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A Matrix-Based Examination of Kepler's Third Law Using RREF and SVD

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Abstract. In the exploration of the solar system or even the Milky way, Kepler's third law provides an important basic concept between planet's orbit and the period. Therefore, this essay will mainly talk about the verification of Kepler's third law, and the he planetary period and orbital radius data used in this essay are sourced from NASA's official solar system database (NASA, n.d.) However, the traditional way to verify this law relied on linear regression ,which is sensitive to the uncertainties. In this case, a combined method will be used in this essay to analyse and verify Kepler's third law in a more rigorous way: the reduced row echelon form (RREF) will be first introduced to demonstrate the ideal rank deficiency, while singular value decomposition is employed to deal with real-world noise. After that, Gaussian noise, which mocks real-world noise, is applied to both the traditional method and the SVD method to facilitate the comparison. Finally, a robust linear correlation will be shown. In comparison with the traditional method—linear regression—this method not only displays the fundamental mechanism of verification but also shows a clear physical meaning in a more fault-tolerant way.

Keywords: Kepler's third law, matrix, reduced row echelon form, linear correlation, singular value decomposition

1. Introduction

Celestial Mechanics, which emphasizes on orbital movement, is one of the core experimental subjects of physics. In this case, people had already investigated this part for many years. However, in the past, without the assistance of sophisticated equipment, people could only conclude the results or hypothesis from what they saw. As for Kepler, who concluded Kepler's third law from massive observation, contends that $T^2 \propto a^3$, with the extended explanation by Isaac Newton, this equation seems perfect. In recent years, many people have given their verification of this equation [1]. However, the linear regression, traditional method, can only take in the uncertainty of single variable. In this case, the method of SVD, which can take both uncertainties in consideration , is used to unravel this issue. Moreover, with the appliance of RREF, this equation will be explained more precisely in both theoretical and practical ways. In order to achieve this, the method from another article [2], which uses MATLAB to perform mock tests, is considered. Therefore, the data of eight planets from the solar system, provided by NASA, are chosen to determine whether the data

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satisfy the equation: $T^2 \propto a^3$. To be more specific, both realistic calculation and comparison with traditional method will be displayed in this essay. After that, the comparison of results is made by inputting data from different planets , whether these are Earth, Jupiter or Mars, to determine whether all results give a consistent conclusion. This investigation can strengthen the foundation of the connection between Newton's law and Kepler's third law. Moreover, with the usage of data from NASA, this essay can show a more accurate result, which explains the gap between realistic circumstance and theoretical hypothesis precisely.

2. Data

The data that are used to verify the Kepler's third law are sourced from the database of NASA, and the subjects of this investigation are the planets in the solar system. This data is gathered from the satellite through years, and then the data is zapped down as telemetry to the science centre, after processing and cataloguing data booklet is produced. To reach uniform results, the orbital period is base on days, and the length of semi-major axis of each planet is based on astronomical unit(AU). The detailed data is listed in the Table 1.

In order to attain the data with the minimum error, the range of data should be considered. For Pluto, it is confirmed that Pluto is not a planet [3], but a dwarf planet. In this case, the result gained by the calculation related to Pluto may not be consistent with other results, so the data of Pluto should not be considered in the verification.

Table 1.Semimajor axis and orbital period data

Name of planet	Mercury	Venus	Earth	Mars	Jupiter	Saturn	Uranus	Neptune
Length of semimajor axis/AU	0.3860	0.7233	1.000	1.523	5.204	9.583	19.20	30.05
Period/day	88.0	224.7	365.2	687	4331	10747	30589	59800

3. Method of calculation

3.1. The transformation of equation

For this essay, the aim is to verify the equation, so it is helpful to change the equation into a more manageable form. For $T^2 \propto a^3$, it can be written as $T^2 = k \cdot a^3 + C$, and logarithm is taken on the both sides, so the equation is written as $\log(T^2) = \log(k \cdot a^3) + \log(C)$. According to the rules of logarithm, this can be written as $2\log(T) = 3\log(a) + \log(k) + \log(c)$, since k is a constant, the final form is $\log(T) = 1.5\log(a) + C$,

where C is a constant. This step is the foundation of the following procedures.

3.2. Rank analysis

After the transformation, three quantities (log(T), log(a), and C) are shown. If the input data of T and a make the equation hold, then the correlation is verified. In this essay, the measure of matrix is introduced.

Firstly, a matrix A is established, the elements inside the matrix is the coefficient of each value, so it is $A = [\log(a), \log(T), 1]$. At the third column, the vector of ones accommodates the coefficient of C.

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After that, the equation holds only when the column vectors have linear correlation. In the vector, there is a specific definition for this situation, which is rank [4]. Rank is the number of linearly independent columns it contains. In this case, If there the rank is equals to two after processing ,the log(T) is linearly related to the log(a). In other words, although there are three columns in matrix A, only two of them are linearly independent.

In order to obtain this, the exploitation of reduced row echelon(RREF) form is preferred. RREF is the simplest form of the matrix ; it satisfies the conditions that the first non-zero element of every non-zero row is one, that other elements in the same row are all zero, and that the zero rows are at the bottom. When only two non-zero row appears, it means that the rank of this matrix A is two. The way that transforms a matrix to its simplest form is by Gaussian elimination, which is adding or subtracting multiples of one row to another to obtain zero elements. This process can be performed more easily in MATLAB using the built-in function RREF.

If all the data input are perfectly close to the theoretical value ,only two non-zero row will appear in the matrix. In this case, the linear correlation is established— $\log(\mathrm{T}) = 1.5\log(\mathrm{a}) + \mathrm{C}$, and the Kepler's third law is verified.

3.3. Singular value decomposition

The method of using RREF indeed give the fundamental way of verifying Kepler's third law by matrix. However, the data from the NASA database, which are observed with inevitable uncertainties, output erroneous answers—the rank always equals three when more than three planets are chosen. Consequently, the linear correlation cannot be shown in the realistic world. In this case, another complementary method using matrices is used to address the realistic problem.

With the uncertainty, this essay will introduce another method to obtain the linear correlation, which is singular value decomposition [5]. For a matrix that is compose of certain planets data, SVD will output three non-negative values, which are called singular value. They are written as sigma1≥sigma2≥sigma3≥0. To be more specific, these values have a more intuitive geometrical meaning. If the matrix is seen as a spatial transformation, then these three singular values respectively represent the lengths of the three principal semi-axes of an ellipsoid after transforming the unit sphere. In this case, sigma1 and sigma2 can be seen as lying in a same plane, and sigma3 is perpendicular to this plane. After that, if the linear relationship is shown, the sigma3 will be zero , so the three dimension ellipsoid will be reduced to a two dimension ellipse. A more geometrical intuitive result is shown in left side of Figure 1. This graph illustrates the ratio of sigma3 and sigma1 against number of planets. It can be seen that the line plotted is closed to the x-axis with all point at a very small numerical value, which is below 10-5. In this case, the linear correlation is proved.

In this case, a mutual verification method can be established with using RREF. If the Kepler's third law holds strictly, the sigma3 will equals to zero, while with the uncertainty in the data, sigma3 will be a value that is far smaller than sigma1 and sigma2,then data is closed to linear correlation. Apart from that, the constant of the equation of Kepler's third law can actually be generated by SVD as well. Because the signma3 is the smallest singular value, so, after processing, the right singular vectors of sigma3 is the coefficients of log(T),log(a) and C, thus the constant can be calculated by the ratio of coefficient of log(T) and log(a),which can then compare with the theoretical value 1.5 to verify the result. .At the same time, the stability of the result can be tested by adding or deleting the data of certain planets, If relatively big deviation is shown, other factors that affect the data will be discovered. The graph on the right side of Figure 1 shows the generated k against number of planets. By adding one planet from Venus to Neptune each time, it can be seen that all values are in small deviation with the theoretical value, so the line plotted is nearly straight and overlap with k=1.5. In

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this case, the result is verified. With the two-side answers given by RREF, SVD is different, it can tolerate the real-world uncertainty.

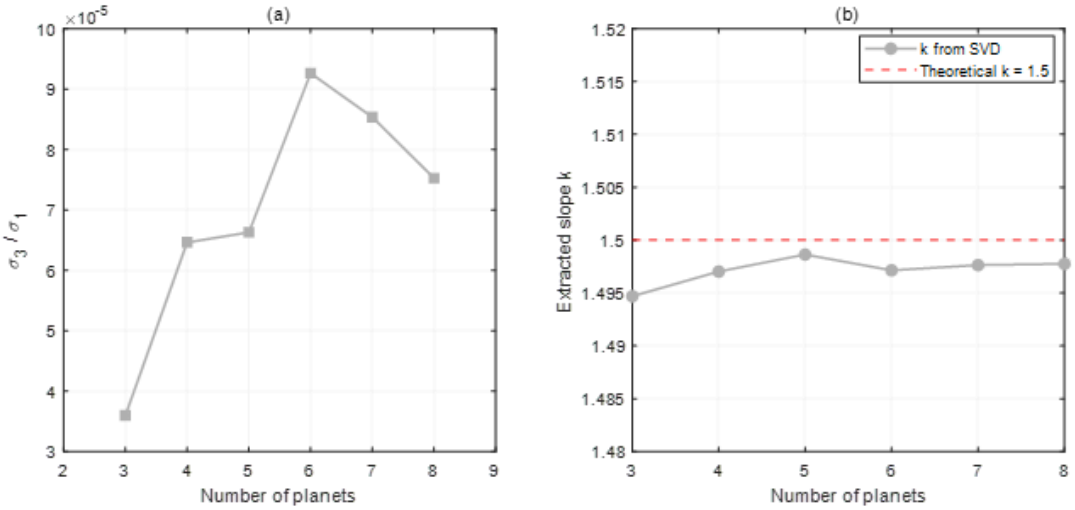


Figure 1. SVD analysis results for the solar system planets. (a) Left: Singular value ratio sigma3/sigma1 against the number of planets. (b) Right: Extracted slope k against the number of planets. The red full line indicates the theoretical value k = 1.5

4. Difference with linear regression

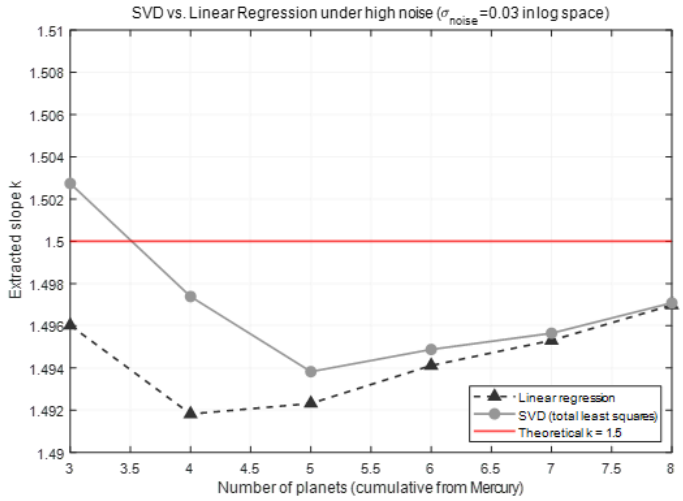


Figure 2. Comparison of SVD and ordinary linear regression under simulated observational noise. The x-axis shows the number of planets ; the y-axis shows the extracted slope k. The red line marks the theoretical value k=1.5

Traditionally, the investigation related to the Kepler's third law is based on linear regression. This method mainly uses the least squares approach, and then determines whether the estimated gradients are located around 1.5–a theoretical value. However, this method is more susceptible to single abnormal data, like systematic errors in observation or deviations in orbital data, while the combined methods of RREF and SVD in this essay possess stronger stability and physical logic, because SVD can take both uncertainties—period and semimajor axis—into consideration. In order to obtain a

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specific comparison, Gaussian noise with percentage uncertainty at 7% is introduced for both period and the length of the semimajor axis. It can be seen that after the noise is considered in both methods, the line representing linear regression deviates from the theoretical line further than the line representing SVD,. In this case, the matrix-based methods are less affected by uncertainty than linear regression. After all, RREF can strictly proof that the Kepler's third law is right under theoretical value, and SVD can attain same result even in realistic world, while in comparison with the value generated by linear regression, the accuracy of the result can be verified. In this case, the combination of three method will give a more rigorous and consistent result. As shown in Figure 2, after introducing 7% Gaussian noise, the slope k extracted by the SVD method remains closer to the theoretical value of 1.5 than that from linear regression, indicating that the matrix-based methods are more robust to observational uncertainties.

5. Conclusion

This essay mainly concentrates on the use of rank deficiency, RREF and SVD, which successfully verifies Kepler's third law. The data in this essay are based on the NASA database and include eight planets in the solar system, from Mercury to Neptune. After inputting the data into the matrix and applying RREF, the appearance of two non-zero rows proves the log(T) and log(a) have linear correlation. However, with the observation error, RREF constantly shows three non-zero row. In this case, SVD is introduced as well, which quantitatively verify the relationship. These methods no longer relies on parameter estimation, but gives a structural verification, which shows the dimension reduction in a physical way, as the three-dimensional matrix is constrained into a two-dimensional plane. Lastly , noise on the data is introduced to justify the advantage of SVD against linear regression. However, there are several limitations that appear in this essay. Firstly, these methods cannot give a statistically consistent result. Secondly, though the SVD has partially alleviate the uncertainty shown in the data and results, the range of sigma3 still need to be confirmed strictly. For these limitations, a range of improvements can be applied. Initially, samples selection and calculation with many times should be carried out, and calculate the standard deviation of the these value, which help us to reach a statistical result. Besides, more subjects outside the solar system can be included in the experiment, which turns the result into a statistically-verified answer, which is more practical. In the future, these methods may be chosen in education, because it perfectly establish a connection between maths and applied physics by connecting matrix, a mathematical tool, with a realistic physics problem. In this case, students can build a stronger mathematical intuition in physics beforehand.

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