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History of Hydromagnetic Turbulence Research: From the Incompressibility and Locally Isotropic Assumption to the Hydromagnetic Equations

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Abstract. This article provides a comprehensive overview of the historical development of research on magnetohydrodynamic (MHD) turbulence, tracing its evolution from the foundations of classical turbulence theory to the formulation and analysis of MHD equations. We examine the key contributions of pioneering works, such as Kolmogorov's theory of isotropic turbulence, as well as subsequent progress in understanding MHD turbulence through the lens of Elsasser variables and spectral analysis. The article emphasises the transition from purely hydrodynamic turbulence to the complex interactions between fluid dynamics and magnetic fields in MHD turbulence, highlighting its significance in the contexts of astrophysics and plasma physics. By summarizing the applicable regimes, key contributions, and main conclusions of various models, this review aims to provide researchers with a concise guide for selecting the most suitable approaches and understanding their theoretical foundations.

Keywords: Magnetohydrodynamic Turbulence, MHD Energy Spectrum

1. Introduction

Modern research on magnetohydrodynamic (MHD) turbulence builds upon studies of purely hydrodynamic turbulence: it investigates how an imposed magnetic field affects the energy spectrum, inertial range, and other characteristics that were first established in the absence of any external field. In astrophysical environments, plasmas such as the solar wind are highly conducting, turbulent fluids whose behavior cannot be described without accounting for magnetic fields; this is precisely the regime addressed by MHD turbulence. Moreover, key cosmic processes—heat conduction, energetic-particle acceleration, magnetic-field generation—are themselves rooted in MHD turbulence, making its theory an indispensable component of astrophysical research [1].

The practical implications of MHD turbulence theory extend across multiple astrophysical scales and phenomena. In the interstellar medium, understanding MHD turbulence is essential for modelling molecular cloud dynamics and star formation efficiency [2]. The propagation and scattering of cosmic rays, which govern the Galactic cosmic-ray spectrum observed at Earth, are directly mediated by turbulent magnetic fields [3]. In stellar contexts, the solar wind turbulence properties inferred from spacecraft measurements provide critical constraints on heating

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mechanisms and the acceleration of the slow and fast solar wind streams [4]. Furthermore, accretion disk turbulence—particularly the magnetorotational instability—determines angular momentum transport and thus mass accretion rates onto compact objects [5]. These applications demonstrate that MHD turbulence theory serves not merely as an abstract mathematical framework but as a quantitative tool for interpreting observational data and predicting physical outcomes in environments inaccessible to direct experimental control.

This review traces the evolution of magnetohydrodynamic (MHD) turbulence research: from the foundations of modern turbulence theory to the analysis of inertial-range energy spectra in MHD. We summarize the applicable regimes, key contributions and principal conclusions of each model, providing researchers with a concise guide for selecting the most suitable approach and understanding its background.

2. Modern turbulence-theory models

In his 1941 paper, Kolmogorov introduced local isotropy, two similarity hypotheses and the Kolmogorov microscale, thereby establishing a statistical framework for small-scale turbulence and laying the theoretical foundation for the inertial subrange in modern turbulence theory [2].

He proposed that in turbulent flow at extremely high Reynolds numbers $(R = LU/\upsilon \gg 1)$, taking a small four-dimensional space-time domain G , whose scale is much smaller than the overall scale L and $T = L/U$ far from the boundary, the probability distribution of velocity increments is independent of the absolute position and time of G as well as the local mean velocity, and is invariant to coordinate rotation and reflection. From this, two similarity hypotheses are derived.

The first hypothesis asserts that at small scales, the statistics are determined solely by the mean dissipation rate ε and the kinematic viscosity ν . Introducing the Kolmogorov microscale $\eta = (\nu^3/\varepsilon)^{1/4}$ and the velocity $\upsilon = (\nu\varepsilon)^{1/4}$, the distance r and the velocity variance $B_{dd}(r)$ are nondimensionalized to obtain a universal function:

$$B_{dd}(r) = \upsilon^2 \beta_{dd}(r/\eta) \tag{1}$$

When $r \gg \eta$, the viscosity effect vanishes. The second hypothesis gives the second universal range: the statistics depend only on ε , yielding the inertial subrange energy spectrum.

$$B_{dd}(r) = C(\varepsilon r)^{2/3} \tag{2}$$

And it derives the ratio of the lateral structure function to the longitudinal one as $B_{nn}/B_{dd} = 4/3$.The paper also presents the exact relationship derived from the incompressibility condition:

$$dB_{nn}/dr = 2B_{dd}/r \tag{3}$$

And the expression of the average dispersion rate is given:

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$$< \varepsilon >= 15\nu a^2 \tag{4}$$

These results decouple turbulent small-scale fluctuations from the complex large-scale motions, predicting the -5/3 energy spectrum and the 2/3 scaling law of structure functions, providing a framework for subsequent experimental and numerical verification [2].

This framework, for the first time, transforms "turbulent small-scale" into a universal object that is reproducible, measurable, and computable.

In his other paper, he first provided the universal evolution law for the entire process of "decaying isotropic turbulence" based on the original foundation, filling in the duration aspect within the same theoretical framework. The core contributions of this paper can be summarized in three points:

Firstly, Kolmogorov established the turbulence lifetime scale as a conserved quantity by using the Loitsyanskii invariant [3]:

$$\Lambda = \int_0^\infty b_{dd}(r,t) r^4 dr = const \tag{5}$$

The initial state of the turbulence is locked into a single value, thereby replacing arbitrary initial conditions and allowing all subsequent evolution parameters to be explicitly expressed as functions of Λ and the kinematic viscosity ν .

Second, he derived the universal power law for energy decay at high Reynolds numbers. With the inertial range constant C given [2], Kolmogorov demonstrated that the mean energy $b(t) = 1/3u^2(t)$ such that:

$$db/dt = -Kb^{3/2}L^{-1},\ K = \sqrt{2/C} \tag{6}$$

Substituting the integral scale $L = (\Lambda/b)^{1/5}$ and integrating, it can be obtained:

$$b(t) = (10/7K)^{10/7}\Lambda^{2/7}(t - t_0)^{-10/7} \tag{7}$$

$$u\,(t) = (10/7K)^{5/7}\Lambda^{1/7}(t - t_0)^{-5/7} \tag{8}$$

$$L(t) = (7K/10)^{2/7}\Lambda^{1/7}(t - t_0)^{2/7} \tag{9}$$

This is the first decay law that does not contain any adjustable coefficients and shares the same constant C with the -5/3 energy spectrum, quantitatively linking spatial small-scale with temporal large-scale decay for the first time.

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$$b_{dd}(r,t) = k(4\nu t)^{-5/2} exp(-r^2/8\nu t) \tag{10}$$

$$b_{nn(r,t)} = k(4\nu t)^{-5/2}[1 - r^2/8\nu t] exp(-r^2/8\nu t). \tag{11}$$

Thus, the root mean square velocity and the scale follow $u(t) \propto t^{-5/4},\ L(t) \propto \sqrt{t}$.

In 1941b [3], the "local universality" of 1941a [2] was upgraded to "lifetime universality": the same constant C determines both the spatial small-scale spectrum and the temporal large-scale decay. This provides a closed, reproducible, and verifiable universal description of isotropic turbulence from its generation to its dissipation. It lays the theoretical foundation for subsequent decay experiments, spectral connection models, and the kinetic energy equations of large eddy simulations.

Kolmogorov's 1941 paper, titled "Dissipation of Energy in Locally Isotropic Turbulence," built upon his earlier work (Kolmogorov 1941a) to further elucidate the statistical properties of turbulent flows, with a particular focus on the energy dissipation mechanism under conditions of local isotropy. This work was crucial as it bridged the gap between theoretical formulations and empirical observations, providing a comprehensive framework for understanding the small-scale behaviour of turbulent flows.

In his previous work, Kolmogorov introduced the concept of local isotropy and defined the second-order structure functions $B_{dd}(r)$ and $B_{nn}(r)$, which describe the variance of velocity differences in the direction of separation r and in a perpendicular direction, respectively. These functions are fundamental for characterizing the energy distribution across different scales in turbulent flows. In the 1941c paper, Kolmogorov extends this framework by considering the third-order structure function $B_{ddd}(r)$, which captures the skewness of velocity differences. This extension is significant because it allows for a more detailed analysis of the energy transfer mechanisms within the turbulent cascade.

The paper begins by presenting the third-order structure function $B_{ddd}(r)$ and its role in the energy dissipation equation:

$$\frac{d}{dr}\left(r^4 \frac{dB_{dd}}{dr}\right) + \frac{d}{dr}\left(r^4 B_{ddd}\right) = 4Er^3, \tag{12}$$

where E denotes the mean rate of energy dissipation per unit mass. This equation is similar to the well-known Karman-Howarth equation for isotropic turbulence but is adapted to the locally isotropic case. By analyzing this equation, Kolmogorov derives a key relationship:

$$6\nu \frac{dB_{dd}}{dr} - B_{ddd} = \frac{4}{3} Er, \tag{13}$$

where ν is the kinematic viscosity. This relationship highlights the interplay between viscous diffusion and energy transfer across scales

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For small r , Kolmogorov shows that the skewness $S = B_{ddd}/B_{dd}^{3/2}$ remains constant, leading to the empirical relation:

$$B_{ddd}\left(r\right) = -\frac{4}{5}S\left(\frac{E}{\nu}\right)^{3/4}r \tag{14}$$

This result implies that the third-order structure function scales linearly with r and is directly related to the energy dissipation rate E . This scaling law is crucial for understanding the inertial subrange of turbulence, where energy is transferred from larger to smaller scales without significant viscous effects.

Kolmogorov also introduces the local scale of turbulence $\eta = \left(\frac{\nu^3}{E}\right)^{1/4}$, which characterizes the smallest scales of motion where energy is dissipated into heat. This scale is pivotal for understanding the transition from inertial to viscous-dominated behavior in turbulent flows.

The paper further discusses the correlation coefficients $R_{dd}(r)$ and $R_{nn}(r)$, which relate to the normalized second-order structure functions. For small r compared to the integral scale L , these coefficients can be expressed in terms of the universal functions $f_{dd}(r/\eta)$ and $f_{nn}(r/\eta)$. Kolmogorov shows that:

$$1 - R_{dd}\left(r\right) \approx \left(\frac{\nu}{E}\right)^{1/2} f_{dd}\left(r/\eta\right) \tag{15}$$

$$1 - R_{nn}\left(r\right) \approx \left(\frac{\nu}{E}\right)^{1/2} f_{nn}\left(r/\eta\right) \tag{16}$$

These expressions provide a bridge between theoretical predictions and experimental measurements, allowing for the determination of the constant C from empirical data.

Kolmogorov references experimental work by Dryden et al, who measured the correlation coefficients in grid-generated turbulence. Using these data, Kolmogorov calculates the coefficient $k = E/(3b)$ and shows that the theoretical predictions align well with the empirical results, validating the proposed scaling laws.

In summary, Kolmogorov's 1941c paper significantly advanced the understanding of turbulent energy dissipation by introducing the third-order structure function and establishing its relationship with the energy dissipation rate. The derived scaling law and the introduction of the local scale η provided a solid theoretical foundation for studying the inertial subrange and dissipation mechanisms in turbulence. This work remains a cornerstone of modern turbulence theory to this day, influencing subsequent research in fluid dynamics, atmospheric sciences, and engineering applications [4, 5].

The development of modern turbulence theory models has distilled the disorder of turbulence into reproducible statistical laws, making Kolmogorov's 1941 three-level theory the basis for MHD turbulence research: the spectral exponent is given by local isotropy, the decay law is given by the lifetime invariants, and the dissipation law is given by the third-order structure function.

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3. Fromturbulence theory to magnetohydrodynamics

In fluid mechanics, the study of turbulence has made significant progress, especially under the theoretical framework of Kolmogorov, where we have gained a deep understanding of the statistical properties of small-scale turbulence. However, in many practical applications, especially in astrophysics and plasma physics, the conductivity of the fluid makes the magnetic field have an important influence on fluid motion. This phenomenon of fluid dynamics in conductive fluids is called magnetohydrodynamics (MHD). The study of MHD turbulence needs to consider not only the dynamic properties of the fluid but also the influence of the magnetic field.

In his 1950 note, Elsasser recasts incompressible MHD as a pair of symmetric scalar-like equations by introducing the Elsasser variables $P = V + \lambda^{1/2}B$ and $Q = V - \lambda^{1/2}B$, where $\lambda = (4\pi\mu\rho)^{-1/2}$. Taking the sum and difference of the momentum and induction equations and neglecting the displacement current yields:

$$dP/dt + (Q \cdot \nabla)P = -\nabla q + \nabla^2(\nu_1 P + \nu_2 Q) \tag{17}$$

$$dQ/dt + (P \cdot \nabla)Q = -\nabla q + \nabla^2(\nu_2 P + \nu_1 Q) \tag{18}$$

with $\nu_1 = \nu + \nu_m$, $\nu_2 = \nu - \nu_m$ and magnetic viscosity $\nu_m = (4\pi\mu\sigma)^{-1}$. For vanishing field $P = Q$ and the system collapses to the Stokes–Navier equation, proving that the new variables smoothly embed MHD into classical hydrodynamics. The explicit symmetry immediately suggests that turbulence will emerge at comparable mechanical or magnetic Reynolds numbers, producing a stochastic "turbulent" magnetic field whose statistics mirror those of the velocity. Elsasser further notes that while Truesdell's flux–conservation theorem extends to compressible flows by replacing B with B/ρ, the same substitution inside the present equations destroys the neat (P, Q) symmetry, leaving compressible MHD without an equally compact canonical form [6].

Based on the turbulence model, Iroshnikov first proved in 1964 that a strong background magnetic field can flatten the energy spectrum from 5/3 to 3/2 by "waving" the turbulence and suppressing local cascades. He also provided a closed-form expression for the energy flux, setting the benchmark for subsequent anisotropic MHD turbulence theories and the −3/2 scaling [6].

Iroshnikov formulated the first quantitative energy spectrum for magnetohydrodynamic (MHD) turbulence permeated by a strong, externally imposed magnetic field B_0 . Assuming a highly conducting, weakly compressible fluid, he decomposed the turbulent velocity and magnetic fluctuations into counter-propagating Alfvén waves whose linear time scale $\omega_a^{-1} = (kv_a)^{-1}$ is much shorter than any non-linear distortion time. Because the wave–wave interaction is weak, the total energy density per unit mass in wavenumber interval dk can be written as a single spectral function:

$$E(k)dk = const. \tag{19}$$

Applying the virial theorem to a weak plane wave gives equal kinetic and magnetic energy densities, so the non-linear coupling is dominated by the compressible component rather than by

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vortical motion. Iroshnikov split the interactions into two types:

1. Non-local coupling between widely separated scales, treated with the adiabatic invariant I .

2. Local coupling between comparable scales, modelled as a diffusion process in wavenumber space.

For the local case the energy flux $S_\omega(k)$ obeys:

$$S_\omega(k) = -D_\omega \frac{\partial}{\partial \omega}\left(\omega^{-2}E_\omega\right). \tag{20}$$

$$D_\omega = B^{'}(N)\omega^6 u_0^{-2}E_\omega \tag{21}$$

where $N = v_{sound}/v_a \approx 1$ and $B^{'}(N)$ is a dimensionless constant of order unity. Stationary turbulence requires $S_\omega(k)$ to equal the constant injection rate ε , yielding the ordinary differential equation:

$$(\frac{\partial E_\omega}{\partial t})_{loc} = B^{'}u_0^{-2}\frac{\partial}{\partial \omega}\left[\omega^6 E_\omega \frac{\partial}{\partial \omega}\left(\omega^{-2}E_\omega\right)\right]. \tag{22}$$

Integration gives the Iroshnikov (IK) spectrum:

$$E_k \sim \varepsilon^{1/2}u_0^{1/2}k^{-3/2}. \tag{23}$$

And was derived to be [7]:

$$E(k) = C_{IK}(\varepsilon B_0)^{1/2}k^{-3/2} \tag{24}$$

with $C = (3/2B)^{1/2} \approx 0.5$. The -3/2 exponent is noticeably shallower than the -5/3 Kolmogorov law because the strong guide field suppresses perpendicular mixing and lengthens the energy cascade time.

To connect the inertial range to dissipation, Iroshnikov added viscous and Joule terms, obtaining the evolution equation:

$$\frac{\partial E_k}{\partial t} = B\Big(N\Big)u_0^{-1}\frac{\partial}{\partial k}\left[k^6 E_k \frac{\partial}{\partial k}\left(k^{-2}E_k\right)\right] - \Big(\nu + \alpha\nu_m\Big)k^2E_k \tag{25}$$

where ν is the kinematic viscosity, $\nu_m = c^2/4\pi\sigma$ the magnetic diffusivity, and α < 1 is a weighting factor.

Thus, the complete spectral model becomes [6]:

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$$E(k) = C\varepsilon^{1/2}v_a{}^{1/2}k^{-3/2}exp\left[-\left(k/k_\nu\right)^{4/3}\right] \tag{26}$$

Although derived under strict isotropy, the IK law became the benchmark for subsequent numerical studies of strong-guide-field turbulence and provided the conceptual seed for later anisotropic extensions such as the Goldreich–Sridhar critical-balance model [7, 8].

In 1965, Krachnan started from the Elasser symmetric from of thr incompressible MHD equations:

$$\left(\frac{\partial}{\partial t} + z\cdot\nabla\right)w = v_+\nabla^2 w + v_-\nabla^2 z - \nabla p, \nabla\cdot w = 0 \tag{27}$$

$$\left(\frac{\partial}{\partial t} + w\cdot\nabla\right)z = v_+\nabla^2 z + v_-\nabla^2 w - \nabla p, \nabla\cdot z = 0 \tag{28}$$

where $w = u + b$ and $z = u - b$ denote the Elsasser fields built from the velocity u and the Alfvén speed normalized magnetic field b. A uniform guiding magnetic field B₀ splits the flow into counter-propagating Alfvén waves whose phase coherence is continuously destroyed on the Alfvén time $(b_0k)^{-1}$. Weak scattering implies asymptotic equipartition of kinetic and magnetic energy, $E(k) = F(k)$, throughout the inertial range. Dimensional analysis then yields the MHD energy spectrum:

$$E(k) = A(\epsilon b_0)^{\frac{1}{2}}k^{-\frac{3}{2}} \tag{29}$$

with ϵ, the constant energy-flux, and A a dimensionless constant. A direct-interaction approximation and its Lagrangian correction further predict a possible two-segment inertial range: a –5/3 hydrodynamic zone followed by a –3/2 MHD zone when $v_0 \gg b_0$. The study thereby provides the first quantitative framework for the –3/2 scaling universally observed in numerical and astrophysical MHD turbulence [9].

4. Recent developments and applications of magnetohydrodynamic turbulence

The theoretical constructs surveyed in preceding sections—Kolmogorov's 1941 framework of local isotropy and universal scaling, the Elsasser reformulation of incompressible MHD, and the Iroshnikov-Kraichnan (IK) theory of wave-modulated turbulence—continue to structure contemporary research, albeit within discernible boundaries of validity. Rather than obsolete historical artefacts, these models function as zeroth-order approximations whose deviations illuminate physical mechanisms absent from the original formulations.

The Kolmogorov 1941 (K41) theory retains foundational status for weakly magnetized, high-Reynolds-number flows where magnetic effects remain dynamically subordinate. Its prediction of the −5/3 energy spectrum and the 2/3 scaling of second-order structure functions remains the benchmark against which numerical simulations and laboratory experiments are calibrated [2, 4]. However, the assumption of local isotropy fails systematically in flows subjected to strong mean

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magnetic fields, rotation, or stratification—conditions ubiquitous in astrophysical and geophysical environments. Contemporary research thus employs K41 not as a literal descriptor but as a null hypothesis: the measured departure from −5/3 scaling quantifies the degree of anisotropy and non-locality introduced by additional physical effects [8, 9].

The decay laws and structure function relations derived in Kolmogorov's 1941b and 1941c papers have acquired renewed significance in the era of high-resolution numerical simulations and spacecraft observations. The Loitsyanskii invariant-based decay exponent of −10/7 provides a critical test for the persistence of large-scale correlations in decaying MHD turbulence [3], while the exact 4/5 law for the third-order longitudinal structure function—generalised to MHD by Politano and Pouquet [10]—serves as the experimental probe of energy cascade directionality in magnetised plasmas. Recent applications to solar wind data by the Parker Solar Probe and Magnetospheric Multiscale (MMS) missions demonstrate the enduring utility of these 1941 theoretical predictions for diagnosing energy dissipation rates in collisionless turbulent environments [11].

The subsequent subsections examine how these theories are operationalised, extended, and transcended in contemporary computational methodologies, engineering systems, and astrophysical observations.

4.1. Computational frontiers

The integration of machine learning with MHD turbulence modelling has led to a paradigm shift from purely physical simulations to physics-data hybrid frameworks. Two complementary approaches are particularly transformative: neural operator learning and generative probabilistic modelling.

In magnetohydrodynamics (MHD) research, multiphase flows introduce significant complexity to the governing partial differential equations due to varying electrical conductivity, viscosity, and density across phases, requiring the coupled solution of Navier-Stokes and Maxwell's equations. Physics-Informed Neural Operators (PINOs) have emerged as a promising approach for MHD numerical simulation. In the 2023 study by Rosofsky and Huerta, PINOs demonstrated high accuracy in the laminar flow regime (Reynolds number $Re \leq 250$), successfully capturing the spatiotemporal evolution of velocity and magnetic fields with high resolution. However, for turbulent flows ($Re > 500$), prediction errors increased significantly for small-scale structures, as deterministic neural operators inherently struggle to capture the stochastic high-frequency components of turbulence due to spectral bias [10]. To address this limitation, Kacmaz and Huerta (2025) proposed a hybrid physics-informed neural operator-diffusion framework that explicitly tackles the spectral bias-induced loss of high-frequency information in PINOs under high-Reynolds-number turbulence [12]. The architecture employs PINOs as a deterministic backbone to predict large-scale coherent structures, while a conditional diffusion model learns the conditional probability distribution of small-scale fluctuations, thereby supplementing high-frequency details through a data-driven generative process. The trained hybrid model maintains physical consistency even under extreme turbulence conditions ($Re = 10{,}000$), achieving prediction accuracy significantly superior to that of pure PINOs [12]. Furthermore, the probabilistic nature of the diffusion component provides inherent uncertainty quantification—a critical capability for experimental validation against noisy satellite or laboratory measurements [12].

Toward digital twins of MHD systems, recent efforts have focused on real-time prediction and control. The fusion of operator learning with variational data assimilation—particularly four-dimensional variational (4DVar) methods—enables the ingestion of streaming observations from magnetospheric multi-scale (MMS) missions or fusion plasma diagnostics into continuously

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updating models [13]. This convergence of physics-informed machine learning, uncertainty quantification, and observational data streams constitutes the emerging frontier of MHD digital twins, with applications ranging from space weather forecasting to active control of fusion plasmas.

Persistent challenges remain in temporal dynamics modelling, where the non-local, non-Markovian nature of MHD turbulence resists standard recurrent architectures. Capturing the complex coupling between velocity and magnetic fields across inertial-range scales—particularly the dynamic alignment and non-linear energy exchange between Alfvénic and compressive modes—requires novel architectures that transcend current operator and diffusion frameworks [12, 13].

The Fourier Neural Operator's spectral representation implicitly preserves the statistical isotropy assumption underlying K41 theory, enabling efficient learning of homogeneous turbulent flows. However, the introduction of physics-informed losses incorporating the induction equation necessarily breaks this symmetry, accommodating the anisotropic energy transfer characteristic of Alfvénic turbulence described by IK theory [12]

4.2. Engineering applications in energy and space

MHD theory is also widely applied in modern engineering fields, with representative areas including space propulsion in aerospace using MHD; magnetic confinement of nuclear fusion in the energy sector based on the physical theoretical foundation of MHD. Both are established on MHD experiments involving liquid metal systems and plasma physics [14].

The important application of MHD in aviation lies in its assistance in overcoming the thermal barrier caused by Mach waves originating from supersonic effects and generated by shock waves. The main principle is that through the action of the Lorentz force, the magnetohydrodynamic phenomenon of wall converters serves as the foundation for magnetohydrodynamic aerodynamic propulsion, and it is predicted to make significant contributions in future research on atmospheric hypersonic flight.

MHD models are applied in tokamak devices and stellarators worldwide. A representative international project among them is ITER. The MHD model is primarily utilised for magnetic confinement of nuclear energy devices to prevent ITER disruptions, thereby avoiding damage to the facility [15, 16]. The MHD model simulates and predicts plasma behaviour, determining the operational boundaries of ITER. Furthermore, MHD provides plasma transport models to simulate the inductive scenarios of ITER: the transient phases of startup and shutdown, enabling rational selection of the voltage applied to the device [17].

In addition, plasma MHD and liquid metal MHD themselves are also widely used and researched as power generation technologies. Plasma power plants use high-temperature plasma (2700K-3000K) as the working fluid, converting the kinetic energy of plasma into electrical energy output as direct current through electromagnetic interaction. This method was once considered an alternative to gas turbines and was heavily invested in, but research enthusiasm has declined due to insurmountable high-temperature resistant material problems and the low electrical conductivity of seeded plasma [18, 19]. Currently, research focus has gradually shifted to liquid metal magnetohydrodynamic power conversion (LMMHD PC) systems for power generation from various heat sources. Its working fluids include sodium, NaK alloy, or gallium-based alloys, etc., with electrical conductivity far exceeding that of plasma. LMMHD can operate at relatively low temperatures (300K-1000K) without requiring superconducting magnets, which significantly reduces material requirements. It holds an important position in the development of renewable energy [18, 20]. Liquid metal cooling channels in fusion blankets and fast reactors operate at low magnetic Reynolds numbers where the Elsasser symmetry is broken by Joule dissipation dominance.

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Here, the relevant theoretical framework descends not from IK theory but from quasi-static MHD approximations that retain only the Lorentz force's dissipative component, effectively returning to a modified Navier-Stokes description with anisotropic viscosity [21].

4.3. Astrophysics implications

MHD is widely used in astrophysics from stellar to galactic scales as a unified framework for describing magnetised plasmas. A representative example is the solar wind, serving as the most accessible MHD experimental example. The frozen-in magnetic field carried by the solar wind forms a highly turbulent environment in interplanetary space, with energy cascading from large-scale coronal structures down to small scales of ion and electron gyroradii, ultimately heating the plasma through dissipation. The statistical characteristics of this energy transfer process can be characterized by power spectra, and observations indicate that solar wind turbulence spectra exhibit power-law forms, but the spectral indices vary with observational conditions, which has sparked competition and reconciliation among classical turbulence theories.

Observational tests of K41 and IK theories provide two benchmark frameworks for understanding solar wind turbulence. Kolmogorov's (1941) theory of locally isotropic turbulence assumes energy transfer through local breakdown of eddies, predicting a power-law spectrum that $E(k) \sim k^{-5/3}$. This spectral shape is widely present in solar wind magnetic power spectra, particularly in near-Sun regions and environments rich with current sheets. In contrast, the IK theory proposed by Iroshnikov (1963) and Kraichnan (1965) for strongly magnetized plasmas emphasizes non-local interactions of counter-propagating Alfvén wave packets, predicting a shallower $E(k) \sim k^{-3/2}$ spectrum. This spectral shape is observed in pristine solar wind far from the Sun (>0.4 AU) and during quiet periods with few current sheets [22].

The apparent contradiction between the two theories is reconciled through Goldreich and Sridhar's (1995) critical balance model. This model points out that under a strong background magnetic field, the nonlinear interaction time of Alfvén waves reaches critical balance with their propagation time along the magnetic field, leading to anisotropic turbulence—the perpendicular cascade follows K41's $k_{\perp}^{-5/3}$, while the parallel direction is suppressed. This framework successfully explains the continuous variation of spectral indices in solar wind observations: when turbulence intensity increases or current sheets dominate, the system approaches K41 characteristics; whereas in weakly perturbed Alfvén-dominated regions, the IK -3/2 spectrum is more prominent. High-resolution observations from missions such as Parker Solar Probe continue to provide new constraints for this theoretical dialogue [22, 23].

Also, recent developments in observation-based measurement techniques, such as synchrotron fluctuation techniques, have significantly promoted MHD turbulence theory and its implications in astrophysics. These techniques have validated the measurement of several properties of magnetic turbulence based on data cubes from MHD turbulence simulations and observations [13, 21].

4.4. Future iredctions

Future research in magnetohydrodynamic turbulence should focus on developing more accurate and efficient models capable of capturing the complex interactions between velocity and magnetic fields. Combining high-resolution simulations with multi-messenger observations will help refine turbulence models and reveal new paradigms of energy cascades under extreme magnetic fields.

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This research is expected to lead to a unified framework for understanding the evolution of magnetic fields in compact objects and the formation of gravitational wave sources [24].

5. Conclusion

The study of fluid magnetic turbulence has significantly evolved from the foundations of classical turbulence theory to a sophisticated understanding of the interplay between fluid dynamics and magnetic fields. Kolmogorov's pioneering work laid the groundwork for understanding isotropic turbulence, providing essential tools for analyzing small-scale fluctuations and energy spectra. Subsequent introductions of the magnetic field through Elsässer variables and the development of MHD turbulence models have further refined our comprehension of this complex phenomenon. Recent advancements, such as the Iroshnikov–Kraichnan spectrum and the Goldreich–Sridhar critical balance model, have offered valuable insights into the behavior of MHD turbulence under different conditions. This review highlights the importance of MHD turbulence in astrophysical and plasma physics contexts, emphasizing its role in key cosmic processes. By summarizing the main contributions and conclusions of pioneering works, this review aims to guide researchers in selecting the most appropriate methods for their studies and in understanding the field's rich history and ongoing developments.

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