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Calculus Analysis Methods for Periodic Motion Patterns of Change and Data Analysis Based on Time Series

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Abstract. Based on experimental materials for a simple pendulum that provide only chart images and include no raw numerical data files, this paper distinguishes theoretical derivations from qualitative image-based observations and explores how time series can be used to represent calculus relationships in periodic motion. The experimental setup described in the materials uses the Arduino Nano 33 IoT development board, the LSM6DS3 inertial measurement unit, and Bluetooth transmission, covering five pendulum lengths from 0.34 m to 1.45 m, with an approximate sampling frequency of 100 Hz. Based on the small-angle simple pendulum model, the analysis establishes the relationship among angular displacement, angular velocity, angular acceleration, period and frequency, and examines the oscillation interval, release shock, decay behavior and multi-axis response in the records. The results show that the images qualitatively support the theoretical trend of "longer pendulum length, slower oscillation", but they are not enough to verify the measured frequency, damping constant or three-dimensional displacement. The study also illustrates how differentiation amplifies noise and why double integration depends on calibration, initial conditions, and drift control. This case provides an intuitive example for establishing links between functions, derivatives, integrals, differential equations, and observable motion characteristics in physics teaching.

Keywords: calculus, periodic motion, simple pendulum, time series analysis, inertial measurement unit

1. Introduction

A simple pendulum is an ideal entry point for studying periodic motion, as its arc of repetitive swings can be represented as a quantity that changes over time, rather than as a set of isolated positions. If angular displacement is expressed as $\theta(t)$, the rate of change is angular velocity, and the angular acceleration can be obtained by taking the derivative again. Thus, pendulum motion connects the language of functions, derivatives, and differential equations with visible physical processes. Spring oscillators, alternating electrical signals, and periodic mechanical motions can also be analyzed in a similar way, and the simple pendulum is particularly intuitive in teaching these concepts [1]. Ideal models alone are not sufficient to interpret instrument records. Sensor curves may include gravity projection, release shock, mounting effects, sampling methods, and noise.

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Therefore, effective classroom analysis should not only explain the mathematical structure of the ideal oscillator, but also identify the evidence that a particular measurement can really provide. Low-cost sensing studies have shown that information related to periods can be recovered under controlled conditions, while also illustrating the importance of timing methods and clear observation intervals. Both the smartphone light sensor study and the Arduino/Hall sensor study used explicit event timing rather than just image-based estimation [2, 3].

This article takes the provided Bluetooth IMU simple pendulum demonstration material as an example. The materials cover five pendulum-length settings and three-axis images, but do not include CSV samples or calibration files. The objective of this study is threefold: to derive the theoretical period predicted by the small-angle equation as a function of the length of the pendulum, to use calculus to explain the salient features of the time series, and to determine which conclusions can be supported by images alone. Based on this, this paper uses experiments to demonstrate analytical reasoning, while avoiding converting visual impressions into numerical results that lack data support [3, 4].

2. Experimental system and data foundation

2.1. Experimental platform and data acquisition

The demonstration material shows that the experiment uses the Arduino Nano 33 IoT development board as the sensing platform. The on-board LSM6DS3 module provides three-axis accelerometer and gyroscope channels, while Bluetooth Low Energy transmits data without a cable attached to the moving device. The experiment used five rope lengths: 34 cm, 68 cm, 95 cm, 119 cm, and 145 cm. Three tests are reported for each setting, and the demonstration material gives an approximate sampling frequency of 100 Hz. This article only analyzes simple pendulum records and does not discuss the door opening examples listed separately in the slides [5-7].

2.2. Scope of records provided

Each pendulum-length folder contains charts in the X, Y, and Z directions. The 34 cm, 68 cm, and 95 cm images are labeled with angular acceleration in deg/s², the 119 cm and 145 cm images are labeled with acceleration in g. Because these signals correspond to different physical quantities and units, direct comparison of amplitudes across the five pendulum lengths is affected simultaneously by channel definitions, measurement ranges, and calibration differences, therefore, no credible amplitude relationship can be obtained. Before verifying the original channel definition, this paper only discusses the time-series patterns under the same signal definition, and comparisons across conditions address only the qualitative trend of oscillation interval. In addition, the materials do not provide sample timestamps, sensor measurement ranges, bias estimates, orientation records, or raw files for each test. Therefore, this article retains the units shown in the diagram and avoids referring to any one diagram as a three-dimensional displacement.

The analysis process separates the calculation from the observation. The first step is to give the reference values of the period and frequency from the small-angle formula, the second step is to investigate the relative interval, release shock, residual oscillation and cross-axis response from the graph, the third step is to classify each statement as a qualitative observation directly supported by the graph, or a quantity that must be obtained by relying on the raw data with time stamps. This order of processing prevents the image from being used as a substitute for experimental fitting. Table 1 lists the theoretical periods, theoretical frequencies, and original image signal labels

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corresponding to the five pendulum lengths, providing a unified reference for subsequent comparisons.

Table 1. Theoretical reference values for a simple pendulum and image signal labels

Pendulum Length	Image Labels (Axes/Units)	Theoretical Period T (s)	Theoretical Frequency f (Hz)
34 cm	X/Y/Z axes, deg/s²	1.170	0.855
68 cm	X/Y/Z axes, deg/s²	1.655	0.604
95 cm	X/Y/Z axes, deg/s²	1.956	0.511
119 cm	X/Y/Z axes, g	2.188	0.457
145 cm	X/Y/Z axes, g	2.416	0.414

Note: The theoretical values are calculated based on the small-angle simple pendulum model, and the gravitational acceleration is taken as $g = 9.81\ m \cdot s^{-2}$.

3. Calculus framework for time series

3.1. Differential equations and period prediction

For a simple pendulum consisting of a concentrated mass suspended by a light string and released at a small angle, the angular coordinate satisfies $d^2\theta/dt^2 + (g/L)\theta = 0$. The solution of the equation is in sinusoidal form, with an angular frequency of $\omega = \sqrt{(g/L)}$, a period of $T = 2\pi\sqrt{(L/g)}$, and a frequency of $f = 1/T$. The equation shows that the angular frequency decreases when the pendulum length increases, while the period increases according to the square root relationship. Taking $g = 9.81\ m \cdot s^{-2}$, the reference values in Table 1 can be obtained: the theoretical period increases from 1.170 s at 34 cm to 2.416 s at 145 cm. These values describe the ideal model, not frequencies extracted from the supplied images [3].

3.2. Meaning of derivatives, integrals and sensor signals

The chain of calculus relationships provides language for interpreting signal shapes. The rapid change in angular position corresponds to a high angular velocity, and the rapid change in angular velocity corresponds to a high angular acceleration. When the angular displacement reaches an extremum, the simple pendulum changes the direction of movement. For an ideal undamped pendulum, the magnitude of the angular velocity is maximum when passing through the equilibrium position. In theory, integration establishes a connection between physical quantities in the opposite direction: acceleration can be integrated to obtain velocity, and then integrated to obtain displacement. However, in practice, this recovery process requires initial conditions, coordinate transformations, filtering, bias removal, and drift control [8, 9].

The inertial measurement unit does not directly output a simple pendulum angle. Accelerometer readings contain both dynamic motion and gravity projections in the sensor coordinate system, while gyroscope readings describe angular velocities about the sensor axes. All three axes show a non-zero response, which may be due to the sensor mounting direction, slight twisting or swinging off a single plane, this phenomenon does not by itself prove that a three-dimensional trajectory has been reconstructed. Subsequent numerical processing should define free-oscillation intervals, filter

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before differentiation, estimate periods using comparable peaks or spectra, and report differences between different experiments. Relevant studies using calibration with a simple pendulum also emphasize that before interpreting the reconstructed quantities, the sensor output should be compared with the independently measured motion [9-12].

4. Results and discussion

4.1. Changes in the supplied curves related to pendulum length

The given image is qualitatively consistent with the pendulum-length effect predicted by the model. The 34 cm and 68 cm records show a tighter oscillation structure after the release event, while the visible activity under the 95 cm condition is more widely distributed in time. This visual comparison is in line with the trend of "frequency decreases when rope length increases" in the presentation material. However, the comparison cannot determine the measured frequency due to the lack of recalculable peak positions and raw timestamps.

It is important to distinguish between reference calculations and observations. The values in Table 1 are calculated from equations, not by fitting image curves. To carry out the experimental verification, it is necessary to specify the analysis interval and the peak- or phase-based criterion, report the results of each test, and provide an uncertainty summary. Within the scope of the available evidence, conclusions are limited to trend-level consistency rather than quantitative validation of the accuracy of the simple pendulum model.

4.2. Multi-axis response and transient behavior and evidence limits

The angular acceleration plots for 34 cm and 95 cm show significant spikes in the vicinity of the manual release, followed by smaller-amplitude oscillations. Since subsequent disturbances are still visible, a single damping coefficient should not be fitted to the entire displayed interval. A more reliable approach is to select a continuous and undisturbed free-oscillation interval for each test before estimating the amplitude envelope. A three-axis diagram helps to identify possible mounting deviations or unexpected motion, but in the absence of orientation compensation and calibrated acceleration data, the multi-axis response does not constitute a spatial trajectory [9, 12]. Table 2 lists the qualitative judgments that the existing images can support, together with their evidence limits, in order to distinguish observable phenomena from unmeasured quantities.

Table 2. Evidence limits for image-based observations

Visible Features in Images	Supportable Qualitative Conclusions	Quantities Not Verifiable From Images Alone
As the pendulum length increases, the oscillation interval appears longer.	The curve is consistent with the predicted direction of the pendulum-length-period relationship.	Measured frequency or period error.
Responses are visible on all three axes, X, Y, and Z.	The response may include orientation effects or out-of-plane motion.	Three-dimensional displacement trajectory.
Release spikes and subsequent smaller movements can be seen.	The transient behavior and possible decay merit further study.	Calibrated single damping coefficient.

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Table 2. (continued)

The densely scaled 145 cm plot lies near a narrow baseline.	The figure shows small fluctuations during quiet intervals.	Sensor accuracy, bias or noise statistics.

Note: The table distinguishes levels of evidence based on the existing image data. All conclusions are qualitative judgments and do not involve quantitative fitting results.

5. Conclusion

The simple pendulum case illustrates three complementary uses of calculus: expressing motion with a function of time, describing the change of motion at a certain moment with a derivative, and establishing theoretical connections between different physical quantities with an integral. The small-angle equation predicts that a longer string produces a longer period, the release, oscillation, and residual motion characteristics shown in the provided chart are also consistent with this relationship. Therefore, this experiment provides a specific way to transition from symbolic relationships to observable time series patterns.

The limitations of this study are that only chart images are available for analysis, and the calibrated raw time series is unavailable, so the measured period, damping parameters, or displacement reconstruction results cannot be reported. Future research can save the raw record of each test, fully document the sensor measurement range, sampling times, mounting orientation, and release conditions, and include calibration, orientation compensation, peak detection or spectral analysis, and uncertainty analysis. After completing the data acquisition, calibration and method validation, the theoretical period and the measured period under different pendulum lengths can be further compared, the damping and multi-axis interaction can be quantified, and the repeatability of the experimental scheme in the teaching of calculus and physics can be assessed.

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