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Application of Financial Mathematical Models in Optimal Pricing Decisions of Monopoly Firms

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Abstract. The pricing decisions of monopoly firms represent a core problem in industrial organization and financial economics, which has significant impaction for market efficiency and consumer welfare. When markets evolve from traditional industrial structures to digital ecosystems, the mathematical modeling of monopoly pricing becomes more sophisticate. Incorporating game-theoretic frameworks, stochastic processes, and optimization techniques. This paper studies how financial mathematical models can support monopoly firms in setting optimal prices. By using the method of comparative analysis, it mainly discusses Cournot competition model, Hotelling spatial competition model, and how these two models are adjusted for adapting to today's market environment. The analysis shows that classical models provide basic understandings into equilibrium pricing under different competitive situations. However, we cannot use these models directly for digital or resources focused markets. People need to change them to consider network effects, algorithm driven dynamic pricing and government rules. The findings suggest that combining stochastic differential equations and game theory can help people to build better pricing models for modern monopoly markets conditions are uncertain. This method also provides some useful suggestions for regulators who hope to keep market efficiency while protecting consumer interests.

Keywords: monopoly pricing, financial mathematical models, Cournot model, Hotelling model, optimal pricing

1. Introduction

Monopoly pricing is one of the foundational problems in microeconomic theory and industrial organization. It studies how firms with market power set prices and get the highest profit under different competitive situations [1]. The theoretical exploration of monopoly pricing has changed a lot since the late 1800s. Early models were static and based on fixed conditions. Later, researchers built dynamic stochastic models to describe the complex modern market [2]. The earliest contributions established the mathematical foundations for oligopoly behavior. After that, scholars added some ideas, such as information asymmetry, dynamic competition and strategic interaction into these models. Despite modeling methods improving greatly, there are still several research gaps. Firstly, most classical models operate under conditions of complete information and deterministic demand, which may not reflect the uncertainty prevalent in digital platform markets [3]. Secondly,

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we still lack enough study on using classic models for new market forms, such as algorithm based pricing on internet platforms [4]. Thirdly, few studies bring financial engineering tools, especially stochastic differential equations, into monopoly pricing analysis, which has received limited attention [5].

This paper examines the application of financial mathematical models to monopoly pricing decisions across three distinct market contexts. The research uses comparative model analysis methodology, systematically reviewing classical oligopoly models and checking whether they still fit modern markets. The research provides a structured framework for understanding how different mathematical models can apply specific monopoly pricing scenarios, offering reference points for future theoretical development and real world regulation design.

2. Mainstream financial mathematical models for monopoly pricing

2.1. Cournot duopoly model

The Cournot model was first proposed by Cournot [1]. It establishes the basic framework for quantity competition between firms. In this model, every firm picks its output at the same time. When a firm makes this choice, it treats the output level of its rivals as fixed. Consider a duopoly market with linear demand function $P = a - bQ$, where $Q = q1 + q2$ represents total market output, and constant marginal cost c applies to both firms. The profit function for firm i is profit function as pi = (a - b(q1 + q2) - c)qi. Maximizing profit with respect to own output yields the reaction function qi = (a - c - bqj)/(2b). Solving all reaction functions together, the Nash equilibrium output per firm can be derived:

$$q^* = \frac{a-c}{3b} \tag{1}$$

The corresponding equilibrium market price is:

$$P^* = \frac{a+2c}{3} \tag{2}$$

This result shows that firms in duopoly market produce less output than competitive output level, but more than the pure monopoly. As a result, the market price sits between competitive and monopoly price. The Cournot equilibrium can be extended to market with n firms, where each firm produces $q^* = \frac{a-c}{(n+1)b}$ and the equilibrium price moves closer to the competitive price as n increases. In the limit as the number of firms approaches infinity, the Cournot result converges to the perfectly competitive equilibrium, where price equals marginal cost. Table 1 lists the key features of classical monopoly pricing models for comparative reference.

Table 1. Comparison of classical monopoly pricing models

Model	Author	Year	Decision Variable	Equilibrium Price	Key Assumption
Cournot	Cournot	1838	Quantity (simultaneous)	P* = (a+2c)/3	Complete info, homogeneous product
Bertrand	Bertrand	1883	Price (simultaneous)	P* = c	Homogeneous product, zero profit
Stackelberg	Stackelberg	1934	Quantity (sequential)	P* depends on leader	First-mover advantage, sequential move
Hotelling	Hotelling	1929	Price + Location	P* = c + t	Spatial differentiation, transport cost

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2.2. Hotelling spatial competition model

The Hotelling model brings product differentiation into spatial competition [6]. It places firms along a "linear city." Consumers are spread across a unit interval [0,1]. Each firm needs to pick two things at the same time: where to locate and what price to charge. There is a transportation cost t for each unit of distance. This cost creates a gap between what consumers are willing to pay and the price firms set. Because of this gap, firms can hold market power even if their products are exactly the same. In the standard two stage game, firms choose locations at first, and then they compete on price. When transportation costs are quadratic, the equilibrium result shows that firms want to make their products different from each other. So it makes price competition less fierce [7]. When transportation costs are linear, the equilibrium price in the Hotelling model is:

$$P^* = c + t \tag{3}$$

This price is higher than marginal cost by the transportation cost parameter. It shows that product differentiation lets firms add a markup on top of the cost. People have used this model widely to study platform competition, geographic pricing and product line management. The Hotelling framework gives a flexible basis for understanding how firms can choose their positions in differentiated markets. It is especially useful for studying internet platform competition. On these platforms, products differ in many ways, not just physical location. Researchers have also extended the Hotelling model. They added network effects, multi-sided markets and dynamic consumer switching behavior. These extensions make the model fit better with today's digital platform ecosystems, where getting new users and keeping old users are key parts of competitive strategy.

3. Application scenarios of financial mathematical models in monopoly pricing

3.1. Application in internet oligopoly markets with differentiated products

The development of digital platforms has changed the way that firms compete in oligopoly markets. Besides traditional price competition, online platforms also use user data, network effects, and pricing algorithms as part of competitive strategies [8]. The Hotelling model has been applied to study competition between two online platforms, especially when the platforms mainly face different groups of users. Some studies have added consumer profiling to the Hotelling model to examine how firms set different prices for different customers. The results suggest that differentiated pricing can reduce firm profits in some cases, when helping firms increase their market sharing. It also does not necessarily to have a negative effect on consumer welfare [9]. Algorithmic personalized pricing is another important development in online markets. By collecting and analyzing consumer data in real time, which firms can adjust prices based on consumers' behavior, making their pricing strategies more flexible [10]. However, actual price differences in online markets are not as large as the theoretical models suggest. Empirical research shows that price differences in online retail markets are about 6 percent on average, while intermediate markups are often below 1.6 percent [11]. This suggests that digital technologies allow firms to use more precise personalized pricing, competition and consumers' ability to compare prices, which still limit the pricing power of firms. Consumer's behavior also plays an important role in the pricing decisions of online platforms. Many consumers use several platforms at the same time, a behavior known as multi-homing. Since these consumers can compare different platforms and switch between them, firms face greater competitive pressure. So platforms cannot rely only on personalized pricing to

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increase their market power. Overall, pricing decisions in Internet oligopoly markets depend not only on firms' costs and market demand, but also on competitors, consumer behavior, and the impact of market information.

3.2. Application in resource monopoly enterprise profit optimization

Resource-based monopoly firms, such as energy and mining companies, face some special challenges when making pricing decisions. Given the finite nature of natural resources, firms must also account for resource depletion, environmental regulations, and long-term profitability. For this reason, the traditional Cournot model can be extended to a dynamic market setting. In this case, firms do not only decide how much to produce at the present time, but also need to consider how much of the resource should be extracted in future periods under the limited resource stocks. For example, in oil markets, the production alliances can be viewed as repeated Cournot games. Member countries decide their extraction levels need to consider how producers outside the alliance may respond. Therefore, pricing decisions in resource industries should not focus only on short-term revenue. They also need to take into account the long-term effects of resource depletion. In practice, factors such as discount rates, extraction costs, and available resource stocks can be included in linear or nonlinear optimization models to determine more reasonable production plans. Compared with the static Cournot model, this dynamic approach provides a more realistic way to analyze production and pricing decisions, when resources are limited. Environmental policies make the problem more complex. For instance, emission limits and carbon pricing can change firms' costs. As a result, it would affect the profitability of resource extraction.

3.3. Application in natural monopoly industry regulation pricing

Natural monopoly industries, such as public utilities and transportation infrastructure, require some government price regulation. If firms in these industries are allowed to set prices just according to their own profit objectives, consumer welfare and overall social welfare may be negatively impacted. In this situation, the Ramsey pricing rule provides a useful way to think about the pricing issue. Its main purpose is to achieve a relatively high level of social welfare that still allows the regulated firm to cover its costs. For industry i, the optimal Ramsey price can be written as:

$$\frac{P_i - c_i}{P_i} = \frac{\lambda}{\varepsilon_i} \tag{4}$$

Here, c_i represents marginal cost, ϵ_i is the price elasticity of demand, and \lambda is the Lagrange multiplier associated with the firm's break-even constraint. The equation shows that the difference between price and marginal cost is related to the elasticity of demand. When consumers are more sensitive to price changes, there is less room for the regulated price to move above marginal cost. On the other hand, products with relatively inelastic demand may allow a larger price markup. In this sense, the Ramsey rule gives regulators a useful theoretical reference when deciding how prices should be set. However, regulated prices do not always match the theoretical Ramsey price. Studies of regulated electricity markets have found noticeable differences between actual prices and Ramsey-optimal levels. This suggests that factors such as regulatory institutions, market conditions, and the way regulation is implemented can affect the final pricing outcome. The Averch-Johnson model also points out a possible problem with rate-of-return regulation. Under this type of regulation, firms may have an incentive to invest more in capital because a larger capital base can increase the return that they are allowed to earn [8]. Therefore, regulation can control monopoly

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pricing, but it may also create other forms of inefficiency. For natural monopoly industries, regulators need to consider several objectives at the same time, including reasonable prices, service quality, cost recovery, and consumer welfare. These decisions become even more difficult when technology changes quickly and the patterns of demand are no longer stable.

Table 2. Application scenarios of financial mathematical models in monopoly pricing

Application Field	Model Used	Key Findings	Limitations
Internet oligopoly markets	Hotelling model	Differentiated pricing increases market share, algorithmic pricing enables near-first-degree discrimination	Limited price dispersion in practice, consumer awareness constrains pricing power, multi-homing reduces platform market power
Resource monopoly enterprises	Cournot model (dynamic extension)	Optimal extraction balances short-term and long-term profits, scarcity rent internalized	Static analysis limitation, uncertainty in resource prices, environmental regulation complexity
Natural monopoly regulation	Ramsey pricing rule	Prices deviate from marginal cost inversely to demand elasticity, welfare maximization under break-even constraint	Systematic deviation from theoretical optimum in practice, A-J effect, technological disruption challenges

4. Existing problems and optimization paths

Table 2 shows that different financial mathematical models are suitable for different monopoly pricing situations, such as internet oligopoly markets, resource monopolies, and regulated natural monopolies. A comparison of these models suggests that each has its own strengths, but no single model can fully describe every type of market.

Several problems still exist in current models. Firstly, many traditional models mainly focus on static or comparative static analysis and pay less attention to dynamic pricing under demand changes and market shocks [5]. The use of stochastic differential equations and optimal stopping theory in monopoly pricing is also relatively limited. In digital markets, network effects and evolving consumer behavior lead to rapid demand fluctuations, rendering traditional deterministic models less effective in handling uncertainty. Secondly, pricing algorithms are creating new challenges for market regulation. As firms increasingly use automated algorithms to set prices, it becomes more difficult for regulators to distinguish normal competition from possible anti-competitive coordination. In addition, traditional Hotelling models usually focus on two competitors, while real digital platforms often involve multiple firms and product characteristics, making the analysis more complicated.

To address these problems, future research could develop dynamic pricing models based on stochastic differential equations. Machine learning could also be combined with regulatory theory to enhance the adaptability of pricing models to shifting market conditions. Hotelling models could be extended to multi-platform and multidimensional markets.

Real options theory may also help firms evaluate investment and pricing decisions under uncertainty. For complex markets with many participants, agent-based modeling and simulation can provide another useful approach. Combining these methods could help narrow the gap between theoretical models and real-world monopoly pricing.

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5. Conclusion

This paper examines how financial mathematical models can be used to study optimal pricing decisions by monopoly firms. The comparison shows that different models can work better in different market situations. The Cournot model is useful for markets with capacity or resource constraints, while the Hotelling model help to explain product differentiation and spatial competition. Ramsey pricing theory is also useful for understanding price regulation in natural monopoly industries. Therefore, no single model can fully explain every monopoly pricing problem.

The main finding is that model selection should depend on market characteristics. For example, Bertrand models are more suitable for price-sensitive markets, while Cournot models can be used when firms face capacity constraints. Stackelberg and Hotelling models are useful, when first-mover advantages or product differentiation are important. In practice, pricing analysis should choose the model that best fits the specific market.

There are also some limitations in this study. The analysis mainly relies on theoretical models and existing research, with limited testing using in real market data. In addition, algorithmic pricing and digital platform competition need further study. Future research could combine real pricing data, computer simulations, and game theory to better understand pricing behavior in complex markets. This could make mathematical models more useful for real-world monopoly pricing decisions.

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