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Review of Full-Waveform Seismic Inversion: Methodological Evolution, Frontier Exploration and Future Directions

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Abstract. Full-waveform inversion (FWI) adopts the dynamic and kinematic information contained in pre-stack seismic wavefields and fits actual observed seismic data using the wave equation. It can realize high-resolution quantitative imaging of physical property parameters of underground media, with a theoretical imaging accuracy reaching half of the seismic wavelength. Since Tarantola and Lailly established the core theoretical framework in the 1980s, FWI has gradually evolved from two-dimensional acoustic approximate algorithms to a three-dimensional elastic and viscoelastic multi-parameter inversion system, and its application scope has expanded from simple geological structures to complex stratigraphic regions. In recent years, the iteration and upgrading of high-performance computing hardware, the application of deep learning algorithms in geosciences, and the popularization of multi-component and wide-azimuth seismic acquisition equipment have jointly driven FWI into a brand-new development stage. This paper systematically sorts out the theoretical system and classical research methods of FWI, and summarizes the development context of core technologies including multi-scale inversion, objective function construction, initial model establishment and computational efficiency optimization. Meanwhile, it concludes the latest achievements of research hotspots such as elastic and viscoelastic multi-parameter inversion, deep learning integrated FWI, uncertainty quantification, 4D time-lapse monitoring and cross-scale imaging. Finally, combined with the industrial development trend, this paper prospects future research directions including deep integration of physics and artificial intelligence, large-scale high-efficiency inversion, multi-physical field coupled imaging, and standardization of result interpretability. It aims to provide a systematic literature review and research reference for researchers engaged in seismic inversion and oil and gas exploration.

Keywords: Full-waveform inversion, seismic imaging, deep learning, multi-parameter inversion, uncertainty quantification, elastic wave inversion

1. Introduction

Accurate characterization of fine underground structures and precise inversion of physical property parameters have always been the core research objectives in the fields of oil and gas exploration, engineering geological investigation, environmental geophysical detection, and crust and mantle structure research. Traditional velocity modeling methods mainly include traveltime tomography

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and migration velocity analysis. These methods only utilize the kinematic feature of seismic traveltime, and their imaging resolution is limited by ray coverage and the radius of the first Fresnel zone, which makes them unable to meet the requirements of high-precision exploration. Against this background, full-waveform inversion emerged and has been continuously improved. The theoretical prototype of FWI was first proposed by Tarantola in 1984. He constructed the theoretical system of seismic waveform inversion from the perspective of functional analysis, and derived the gradient calculation formulas for bulk modulus and density parameters under the acoustic medium assumption [1]. In the same period, Lailly independently established the theoretical framework of adjoint state inversion based on the wave equation [2].

Different from traditional imaging methods that only rely on traveltime information, FWI fully matches the amplitude and phase information of seismic records through full wavefield numerical simulation, and integrates both kinematic and dynamic characteristics. Its theoretical imaging resolution can reach half of the seismic propagation wavelength, so it is regarded as a core technology with great potential in the field of seismic imaging. From the mathematical perspective, FWI is a highly nonlinear and ill-posed large-scale inversion problem, which takes minimizing the residual between observed data and simulated data as the optimization objective. Its practical solution has long been faced with three core problems that restrict the field application of the technology due to mutual constraints.

The first problem is the strong nonlinearity of the objective function and cycle skipping. The commonly used L2-norm objective function of FWI has a large number of local minima. When there is a large deviation between the initial model and the real underground stratigraphic structure, the phase difference between simulated waveforms and actual waveforms will exceed half a cycle, causing the iterative optimization to converge to a wrong local optimal solution and resulting in cycle skipping. In the classical research on multi-scale inversion, Bunks et al. verified that traditional iterative algorithms can hardly achieve effective convergence even with the Marmousi standard model, because the objective function is distributed with numerous dense local minima in the multi-scale space [3]. To solve this problem, scholars proposed the multi-scale inversion idea of gradual iteration from low frequency and large scale to high frequency and small scale. This method uses the sparse local minima of the large-scale objective function to guide the algorithm to approach the global optimal solution. After reviewing existing studies, Virieux and Operto pointed out that there are two main ways to suppress cycle skipping: one is to construct a high-precision initial velocity model, and traveltime tomography is the commonly used method; the other is to ensure that seismic data contains complete and reliable low-frequency components [4]. Métivier et al. adopted the optimal transport distance to construct a new objective function. Studies have proved that compared with the traditional L2 norm, the Wasserstein distance has better convexity and a stronger suppression effect on cycle skipping [5, 6]. In recent years, Zhang et al. tried to combine well logging data and migration imaging results, and adopted generative diffusion models to build high-precision initial models, providing a new idea for solving cycle skipping [7]. Up to now, cycle skipping is still a key bottleneck restricting the practical engineering application of FWI.

The second problem is the excessive computational cost of forward modeling and gradient calculation. Each iteration of FWI requires seismic wave forward modeling and adjoint gradient solution. The adjoint state method was introduced into seismic inversion by Lailly, which can calculate the gradient of millions of model parameters with the computational cost of approximately two forward simulations [2]. Even with this optimized algorithm, large-scale FWI calculation under three-dimensional elastic and viscoelastic media still puts forward extremely high requirements on computing resources. Pratt first introduced the frequency-domain theory into waveform inversion,

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simplified the inversion expression in matrix form, improved computational efficiency by adopting finite difference and finite element forward algorithms, and realized sub-wavelength resolution imaging in physical scale models through gradual iteration from discrete low frequency to high frequency. He established the application paradigm of frequency-domain FWI [8].

With the rapid development of high-performance computing technology, three-dimensional FWI has achieved the leap from theoretical research to practical application. Tromp et al. adopted GPU-accelerated spectral element method to simulate seismic wave propagation in three-dimensional inelastic crust and mantle, and built the first global three-dimensional mantle model based on adjoint tomography, which verified the feasibility of large-scale FWI iteration on supercomputing platforms [9]. Du et al. developed the open-source toolkit Cube2sph-GPU and built a GPU-accelerated FWI framework suitable for regional and teleseismic detection, which further expanded the application scenarios of high-performance computing in multi-scale seismic inversion [10]. Despite the continuous optimization of algorithms and hardware, the high computational cost of three-dimensional multi-parameter FWI still hinders its large-scale industrial promotion.

The third problem is the ill-posedness caused by noise interference, data loss and multi-parameter coupling. Actually acquired seismic data are inevitably affected by environmental noise. In addition, the limited aperture of observation systems and narrow effective frequency band lead to inherent ill-posedness and non-uniqueness of FWI inversion. When FWI expands from single acoustic velocity inversion to synchronous inversion of elastic, anisotropic, attenuation and other multiple parameters, the coupling response of different physical parameters to the seismic wavefield will produce crosstalk effect, which further increases the complexity of inversion. In the research on multi-parameter FWI, Operto et al. pointed out that different stratigraphic parameters have obvious differences in physical dimensions and wavefield sensitivity, and the parameter coupling characteristics are prominent under the propagation mechanism of transmitted and reflected waves. Unreasonable parameterization schemes and scale processing will greatly aggravate the ill-posedness of inversion [11].

To address this issue, scholars tried to use the Hessian matrix to realize multi-parameter decoupling, suppress parameter crosstalk, and establish the FWI workflow suitable for visco-acoustic anisotropic media [11]. Meanwhile, uncertainty quantification has gradually become a research hotspot of FWI. Based on the Bayesian theoretical framework, it quantifies the error range and reliability of inversion results by solving the posterior probability density of model parameters. However, limited by the high computational complexity and strong ill-posedness of FWI, high-precision uncertainty quantification has long been difficult to implement. Marchner et al. combined Monte Carlo sampling and variational inference methods, and constructed a unified analysis framework for FWI uncertainty quantification based on optimal transport and gradient flow theories [12].

The above three core problems have restricted the development of FWI for a long time, and also promoted the continuous iteration of its theories and algorithms for more than forty years. Three remarkable trends have emerged in the industry in recent years. First, the exponential improvement of high-performance computing capability has enabled three-dimensional elastic and viscoelastic FWI to move from theoretical model tests to practical geological applications. As summarized by Tromp et al., the combination of modern numerical algorithms and supercomputing platforms can realize imaging across nine orders of magnitude in frequency and wavelength scale, covering near-surface detection to global mantle structure research. Exascale supercomputers are expected to realize global mantle FWI imaging with a resolution below 10 kilometers at the frequency of 1 Hz [9]. Second, deep learning technology is deeply integrated with FWI, breaking the traditional purely

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physics-driven optimization mode. Methods such as end-to-end velocity modeling, physics-constrained neural networks and deep generative prior regularization have emerged one after another. In the research of marine seismic imaging, Chen mentioned that intelligent interpolation, multi-source joint inversion and physics-guided deep learning are promoting the transformation of FWI towards the integration mode of physical constraints and data driving [13]. Singh et al. introduced transfer learning into FWI, and effectively improved iterative convergence speed and imaging accuracy by pre-training to optimize the initial network weights [14]. Third, research focus has shifted from "whether inversion can be realized" to "whether inversion results are reliable". Multi-parameter joint inversion, Bayesian uncertainty quantification and cross-scale integrated modeling have become the mainstream research directions, with more attention paid to the practicability and transferability of imaging results.

2. Basic theories and classical methods of full-waveform inversion

2.1. Mathematical foundation: from Tarantola To Lailly

The mathematical basis of full-waveform inversion (FWI) can be traced back to the pioneering work of Tarantola and Lailly. Tarantola expressed seismic waveform inversion as a nonlinear optimization problem under the generalized least squares framework. Starting from the inverse scattering theory, Lailly revealed the internal relationship between FWI gradient calculation and the adjoint state method. Together, they established the basic computational paradigm of FWI: conducting forward modeling using the wave equation, efficiently calculating the gradient of the objective function via the adjoint equation, and iteratively updating underground model parameters combined with local optimization algorithms. The discrete wave equation is written as:

$$\mathrm{F(m)u = s} \tag{1}$$

where m stands for model parameters, u represents wavefield, and s is the source term. The simulated data at receiving points are extracted by the receiver operator R, namely:

$$\mathrm{d^{cal} = Ru} \tag{2}$$

FWI is generally formulated as the following nonlinear least squares problem:

$$\min_{m} \Phi(m) = \frac{1}{2}\,||Ru(m) - d^{obs}||_2^2 \tag{3}$$

Let the data residual be $\mathrm{R(m) = Ru(m) - d^{obs}}$, then:

$$\Phi(m) = \frac{1}{2}\,r(m)^T r(m) \tag{4}$$

To avoid explicitly constructing a large-scale Fréchet derivative matrix, the adjoint state method is usually adopted to calculate the gradient in FWI. The Lagrangian function is introduced as:

$$\mathscr{L}(\mathrm{u, m, \lambda}) = \frac{1}{2}\,||Ru - d^{obs}||_2^2 + \lambda^T(\mathrm{F(m)u - s}) \tag{5}$$

Taking variation with respect to the wavefield variable u, we can obtain the adjoint equation:

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$$F(m)^T\lambda = R^T(Ru - d^{obs}) \tag{6}$$

Further taking the derivative with respect to model parameter m, the gradient of the objective function can be obtained:

$$\nabla_m \Phi(m) = -\lambda^T \frac{\partial F(m)}{\partial m} u \tag{7}$$

This formula indicates that the FWI gradient is jointly determined by the forward wavefield and the adjoint wavefield, and its physical meaning can be understood as the correlation effect of the two wavefields. Based on this gradient, model parameters can be updated by optimization algorithms such as gradient descent, nonlinear conjugate gradient method or L-BFGS, so as to gradually approach the real underground model.

2.2. Forward modeling

Forward modeling is the core foundation of full-waveform inversion. Its simulation accuracy, computational efficiency and ability to characterize complex wavefield effects such as scattering, dispersion and anisotropy directly determine the imaging resolution and reliability of FWI. At present, mainstream numerical forward methods are divided into three categories. The Finite Difference (FD) method has a simple algorithm structure and strong parallelism, and is widely used in three-dimensional acoustic and elastic wave simulation [8]. The Finite Element Method (FEM) and Spectral Element Method (SEM) adopt unstructured meshes, which can accurately fit undulating surfaces and complex structural boundaries, and have outstanding advantages in regional and global-scale seismic simulation [15, 16]. The Pseudospectral Method (PSM) solves spatial derivatives by fast Fourier transform and achieves ultra-high simulation accuracy in frequency-domain FWI [8].

Frequency-domain and mixed-domain forward algorithms have attracted increasing attention in recent years. Frequency-domain FWI gradually iterates from discrete low frequency to high frequency, which reduces computational cost and effectively alleviates the nonlinearity of inversion [8]. The Laplace-Fourier domain method uses the damping characteristics of complex frequency to extract the early arrival information of seismic waves. It can stably recover the large-scale background velocity of strata from observed data with strong attenuation and missing low-frequency components, and has good noise resistance and convergence stability [17].

2.3. Objective function and optimization methods

The construction form of the objective function directly determines the iterative convergence characteristics of FWI. The traditional L2-norm objective function is easy to calculate, but it is highly sensitive to waveform phase deviation and easily causes cycle skipping and trapping in local minima. To expand the convergence domain of algorithms, scholars have proposed a variety of new objective functions one after another. The cross-correlation traveltime objective function extracts traveltime residuals through waveform cross-correlation, and transforms the strongly nonlinear phase distortion problem into a weakly linear traveltime perturbation problem [18]. The instantaneous phase objective function extracts phase features based on Hilbert transform and reduces the interference of amplitude distortion on inversion [19]. The envelope objective function preferentially fits the waveform envelope, and can stably recover large-scale velocity structures under the condition of missing low-frequency components [20, 21]. The Wasserstein distance

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objective function based on optimal transport has become one of the mainstream schemes to suppress cycle skipping due to its better convexity [5, 6].

In terms of optimization algorithms, the nonlinear conjugate gradient method and L-BFGS quasi-Newton method have become mainstream choices for actual data processing because of low memory consumption and stable convergence. Such first-order algorithms can realize superlinear convergence only relying on gradient information. The Gauss-Newton method and truncated Newton method introduce objective curvature and Hessian matrix information, which can effectively decouple parameter crosstalk and accelerate convergence, but bring a significant increase in computational and storage costs. In practical engineering, matrix-free algorithms combined with low-rank approximation are often adopted to balance imaging accuracy and computational efficiency.

2.4. Multi-scale inversion strategy and classical workflow

The multi-scale inversion strategy proposed by Bunks et al. is a methodological innovation of great practical value in the development of FWI [3]. Studies have found that the objective function surface corresponding to low-frequency components of seismic data is gentler with sparse local minima, while high-frequency components will lead to drastic fluctuations of the objective function. Based on this conclusion, the progressive inversion idea of "recovering large-scale structures with low frequency and fine structures with high frequency" is formed. The inversion results of low frequency are taken as the initial model for high-frequency iteration to fundamentally suppress cycle skipping. This strategy is complementary to the progressive idea of "from far offset to full angle" in the Laplace-Fourier domain, and has become a standard paradigm to ensure the robust convergence of FWI [3, 17].

The standard workflow of conventional FWI is summarized into four parts. First is seismic data preprocessing, including basic steps such as denoising, static correction and source wavelet estimation. Second is initial model construction, which obtains the long-wavelength background structure of strata relying on traveltime tomography and migration velocity analysis. Third is multi-scale iterative inversion, including frequency grouping, wavefield forward modeling, residual calculation, adjoint gradient solution and model updating in sequence. Fourth is post-processing and accuracy evaluation of inversion results. The reliability of imaging results is evaluated by combining well logging data calibration, migration gather analysis, uncertainty test and other methods.

3. Evolution of core methods and key challenges

3.1. Multi-scale strategies in data domain and model domain

The multi-scale idea is a core means to solve the nonlinearity and cycle skipping problems of FWI. After years of development, it has expanded from the traditional frequency scale to a multi-dimensional optimization system covering data domain and model domain. The classical frequency-domain multi-scale strategy adheres to gradual iteration from low frequency to high frequency, and uses the smooth characteristics of large-scale objective functions to guide global convergence, which is still a general industrial standard up to now [3].

In the data domain, source encoding technology provides a new way to improve the computational efficiency of multi-scale inversion. Krebs et al. proposed the random phase-encoded super shot strategy, which combines multiple seismic sources into virtual super shots. This method decouples the iterative computational cost from the number of sources and realizes large-scale

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acceleration of multi-source FWI [22]. Aghazade K et al. further established a unified framework of randomized source sketching. It compresses the solution space through random projection in the source dimension and greatly reduces the number of partial differential equation solutions. Numerical tests verify that this method can reduce the cost of large-scale inversion by more than one order of magnitude [23].

In the model domain, the Laplace-Fourier domain multi-scale method has unique characteristics. It recovers velocity structures of long, medium and short wavelengths in turn by adjusting complex frequency parameters, and has stronger adaptability under complex observation conditions such as missing low-frequency components and limited offset. In recent years, multi-scale feature extraction technology of deep learning has been introduced into FWI. The multi-scale dual-encoding diffusion model accurately extracts multi-scale features of complex faults and strata with abrupt velocity changes through noise scheduling and attention mechanism, and significantly improves the inversion accuracy of fine structures [24].

3.2. Initial model construction: from tomography to generative models

The occurrence of cycle skipping essentially stems from the extremely high dependence of full-waveform inversion on the accuracy of the initial model. Gradient-based iterative algorithms can converge normally only when the initial velocity model falls within the convergence range of the global optimal solution of the objective function. Once deviating from this range, the inversion results are very likely to fall into local minimum solutions far away from real strata. In the past, initial models were mostly constructed by two conventional methods: traveltime tomography and migration velocity analysis.

With the continuous application of intelligent algorithms in geophysical inversion, modeling methods represented by Generative Diffusion Model (GDM) have gradually become a new research hotspot. This method can integrate well logging data and prior geological structural constraints to generate initial velocity fields conforming to regional geological laws, and effectively improve the iterative convergence stability and final imaging accuracy of FWI. It also marks that data-driven prior information has been deeply integrated into traditional physics-based inversion systems.

From the perspective of theoretical mechanism, the FWI objective function has strong non-convexity. Pure local optimization via gradient is highly dependent on the starting position of the initial model. For measured data from complex structural areas or seismic data missing low-frequency components, traditional traveltime-based modeling methods can hardly provide long-wavelength background structures that meet the requirements of inversion convergence. How to construct high-precision initial models has always been a prominent problem restricting the field application of FWI.

In recent years, deep learning has provided new ideas for initial modeling. In particular, generative diffusion models have attracted extensive attention from scholars due to their powerful feature learning and uncertainty characterization capabilities. Zhang et al. [7] proposed an initial modeling scheme of diffusion model constrained by well logging and structures, which integrates prior geological information into velocity modeling, effectively characterizes the details of faults and stratum boundaries in complex work areas, and plays an obvious role in suppressing cycle skipping. Wang et al. [25] introduced the diffusion model as a regular prior into the FWI framework. It learns the general distribution rule of underground velocity through unsupervised pre-training and is embedded into the inversion sampling process. Reliable underground structures can be reconstructed even under sparse observation and strong noise conditions.

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Meanwhile, intelligent networks such as Fourier Neural Operator can directly predict the background velocity model from raw shot gathers in an end-to-end manner, and can adapt to actual seismic data only through training with simulated data [26]. Basic model frameworks such as LAUNet combine low-rank adaptation and attention mechanism, and realize small-sample fine-tuning via transfer learning, possessing excellent cross-regional generalization modeling capability [27]. In general, deep generative models can not only provide richer geological prior constraints than traditional tomography, but also output modeling uncertainty synchronously, which provides a new technical path for the reliability evaluation of inversion results [28, 29].

3.3. Mitigation strategies for cycle skipping

Besides the multi-scale iterative framework, reconstructing the measurement form of the objective function and expanding the inversion optimization space are another important technical route to reduce the nonlinearity of FWI and avoid cycle skipping.

Wavefield Reconstruction Inversion (WRI) is a representative method in this direction. Van Leeuwen and Herrmann [27] first put forward its core idea. Instead of taking the wave equation as a rigid constraint, they incorporated it into the optimization objective in the form of a penalty function, allowing wavefield state variables and underground model parameters to be updated alternately. This constraint relaxation method greatly expands the search space of the algorithm. Even if the kinematic accuracy of the initial model is low, it can realize effective fitting of seismic data and significantly reduce the probability of cycle skipping.

Subsequent studies expanded and optimized WRI from two dimensions of computational acceleration and theoretical improvement. Fang and Wang [28] introduced curvelet sparse constraints combined with GPU parallel acceleration, realized adaptive inversion of source functions synchronously, suppressed imaging artifacts, and lowered the computing threshold for practical application of WRI. Liu et al. [29] introduced approximate expression of Hessian matrix into the Bayesian WRI system, which provided a solid theoretical support for crosstalk suppression of multi-parameter inversion and quantitative analysis of resolution.

In addition, the combination of wavefield reconstruction idea and implicit parameterization of neural networks has spawned a new research direction. Chen et al. [30] defined neural sensitivity kernel and wave tangent kernel, and explained the internal mechanism of optimized convergence effect of neural reparameterized FWI relying on spectral filtering and gradient modulation from the theoretical level. Song et al. [31] combined meta-learning with SIREN implicit modeling. It learns general inversion prior through multi-task pre-training, and can quickly adapt to new work areas with only a small number of gradient iterations. This method not only accelerates convergence speed, but also improves the cross-regional generalization performance of the model.

3.4. Computational efficiency and high-performance computing optimization

Excessively high computational cost has always been the core bottleneck restricting the large-scale industrial application of three-dimensional multi-parameter FWI. Industrial-level practical inversion usually contains thousands of seismic source data, billions of unknown model parameters and hundreds of iterative updates. Even if the adjoint state method reduces the computational cost of a single gradient calculation to twice the cost of forward modeling, the overall computation volume is still huge, which sets strict requirements on both numerical algorithms and hardware architectures.

In terms of algorithm optimization, stochastic optimization strategies estimate gradients by extracting partial data subsets in each iteration, and greatly reduce computational cost at the cost of

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statistical approximation. Song et al. first introduced Stochastic Gradient Descent (SGD), Adam and other stochastic gradient algorithms into the FWI solution system. Settgast et al. [32] further constructed an elastic stochastic FWI scheme under CPU/GPU heterogeneous architecture, which fully taps the peak computing power of high-performance clusters.

In terms of hardware acceleration and engineering implementation, multi-machine GPU parallelism has become the standard architecture for industrial FWI. Aiming at complex undulating surface scenarios, the ElasWave3D platform adopts irregular subdomain indexing strategy to realize GPU-accelerated simulation of elastic wave finite difference, which has an obvious speed advantage compared with the 24-core CPU parallel version [33]. Targeting the computing bottleneck of three-dimensional WRI, Fang and Zong [34] developed an adaptive GPU acceleration algorithm. It realizes an extremely high acceleration ratio through dynamic adjustment of simulation accuracy and optimization of hardware resources, and truly promotes the practical engineering application of three-dimensional WRI.

In actual oil and gas exploration work areas, the application of elastic FWI in pre-salt carbonate reservoirs of the Santos Basin accurately characterizes salt rock boundaries and abrupt reservoir velocity changes relying on GPU operator optimization and time-lapse objective function design [35]. The Cube2sph-GPU toolkit developed by Du et al. provides a flexible heterogeneous acceleration scheme for FWI at continental and teleseismic scales [10].

Overall, the improvement of FWI efficiency is the result of multi-dimensional coordination of algorithms, coding and hardware. Stochastic optimization and mixed precision reduce the computational cost of iteration; source encoding and phase encoding cut down the number of forward simulations; distributed GPU parallelism shortens the time of a single simulation. The superposition of multiple technologies has compressed the operation cycle of industrial three-dimensional elastic FWI from several weeks to several days or even hours, which effectively promotes the transformation of full-waveform inversion from scientific research experiments to industrial standardized business processes.

4. Current research frontiers and hot topics

With the continuous maturity of computing hardware, seismic observation technology and intelligent algorithms, FWI has developed from traditional single-parameter acoustic inversion into a comprehensive imaging technology oriented to complex media, multi-parameter coupling, probabilistic evaluation and cross-scale monitoring. Current research hotspots are mainly divided into four categories.

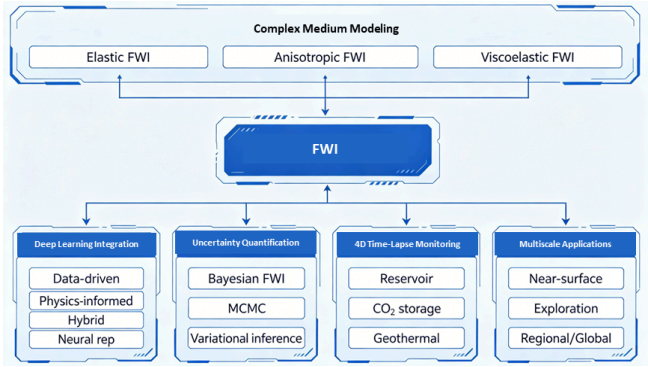

Figure 1. Schematic diagram of current research frontiers and hot topics of Full-Waveform Inversion (FWI)

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4.1. Elastic, anisotropic and viscoelastic FWI

Traditional acoustic FWI ignores shear wave propagation, stratigraphic attenuation and anisotropic characteristics, and can no longer meet the processing requirements of new seismic acquisition data such as wide-azimuth and ocean-bottom node data. Elastic FWI can invert P-wave velocity, S-wave velocity and density parameters synchronously and extract abundant petrophysical information, so it has outstanding application value in reservoir identification and fluid prediction. Actual strata generally have inclined bedding and energy dispersion and attenuation, which drives the rapid development of VTI, TTI anisotropic FWI and viscoelastic FWI. By introducing anisotropic parameters and quality factor Q, these methods are more consistent with the propagation rules of seismic waves in real underground media.

The core problem faced by synchronous multi-parameter inversion is parameter crosstalk and insufficient identifiability. Different physical parameters have similar response characteristics to wavefields, which easily leads to gradient coupling and blurry imaging. Current mainstream solutions include parameter sensitivity analysis, Hessian preconditioning decoupling, phased and layered inversion, and joint inversion constrained by petrophysics [36]. Despite continuous relevant research, the iterative stability, high computational cost and parameter decoupling accuracy of multi-parameter FWI are still practical problems restricting large-scale application.

4.2. Integration of deep learning and full-waveform inversion

Deep learning has provided a new modeling and optimization paradigm for FWI. Existing research can be divided into three types: data-driven, physics-guided and hybrid integration [37]. Data-driven methods directly construct end-to-end mapping from seismic data to velocity models. Physics-guided neural networks embed the wave equation as constraints into network training to ensure that prediction results conform to physical laws. Hybrid methods give consideration to the learning advantages of data and the rigor of physical equations.

Neural network implicit parameterization has become a research hotspot. Instead of inverting model parameters grid by grid, it characterizes underground structures via network weights. Relying on its own smoothing and multi-scale prior, it achieves better imaging effects than traditional FWI under the conditions of deviated initial models, incomplete data coverage and strong noise [38]. Deep generative methods such as Generative Adversarial Network and diffusion model are also used to construct geological prior and conduct posterior sampling, providing new approaches for characterizing inversion multiplicity and uncertainty [39]. Although Physics-Informed Neural Networks and neural operators show potential in wavefield solution, their stability under high-frequency fluctuation, complex boundaries and large-scale three-dimensional scenarios still needs further verification. In general, deep learning will not replace traditional FWI, but acts as an enhancement tool to play an important role in accelerating convergence, adding prior constraints and improving accuracy.

4.3. Uncertainty quantification

Traditional deterministic FWI only outputs a single optimal imaging model, and cannot quantify the uncertainty of results caused by data noise and model errors, which fails to meet the reliability requirements of engineering decision-making and reservoir evaluation. Therefore, uncertainty quantification (UQ) has become an indispensable research branch of FWI. The Bayesian framework is the theoretical basis of UQ. It regards underground models as random variables, and solves

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posterior probability combining prior distribution and observation likelihood, so as to obtain both the optimal model and confidence interval.

However, the high-dimensional and nonlinear characteristics of three-dimensional FWI lead to excessive computational cost of accurate sampling via Markov Chain Monte Carlo (MCMC), which is difficult to implement in practice. For this reason, scholars have developed approximate inference methods such as variational inference and Laplace approximation, as well as Ensemble Kalman Filter (EnKF), to estimate posterior statistical features under controllable computational cost [40]. Meanwhile, ideas such as low-dimensional representation, common image gather features and generative model dimensionality reduction are constantly emerging to reduce computational complexity while retaining effective inversion information. UQ technology can not only improve the interpretability of FWI results, but also provide reliable basis for risk decision-making in carbon dioxide storage monitoring, geothermal development and oil and gas reservoirs.

4.4. 4D time-lapse FWI and monitoring applications

4D time-lapse FWI focuses on characterizing the dynamic changes of underground medium physical parameters over time, and it is a core monitoring tool in the fields of oil and gas reservoir development, CO_2 geological storage and geothermal resource development. Compared with traditional 4D reflection seismic methods, 4D FWI can directly invert the time-varying differences of elastic parameters, and is more sensitive to subtle stratigraphic responses caused by changes in pressure, fluid saturation and temperature, showing remarkable advantages in quantitative monitoring [41].

In practical application, there are noise differences, acquisition geometry mismatch and non-repetitive observation errors between baseline observation and monitoring observation, which are easy to cover up real time-lapse signals. Meanwhile, differential inversion is vulnerable to small singular values and null space effects, which amplify noise interference. At present, noise is mainly suppressed through spatio-temporal regularization and model constraints, and data-driven algorithms are combined to separate time-lapse anomalies induced by production. With the improvement of repeated seismic acquisition technology, the application prospect of 4D time-lapse FWI in the dynamic management of underground energy systems will continue to expand.

Overall, current FWI research presents four transformation characteristics: from simple acoustic media to complex elastic and anisotropic media, from purely physics-driven optimization to the integration of physics and data driving, from deterministic imaging to probabilistic uncertainty evaluation, and from static structural imaging to dynamic time-lapse monitoring. Nevertheless, problems such as multi-parameter crosstalk, initial model dependence, high computational cost and insufficient practical application of uncertainty quantification remain to be solved.

5. Future development directions

Facing the demand of geophysical exploration for underground models with higher accuracy, higher efficiency and higher credibility, full-waveform inversion is accelerating to evolve towards four directions: collaborative driving of physics and data, real-time processing, multi-physical field fusion and interpretable evaluation. This section outlines the key paths for FWI to develop from "usable" to "practical" and "reliable" from four aspects: deep integration of physics and artificial intelligence, large-scale real-time inversion, multi-physical field joint imaging, and standardization of interpretability and uncertainty quantification.

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5.1. Deep integration of physics and artificial intelligence: toward the fourth paradigm

At present, deep learning and FWI are still in shallow integration. The core of future development is to realize deep collaboration between physical equations and intelligent algorithms. The wave equation and adjoint equation will be embedded into neural network training as hard constraints to construct physics-informed neural networks and operator learning frameworks, so that model outputs naturally follow the physical laws of wavefield propagation. Meanwhile, networks will be used to adaptively learn the optimal objective function, regularization strategy and optimization path, forming a fourth-paradigm inversion system with both physical fidelity and intelligent efficiency.

5.2. Large-scale and real-time FWI

With the further application of FWI in the development stage, realizing real-time or near-real-time full-waveform inversion has become an emerging rigid demand. Therefore, comprehensive optimization must be carried out in the computational process, including adaptive data acquisition design based on compressed sensing, online inversion architecture based on streaming computing, and the use of surrogate models of neural networks to realize millisecond-level forward modeling and gradient estimation. The practice of rapid imaging of elastic multi-parameter FWI in the industrial field in recent years shows that the inversion delay can be reduced by several orders of magnitude through collaborative acceleration of algorithms and hardware. This will promote FWI to enter a new era of high-frequency real-time monitoring and decision support.

5.3. Multi-physical field coupling and joint inversion

The underground strata are a complex system with coupled multi-physical fields, and single seismic inversion has the limitation of multiple solutions. In the future, FWI will be integrated with multi-source data such as electromagnetic exploration, gravity survey and downhole well logging. Joint inversion will be carried out relying on petrophysical relationships and structural constraints to realize integrated imaging of underground structures and physical properties from multiple dimensions. It will improve the stability and resolution of imaging, and provide multi-dimensional parameter support for reservoir sweet spot prediction and fine characterization of geological bodies.

5.4. Interpretability and standardization of uncertainty quantification

FWI has gradually become the basis for industrial decision-making, so it is urgent to establish unified evaluation standards for result interpretability and uncertainty. Future research will focus on interpretable methods combining physical mechanisms and statistics, balance the accuracy and efficiency of UQ technologies such as Bayesian inference and variational inference, and formulate universally accepted industrial specifications for uncertainty quantification. These efforts will make FWI imaging results credible and traceable, so as to better serve engineering practice.

6. Conclusion

Since Tarantola and Lailly laid the theoretical foundation, full-waveform inversion has developed for more than forty years, evolving from a single theoretical algorithm into a multi-scale, multi-parameter and intelligent comprehensive seismic imaging technology. At present, FWI is in a critical stage of transformation from cutting-edge scientific research technology to mainstream industrial

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tools. The popularization of elastic multi-parameter inversion, the in-depth empowerment of deep learning and the practical application of uncertainty quantification have become the mainstream trends of the industry. Although inherent problems such as cycle skipping, parameter crosstalk and high computational cost have not been completely solved, with the continuous breakthrough of technologies such as physics-AI collaboration, multi-scale joint modeling and multi-physical field fusion, full-waveform inversion will play a more important role in deep earth exploration, oil and gas resource exploration, energy storage monitoring and other fields.

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