photon. When considering $v_t=c$, the hyperbolic trajectory's asymptote angle is given by [1]:

$$\cos(\beta)=\frac{1}{e} \tag{3}$$

Thus, the total angular deflection is written as [1]:

$$\delta_{NG}=\pi-2\beta=\pi-2\cos^{-1}\left(\frac{GM}{c^2r_p-GM}\right)\approx 2\frac{GM}{r_s c^2} \tag{4}$$

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And in Einstein's General Relativity, considering the curvature of space-time, the result would be the product of the Newtonian Mechanics' result and a factor of 2 [1]:

$$\delta_{GR}=4\frac{GM}{r_s c^2} \tag{5}$$

While the formula above provides a general expression for the deflection angle of light as it passes through a massive celestial body, a lens approximation formula can also be derived [2]. Assuming that the distance between the distant light source and the massive lensing object is D_{ds}, and the distance between the massive lensing object and the observer is D_d —the distance between the observer and the distant light source is D_s. Additionally, the lensing equation of each individual source plane β_s is independent of other source planes:

$$\beta_s=\theta-\frac{D_{ds}}{D_s}\alpha(\theta) \tag{6}$$

The equation gives a direct relationship between the angular distance of the source β_s, the deflection angle α and the angle between the line perpendicular to both planes of source and lens and the emergent light. A notable point of the equation is that the lensing mass on a random minor plane is indeed equivalent to that on the main plane, allowing for an effective conversion [2].

Another significant property of gravitational lensing is the famous time delay $t(\theta)$, also known as the arrival time or the Fermat Potential [2]. Time delay occurs when a distant light source emits photons, and each individual photon may take on a different light path traveling past the massive lensing object and being deflected differently. When these photons ultimately reach the observer plane, the arrival time of each individual photon is different, and this difference is known as the time delay.

Therefore, for a Sun-like deflectors at distance D, the combined geometrical and gravitational time delay is given by [2]:

$$t(\theta)=\frac{D}{2c}|\theta-\beta|^2-\frac{G}{c^3}\sum_i M_i \ln|\theta-\theta_i| \tag{7}$$

3. Current feasible proposed applicable scenarios of Gravitational Lensing

3.1. Different types of Gravitational Lensing

In the previous part of this paper, Gravitational Lensing (GL) is defined as a phenomenon that occurs when a distant light source, a massive lensing object, and an observer are aligned. Nevertheless, the occurrence of gravitational lensing doesn't necessarily depend on the rigorous condition; instead, it could be achieved regardless of that condition. Considering mass of lensing objects and alignment degrees, different conditions could result in various types of lensing. And the three major types of lensing are Strong Lensing (SL), Weak Lensing (WL) and Micro Lensing (ML) [2-4]—each corresponds to different possible applications [5-7].

SL occurs when the light source, the massive lensing object, and the observer achieve a nearly perfect degree of alignment, and the lensing object is a massive celestial body such as galaxies or galaxy clusters. The deflection angle of SL is usually large, and the most captured phenomenon includes Einstein Rings, Einstein Crosses, Giant Arcs, and multiple images. As one noticeable phenomenon of SL, Einstein Ring corresponds to a special solution to the former lens equation (6) —the high alignment degree. Nevertheless, Einstein Ring doesn't only occur in SL, rather, it would also appear in WL, but Einstein Ring caused by SL is brighter and clearer—basically these are different forms of the same gravitational deflection effect.

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Applying "thin lens equation" $\frac{1}{d_1}+\frac{1}{d_2}=\frac{1}{f}$, in which d_1 represents the distance between source and lens, d_2 represents the distance between lens and observer, the Einstein Radius could be obtained [2]:

$$b_E=\sqrt{4\frac{GM}{c^2}\frac{(d_1+d_2)}{d1d2}}=\sqrt{4\frac{GM}{c^2}\frac{D_{LS}}{D_L D_S}} \tag{8}$$

WL occurs when a relatively low degree of alignment is achieved, and common lensing objects are large-scale structures such as galaxy groups. Usually, WL's phenomenon includes relatively small deflection angle, small shear and changes in ellipticity. Specifically, the distortion of a single galaxy is difficult to capture whilst the overall slight shear of a large quantity of galaxies could be statistically determined.

ML occurs when the lensing object is a low mass object such as planet, star and stellar-mass black hole, inducing a change of brightness of background stars—graphing the instantaneous change of brightness indicates a single, smooth and symmetric peak—a bell-like curve.

3.2. Applications of Gravitational Lensing in cosmological tests, dark matter detection, and its deflection of Gravitational Waves

Strong Lensing (SL) provides a feasible method for determining the Hubble Constant.

H_0 and estimating the expanding rate of the universe precisely. The two major cosmological models discussed in the paper are ΛCDM and $R_h=ct$, which estimate the angular diameter distance between red shifts z_1 and z_2 in different ways [3]:

$$D^{\Lambda CDM}(z_1,z_2)=\frac{c}{H_0}\frac{1}{1+z_2}\int_{z_1}^{z_2}\left[\Omega_m(1+z)^3+\Omega_r(1+z)^4+\Omega_\Lambda\right] \tag{9}$$

$$D^{R_h=ct}(z_1,z_2)=\frac{c}{H_0}\frac{1}{1+z_2}\ln\left(\frac{1+z_2}{1+z_1}\right) \tag{10}$$

Here, $\Omega_m=\frac{\rho_m}{\rho_c}$, $\Omega_r=\frac{\rho_r}{\rho_c}$, $\Omega_\Lambda=\frac{\rho_\Lambda}{\rho_c}$, and assume in a zero spatial curvature universe that $\Omega_m+\Omega_r+\Omega_\Lambda=1$. Here, ρ_m, ρ_r and ρ_Λ refer to current mass, radiation and dark energy density respectively, and parameter $\rho_c=\frac{3c^2H_0^2}{8\pi G}$ represents the critical density of universe.

To constrain the cosmological models, there are mainly three methods—the time delay $\Delta\tau$ based on SL, the velocity dispersion σ_{SIS}^2 obtained from Einstein Radius Equation (8), and a combination $\frac{\Delta\tau}{\sigma^2}$, thus leading to a conclusion about proportionality [3]:

$$\Delta\tau\propto\frac{D_S D_{LS}}{D_L},\ \sigma_{SIS}^2\propto\frac{D_{LS}}{D_L},\ \frac{\Delta\tau}{\sigma_{SIS}^2}\propto D_s \tag{11}$$

After applying 36 lensing systems and the Akaike Information Criterion (AIC) in TDL test, the estimated result is obtained— $AIC_{\Lambda CDM}=9.35$, $AIC_{R_h=ct}=7.71$, indicating that the likelihood of $R_h=ct$ being correct equals 74.8 per cent whilst the likelihood of ΛCDM being correct equals 25.2 per cent. Nevertheless, more data is required for further ruling out one model at 99.7 per cent level confidence [3].

Additionally, research indicates that $\frac{\Delta\tau}{\sigma^2}$ is more effective than $\Delta\tau$ in the constraint of Ω_m. $\frac{\Delta\tau}{\sigma^2}$ and σ^2 are not independent, while $\frac{\Delta\tau}{\sigma^2}$ and $\Delta\tau$ are independent. The overall result suggests that the $\frac{\Delta\tau}{\sigma^2}$ model would become one powerful tool for Cosmological Model Selection [3].

Furthermore, Gravitational Lensing could also help test General Relativity (GR) and constrain spacetime parameters for black holes.

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Starting from the initial equation estimating a light ray that comes from infinity and then travel past the Schwarzschild black hole at a minimum radius $r=r_{min}$ and then escape [4]:

$$\frac{1}{\lambda^2}=\frac{1}{r_{min}^2}-\frac{2M}{r_{min}^3} \tag{12}$$

A further formula of deflection angle could be derived by appropriate approximation:

$$\alpha=\frac{4M}{r_{min}} \tag{13}$$

The closest approach to Kerr-Newman black holes is given by [4]:

$$r_0(\lambda)=\frac{\lambda}{\sqrt{6}}\sqrt{1-\omega^2}\left[\sqrt{1+\gamma}+\sqrt{2-\gamma-\frac{3\sqrt{6}m\left(1-\omega^2\right)}{\lambda\left(1-\omega^2\right)^{\frac{3}{2}}\sqrt{1+\gamma}}}\right] \tag{14}$$

This indicates that the non-zero charge of black holes repels light rays, while their spin attracts light rays toward the black hole.

In addition to cosmological tests, Gravitational Lensing is also highly applicable to dark matter probing, as dark matter contributes significantly to light deflection, inducing additional deflection angles. These additional deflection angles could facilitate the estimation of distribution of dark matter, further helping to develop the origin and formation of universe since dark matter has long been considered as one dominant element in the initial formation of universe.

Specifically, the change in distribution of matter, the existence of dark matter, impacts the strong lensing as it induces image magnification, image positions, image time delay, and polarization angle of image [5]; the creation of relative magnification of unresolved lensed images is also named flux-ratio anomalies and could only be applied in probing dark matter when observing at wavelengths in which angular size of lensed objects is larger than the scale of ML, and dust and free electron absorption/scattering is low. For resolved sources, these anomalies—the local changes in the relative positions of multiple lensed images—can only be caused by a gravitational overdensity rather than microlensing, resulting in highly magnified arcs, Einstein Rings, and occasionally surface-brightness anomalies. Image time delay is the weakest phenomenon of change in distribution of matter because the anomalies induced by subhaloes are in the order of only a fraction of one day, which is far smaller than typical time delay uncertainty (which could be equal to or larger than one day), thus requiring more precise measurement. Based on time delay measurements, multiple lensed images undergo varying levels of rotation during propagation, leading to differential birefringence—which can thus be used to estimate the particle mass and its coupling to photons.

Gravitational Lensing (GL) also could deflect Gravitational Waves (GW) since the wavelength of GW is relatively small compared to the mass scale of the lens, and the propagation of waves behave like light in classical Gravitational Lensing Theory. Starting from line element of space-time disturbed by the lens [6, 7]:

$$ds^2=-(1+2U)dt^2+(1-2u)dr^2=g_{\mu\nu}dx^\mu dx^\nu \tag{15}$$

Thus, the amplification when a waveform encounters the lens is given by:

$$F(r_0)=\left[\prod_{j=1}^{N-1}\int\frac{d^2\theta_j}{A_j}\right]\exp\left\{i\omega\left[\epsilon\sum_{j=1}^{N-1}\frac{r_j r_{j+1}}{2}\left|\frac{\theta_j-\theta_{j+1}}{\epsilon}\right|^2-\psi(\theta_L)\right]\right\} \tag{16}$$

4. Analysis of plasma's impact on gravitational lensing

Initially, plasma's influence on gravitational lensing was considered to be evident, but further research indicates that the influence is only phenomenal in certain scenarios. The presence of plasma can induce several lensing effects, including: the positions and magnifications of lensed images,

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time delays between multiple images, and shear. Specifically, Magnification ratio of multiple images is determined by both mass distribution and plasma inhomogeneity.

Incorporating plasma lensing also provides a method for calculating plasma density; a higher plasma density occurs when the separation between lensed images is greater. However, the influence of plasma on lensing is only significant when observing at a low frequency (~100MHz) and when plasma inhomogeneity is high [8].

Consequently, people estimating Hubble Constant at very low frequency should take the effect induced by plasma into consideration.

5. Conclusion

The nature of Gravitational Lensing (GL) is an extrinsic property of light: its tendency to propagate along spacetime geodesics, which appear curved to an observer when light passes through a spacetime region with non-zero curvature. Specifically, the deflection angle of light ray induced by a lensing object in General Relativity is indeed twice the result derived from Newtonian Mechanics, and a lens equation could be obtained describing the relationship between source, lens, and observer.

GL has three main different types of lensing—Strong Lensing (SL), Weak Lensing (WL), and Micro Lensing according to the alignment degree of source, lens and observer and also the lens mass. Strong Lensing is associated with prominent observational phenomena, including large deflection angles, multiple images, Giant Arcs, Einstein Rings, and Einstein Crosses. Different types of lensing lead to different application scenarios respectively.

Gravitational Lensing could be applied in different fields; SL could facilitate cosmological test, test of General Relativity and also the distribution of small-scale dark matter. GR could also be influenced by high plasma inhomogeneity when observing at low frequency, actually providing one feasible method to estimate the density of plasma.

Ultimately, special characteristics and multiple advantages of Gravitational Lensing (GL) suggests that it would be widely applied and thoroughly studied in future—especially when AI technology could associate with existing GL to achieve AI lensing—the potential applications of GL is unpredictable. Also, further study would require more available data to testify certain cosmological model or to constrain cosmological parameters.

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