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Singular Value Decomposition for Image Processing: Methods and Applications

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Abstract. Singular value decomposition (SVD) has significant practical value in image processing. It decomposes the image into different levels of information with varying amounts of information. The most important application is image compression. At the same time, it is also the mathematical basis for image denoising and watermark embedding, and is a classic tool for the underlying processing of digital images. This paper first introduces the mathematical form and geometric meaning of SVD. Subsequently, at the application level, it reviews six typical methods: fingerprint recognition, image denoising, watermark embedding, image compression, super-resolution enhancement, and ground-penetrating radar images. Each method is based on the energy concentration characteristic and low-rank approximation ability of SVD, and designs corresponding singular value processing strategies for different tasks. As a core tool of linear algebra, SVD demonstrates strong low-rank representation and feature extraction capabilities in image compression, denoising, watermarking, and enhancement, with remarkable effects. However, computational efficiency, geometric robustness, and adaptive threshold selection remain key bottlenecks. In the future, it is necessary to combine randomized algorithms, GPU parallelization, invariant feature alignment, and data-driven threshold learning to promote the in-depth application of SVD in real-time and complex scenarios.

Keywords: Singular value decomposition, image processing, linear algebra

1. Introduction

Linear algebra is a bridge connecting elementary mathematics to professional applications, primarily aimed at cultivating abstract thinking and spatial imagination. It focuses on vectors, matrices, and their operations, serving as the mathematical language for describing linear relationships and spatial transformations. It is a fundamental tool for many practical problems, especially in fields like artificial intelligence, signal processing, structural mechanics, where a key technique is Singular Value Decomposition (SVD). The role of SVD is to decompose any matrix into the product of multiple simple matrices, which reveal its inherent low-dimensional structure and key features. It has been widely applied in many fields, and the most representative among them is image processing， such as compressing storage space and accelerating computations. SVD is a versatile matrix analysis tool and the foundation of data science and machine learning. Therefore, it is necessary to further analyze it.

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The concept of Singular Value Decomposition (SVD) was first proposed by Beltrami in 1873 [1]. Later, in 1939, Eckart and Young generalized it to rectangular matrices and regarded it as the formal proof of SVD [2]. Singular value decomposition plays a significant role in various fields. For instance, Edgar et al. proposed using singular value decomposition based on deep learning features to perform visible fingerprint recognition on epoxy resin X-ray images [3]. Additionally, Montagnon mentioned using singular value decomposition to decompose time series, and then using ARMA and Fokker Planck equations for prediction [4]. Moreover, singular value decomposition has many contributions in image processing. Liu proposed performing low-rank approximation on image block matrices through singular value decomposition, filtering out noise components by shrinking singular values, and thereby restoring the image [5]. In digital watermarking technology, Gao proposed that singular value decomposition can be used to decompose image sub-blocks and utilize the stability of its singular vectors to embed watermarks by fine-tuning these coefficients [6]. In image compression, Wu proposed that it can be achieved by selecting some singular values and singular vectors to reconstruct the matrix [7]. In super-sampling image detail enhancement, Xiao et al. proposed eliminating pseudo high-frequency noise through singular value decomposition, resolving the phenomenon of patches or lines, and thereby achieving detail enhancement [8]. Furthermore, Wen et al. proposed that singular value decomposition can be used for ground-penetrating radar images [9]. By using the singular value decomposition method, the subspace of the detection target, noise, and clutter in the B-scan images obtained by the detection radar can be distinguished, thereby more efficiently filtering clutter and noise and obtaining a higher peak signal-to-noise ratio and more accurate target images. In light of its importance in various fields and the significant progress made in image processing using SVD in recent years, it is necessary to conduct a review and summary.

This paper will provide a comprehensive overview.The structure of the remaining parts of this article is as follows. First, in Section 2, this paper will explain SVD and how it is applied in image processing in other literature. Secondly, in Section 3, this study will discuss the shortcomings of SVD in image processing and future development directions. Finally, Section 4 concludes the paper with a summary of key findings.

2. Introduction of SVD and its application in image processing

2.1. Preliminaries of SVD

Singular value decomposition is one of the most fundamental matrix decomposition tools in linear algebra. Its core idea is that any $m \times n$ matrix A over the complex number field can be decomposed into the product of three matrices with specific structures and properties [10]. It reveals the essential information of the matrix from aspects such as rank, norm, and condition number, and by retaining only the largest singular values and their corresponding vectors, it provides the optimal solution for data dimensionality reduction and low-rank approximation.

In terms of formulas,for any $m \times n$ matrix A, the standard mathematical form of its singular value decomposition is: $A = U \sum V^*$,where U is an m*m unitary matrix and $\sum$ is an m*n diagonal matrix. In addition, V denotes an n*n unitary matrix while V* is the conjugate transpose of V.

The geometric principle of Singular Value Decomposition (SVD): Any matrix multiplication Ax can be regarded as a linear transformation on the vector x. SVD decomposes this transformation as follows: first, it rotates or reflects using V*, then it independently stretches or compresses in

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different coordinate axis directions using $\sum$, and finally it rotates using U. This decomposition makes the effect of the matrix on the vector clear and intuitive.

The algebraic principle of Singular Value Decomposition is,for an $m \times n$ matrix A, construct the symmetric semi-definite matrix $A \times A$. Since $A \times A$ is a real symmetric matrix, it can be orthogonally diagonalized: $A \times A = VBV^*$, where V is the standard orthogonal eigenvector matrix, and B is a diagonal matrix with diagonal elements being the eigenvalues λi.

Define the right singular vector matrix V as the eigenvector matrix of $A \times A$, and the singular value $\sigma i = \lambda i$.

Using the relation $Av_i = \sigma_i u_i$, the left singular vector ui can be derived, and finally an orthogonal matrix U and a diagonal matrix $\sum$ are constructed, making the decomposition valid.

2.2. Application in image processing

2.2.1. Fingerprint recognition

Edgar et al. proposed a visualization method that combines deep learning with Singular Value Decomposition (SVD) to extract discriminative feature fingerprints from epoxy resin X-ray images [3]. They Firstly, the original image is preprocessed with Sobel gradient to enhance the structural information. Then, a CNN is trained for classification, and the response maps of the early convolutional layers are extracted to form a matrix. Finally, SVD is used to obtain the principal left singular vectors as the most representative feature patterns. The results show that the features extracted by this method are consistent with the material toughness ranking (Sample 3 is the best), and they exhibit high robustness under different network architectures and initialization conditions. Compared with the single strongest activation channel method, it can provide clearer and more reliable visual feature representations.

2.2.2. Restore the image

Liu. proposed to use SVD for image denoising [5]. This article is mainly about utilize the non-local self-similarity of the image, converting similar image blocks into low-rank matrices, thereby transforming the denoising process into a low-rank estimation problem. In principle, the SVD decomposition of the similarity block matrix is performed, and the singular values are adaptively shrunk according to the relationship between the singular values and the noise level, separating the signal from the noise, and then the image is reconstructed through weighted fusion and iterative back-projection. The results show that this method is comparable to WNNM in terms of PSNR and FSIM indicators, but the speed is increased by approximately 7 times, and it can effectively suppress artifacts, with a visual effect superior to methods such as LAR_SVD.

2.2.3. Embedding watermark

Gao proposed a digital image blind watermarking algorithm by utilizing SVD [6]. The core idea is to utilize the stability of the first column vector of the left singular matrix U after image SVD under attack conditions. By fine-tuning the coefficients with high similarity, $U_\{21\}$ and $U_\{31\}$ are used to embed watermark information. In principle, the carrier image is first subjected to a first-level DWT and divided into blocks. For each sub-block, its SVD is performed. Depending on whether the watermark bit is 0 or 1, the mean values of the coefficients are respectively fine-tuned in different directions for $U_\{21\}$ and $U_\{31\}$. During extraction, simply comparing the sizes of the two

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can achieve blind extraction of the watermark. The results show that after embedding the watermark, the PSNR is approximately 50 dB. The algorithm has good robustness against noise and JPEG compression attacks. The NC value remains above 0.7 even under strong noise conditions.

2.2.4. Image compression

Wu proposed an image compression method based on SVD [7]. The core idea is to decompose the image matrix into singular values and singular vectors using SVD. By selecting some larger singular values and their corresponding vectors to reconstruct the image, and discarding the smaller singular values to achieve compression. In principle, for an n×n image matrix, performing SVD decomposition, retaining the first k singular values, only $k(2n+1)$ pieces of data need to be stored, and the compression ratio is $n^2/[k(2n+1)]$. The results show that after dividing the 256×256 Lena image into blocks and selecting the k value based on the energy contribution (such as 99.9%), a visually satisfactory reconstructed image can be obtained at a compression ratio of approximately 2.44.

2.2.5. Upsampling image detail enhancement

Xiao et al addressed the "block or line" artifacts problem in the super-resolution algorithms based on local self-similarity, proposing to enhance the details by using SVD thresholding [8]. The core idea is that after obtaining high-frequency information through block matching, the high-frequency block matrix is decomposed using SVD, and the low-energy pseudo-high-frequency components are eliminated. In principle, the SVD decomposition of the high-frequency block is performed, and the soft thresholding of the singular value matrix is carried out ($\lambda_i = \lambda_i - \tau$, those greater than the threshold are retained), and then the effective singular values are used to reconstruct the high-frequency block, thereby removing the pseudo-high-frequency noise. The results show that this algorithm effectively eliminates the face block phenomenon, and the PSNR and SSIM indicators are superior to those of Freedman et al. methods, and it can be extended to various super-resolution algorithms for detail enhancement.

2.2.6. Ground-penetrating radar image

Wen et al. combined SVD with FCM to process GPR images [9]. The core idea was to use SVD to decompose the B-scan images into three subspaces: target, clutter, and noise. Then, through FCM clustering, the overlapping areas of these three were separated. In principle, the GPR image matrix was subjected to SVD decomposition, and the magnitudes of the singular values corresponded to the energy of different components. The FCM algorithm clustered the singular values, using the membership function to determine whether each singular value belonged to the target or noise category, thereby determining the number of singular values k _ 2 in the target subspace. The results showed that this method could effectively filter out clutter and noise, with a PSNR of 29.45 dB and an MSR of 0.125. It outperformed the comparison methods in the literature and resulted in clearer and more accurate target images.

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3. Challenges and future prospects of SVD in image processing

3.1. Challenges

3.1.1. Computational burden precluding real-time operatio

The time complexity of SVD decomposition (especially for large-scale matrices) is approximately $O(mn^2)$ or higher. When the image size is large or block processing is required, the computational load significantly increases. As Wu pointed out, "The factor affecting the computing speed is that the SVD transformation operation is relatively large [7]." Gao also mentioned that SVD participating in multi-matrix operations indirectly increases the computational complexity [6]. Xiao et al mentioned that SVD threshold processing is only used as a local enhancement step, but if it is globally applied, there is still an efficiency bottleneck [8]. This limits the application of SVD in real-time video processing, embedded systems, and other scenarios.

3.1.2. Poor robustness against geometric attacks

The SVD decomposition is sensitive to the geometric transformations (rotation, translation, scaling) of images. Although the singular values have a certain degree of stability, the singular vectors (especially the higher-order vectors) will undergo significant changes due to the rearrangement of pixels. Yuan clearly pointed out that the proposed DWT-SVD algorithm "has a relatively poor resistance to rotation attacks... Wavelet transformation is an operation of arbitrary angle transformation, and the watermark information after being attacked by rotation loses consistency with the original watermark information". In addition, Wen et al. also faced the dilemma that SVD was difficult to distinguish alone when the target hyperbola overlapped with the ground reflection [9]. This limited the application of SVD in scenarios that need to resist geometric attacks (such as robust watermarks, image registration).

3.1.3. Difficulty in adaptive threshold selection for singular value truncation

One of the core operations of SVD is to truncate (or reduce) some of the singular values to separate the signal from noise/redundancy. However, the selection of threshold k or τ often depends on experience or fixed values, lacking a universal adaptive criterion. Liu mentioned that "When the noise level varies, the appropriate truncation order l needs to be adjusted according to the noise changes", and his paper determined the linear relationship $r = [0.08*\tau + 6.2]$ through a large number of experiments [5]. Xiao Jinsheng used a fixed value of 3.5 for the soft threshold, and Wu used energy contribution threshold values (such as 99%, 99.6%, 99.9%) that also needed to be set manually [7, 8]. For different image contents or noise levels, fixed thresholds may lead to loss of details or residual noise.

3.2. Future prospects

3.2.1. Introducing randomized SVD or block parallel optimization

In the future, the Randomized SVD algorithm can be adopted. By randomly projecting and reducing the dimension of the image matrix, the decomposition time can be significantly reduced while maintaining approximately optimal accuracy. Additionally, by combining GPU parallel computing and adaptive block strategy (such as the idea of variable step size block iteration mentioned by Liu),

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using a large step size for flat areas and a small step size for complex texture areas, the efficiency can be significantly improved while ensuring quality.

3.2.2. Combining invariant features or spatial transformation alignment

Future research can explore the combination of SVD and scale-invariant feature transform (SIFT) or affine invariant moments. Before the SVD decomposition, geometric correction can be performed on the image to align the singular vectors to a unified reference frame. Another approach is to use Log-polar transformation or Radon transformation to convert rotation/scaling into translation. Then, SVD watermark embedding or feature extraction can be carried out in the transformed domain, fundamentally decoupling the influence of geometric changes on the singular vectors.

3.2.3. Introducing a data-driven adaptive threshold selection mechanism

In the future, deep learning or Bayesian inference can be introduced to automatically estimate the optimal truncation threshold. For instance, a lightweight CNN can be trained to predict the optimal number of singular values to retain based on the image content; or statistical criteria such as Stein's unbiased risk estimation (SURE) can be employed to automatically determine the threshold that minimizes the reconstruction error in the absence of Ground Truth, achieving a truly "blind adaptive" SVD processing.

4. Conclusion

This article provides a comprehensive review of the application of Singular Value Decomposition (SVD) in images. Through discussion and analysis, it can be concluded that currently, the SVD technology mainly faces three challenges: heavy computational burden, its high complexity is difficult to meet the requirements of real-time processing; weak geometric robustness, sensitive to attacks such as rotation and scaling, prone to causing feature failure; difficult threshold selection, the truncation parameters mostly rely on empirical settings, lacking universality, and prone to causing detail loss or residual noise. To address these bottlenecks, future research can break through from three aspects: introducing randomized SVD and GPU parallel computing to significantly improve the decomposition efficiency; integrating invariant features such as SIFT or logarithmic polar coordinate transformation, aligning geometric deformations before decomposition, fundamentally enhancing robustness; exploring data-driven adaptive threshold mechanisms, using lightweight neural networks or SURE statistical criteria, automatically optimizing the truncation order based on image content, achieving intelligent and blind adaptive SVD processing, and expanding its practical value in video, embedded systems, and other scenarios.

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