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Competing Models of Cosmic Acceleration: A Comparative Review Using a Late-Time, Model-Independent Hubble Constant

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Abstract. In this review, we compare the H_0 values derived from four representative cosmic acceleration models—ΛCDM, w_0w_aCDM, βϕCDM, and Hu–Sawicki $f(R)$ gravity—against a unified late-time, dark-energy-model-independent calibration of H_0. This work does not conduct a new likelihood analysis, but rather collects results from completed studies and synthesizes data to provide a systematic comparison of the considered cosmological models. The reference H_0 is derived using the inverse distance ladder method, which yields a value of H_0=66.82±0.74 $kms^{-1}Mpc^{-1}$. This establishes a common benchmark against which the estimations of the different cosmological models can be evaluated. For each model, a two-metric evaluation is conducted: the deviation of its inferred from this reference is expressed by the Gaussian tension, and the Bayesian evidence is considered simultaneously to account for the trade-off between goodness of fit and model complexity. In terms of the tension, none of the four models exhibits a statistically significant departure from the reference value, with all inferred H_0 values remaining within approximately 1.3σ. Similarly, the Bayesian evidence also indicates no strong preference toward any beyond-ΛCDM models. When combined for an integrated analysis, these results do not provide compelling evidence that any of the evaluated late-time extensions offers a decisive improvement over the standard ΛCDM model; thus, this baseline framework remains as the most parsimonious and well-supported reference model at present. Meanwhile, the Hubble tension between the local measurement of H_0 and model dependent H_0 remains unsolved. Therefore, in the future, more comprehensive observation and analysis are needed to obtain more effective solutions for current issues and more robust conclusions.

Keywords: cosmic acceleration, cosmic acceleration models, Hubble tension, Bayesian evidence, dark energy

1. Introduction

The discovery of the accelerated expansion of the universe is regarded as one of the most profound paradigm shifts in modern cosmology. For a long period of time before the 21st century, the common expectation was that after the Big Bang, the universe's expansion should gradually decelerate under the influence of the gravitational self-attraction of matter. Nevertheless, this picture

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was overturned in 1998; two independent research teams, using Type Ia supernovae as standardized candles, found that the brightness of distant supernovae was substantially lower than expected in a matter-dominated universe. This observation indicated that the expansion of the universe had been indeed accelerating over the past several billion years. The significance of this discovery was later recognized with the 2011 Nobel Prize in Physics; simultaneously, it launched an era of precision cosmology dedicated to unveiling the physical motivation behind this phenomenon. At present, one of the most prominent developments arising from this discovery is the introduction of dark energy, a new component with negative pressure that drives the accelerated expansion.

Within the framework of the Friedmann–Lemaître–Robertson–Walker metric, which describes a homogeneous and isotropic universe, the state of the accelerated expansion is described in the second Friedmann equation. For acceleration to occur, the total equation-of-state parameter w must satisfy $w<-1/3$, a threshold value that determines cosmic expansion rate. The simplest, and to date, the most broadly accepted candidate is the cosmological constant Λ, corresponding to a perfect fluid with $w=-1$ and a constant energy density. Combined with the fundamental framework of cold dark matter (CDM), a simple, baseline ΛCDM model was established. The ΛCDM model serves as a basis acceleration model that is currently the most generally accepted and has the highest degree of matching with observed data. For instance, precision measurements of the cosmic microwave background anisotropies, most recently by the Planck satellite, datasets constrained the present-day dark energy density parameter $\Omega_\Lambda \approx 0.68$ with sub-percent accuracy. The model also accurately reproduces the large-scale structure of the galaxy distribution, and the baryon acoustic oscillation feature mapped by surveys such as the Sloan Digital Sky Survey and the Dark Energy Survey [1].

Despite its phenomenological success, ΛCDM also faces persistent theoretical and observational challenges in recent years, and its status as the definitive "standard model" is increasingly scrutinized. Issues such as the fine-tuning of the cosmological constant and the coincidence of the current period's dark energy and matter density are among the fundamental problems. Beyond these, a series of ongoing observational tensions have also driven the exploration of alternative models. The most prominent one is the Hubble tension, which refers to the significant discrepancy between the local measurement and the calculated rate of cosmic expansion in the early universe. Additionally, the measurement of the amplitude of matter clustering also presents a relatively mild but equally notable tension, and weak gravitational lensing surveys tend to give values lower than those extrapolated from the cosmic microwave background. Whether these tensions are manifestations of systematic errors or signals of cracks in the standard model are currently being vigorously debated. However, the accumulating evidences suggest that ΛCDM may no longer be the uniquely preferred solution.

Driven by these difficulties, theoretical efforts have followed two complementary routes to explore alternatives to ΛCDM. Firstly, a dynamic dark energy model w_0w_a CDM retains dark energy as an independent component but relaxes the rigid assumption that its equation of state is exactly $w=-1$, allowing dynamical evolution. The most widely adopted parametrization in this class is the Chevallier–Polarski–Linder (CPL) form, which writes the dark energy equation of state as a function of scale factor a: $w(a)=w_0+w_a(1-a)$. The main motivation of this model is aiming to explain the hints from recent observations that show w may deviate from -1 and why w happens to be so close to -1 today [2]. A second route retains dark energy as a dynamical scalar field – the quintessence model – rather than a rigid cosmological constant. $\beta\varphi$CDM introduces a canonical scalar field ϕ with a slowly evolving potential $V(\phi)$, which yields a time-dependent equation of state $w_\phi=p_\phi/\rho_\phi$ that can deviate from -1. This framework was originally proposed to address both the fine-tuning and cosmic coincidence problems, as many quintessence potentials exhibit tracker

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behavior—the field energy tracks the dominant background component during radiation and matter eras, then overtakes it at late times to drive acceleration. Unlike the phenomenological CPL parametrization, which fits $w(a)$ with two free parameters, quintessence provides a physical mechanism grounded in scalar-field dynamics, with specific potentials predicting distinct evolutionary histories [3]. It replaces the static Λ with an active field, offering a testable alternative to ΛCDM through observable effects on structure growth and expansion history.

In contrast to the models that introduced a new energy component outlined above, a fundamentally different avenue is to modify the gravitational concepts on cosmological scales. In these theories, the acceleration is not attributed to a new substance, but rather to a distinct modified gravity framework itself. This theory suggests that Einstein's general theory of relativity model is inaccurate and requires fundamental modifications to adapt to the expansion of the universe. The major difference between this framework and traditional dark energy framework is that it abandons the concept of dark energy and is established through a re-consideration of space or observed matter [4]. Beyond that, although not within the mainstream, more diverse and radical proposals do exist. The categories of these theories include, but are not limited to, completely different cosmic structures, material compositions, and observation perspectives. Despite the fact that these models usually struggle with their own vulnerabilities and are inconsistent with observed data, such studies still help researchers narrow down and examine existing models of cosmic acceleration. Altogether, this work can provide useful insights for developing a more comprehensive understanding of the accelerated expansion.

Given this background, a systematic and objective comparison of competing models has become both timely and necessary. Most existing studies examine distinct frameworks using different datasets and conduct analyses independently, often using different parameters or calibration schemes, which complicates a fair judgement of their relative performance. In particular, the Hubble tension offers a sharp discriminant: a viable model must simultaneously accommodate the existing observational data and the local measurement of expansion rate; yet the degree to which different models can alleviate the tension remains unclear without a common, late-time dark-energy model-independent calibration standard. This review evaluates the model-dependent estimation of H_0 by employing the inverse distance ladder—a method that uses the CMB-derived sound horizon as an absolute ruler and Type Ia supernovae redshift without assuming any late-time specific acceleration model—to establish a reference H_0 value. Against this anchor, the four representative frameworks were chosen: the standard ΛCDM model, the Chevallier–Polarski–Linder (CPL) w_0w_a CDM dynamical dark energy model, the $\beta\varphi$CDM quintessence model, and the Hu–Sawicki $f(R)$ modified gravity model. For each, the model-dependent prediction of H_0 is derived using the baseline Planck CMB and DESI BAO datasets. The comparison depends on a two-metric criteria: quantify its tension through Gaussian tension (σ value) and compare it with the reference value; use Bayesian evidence to penalize unnecessary model complexity and determine the relative preference among models. The core of the essay thus provides a structured comparative analysis, highlighting which model offers a more consistent description of the cosmic expansion and, consequently, provides a viable resolution toward alleviating the Hubble tension; ultimately, this analysis contributes to a deeper understanding of a generalization of the latest research results and a further insight the fundamental physics behind the accelerating universe.

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2. Methodology

2.1. Research background

This present review does not perform a new, independent study with statistical analysis; instead, we adopt a meta-analytic strategy. The core tool used is the inverse distance ladder, a technique that exploits the sound horizon imprinted in the cosmic microwave background (CMB) as an absolute standard of length to calibrate low-redshift distance indicators without assuming any particular late-time acceleration mechanism, thus ensures that each model's distinct estimation of H_0 is evaluated against the same independent reference H_0. For each model, an exclusive prediction of H_0 is obtained from a combined fit to the CMB, baryon acoustic oscillation and supplementary datasets. Two complementary evaluation metrics are used to comprehensively evaluate the effectiveness of each model. The agreement between reference H_0 and each model-dependent H_0^{model} is quantified, and model complexity is then considered. The resulting analysis is conducted to present a systematic comparison of the alternative models from multi-dimensions. This section details the competing models, the extraction of the relevant data, and the quantitative criteria employed to assess and rank their performance.

2.2. Classification of cosmological models under examination

2.2.1. Motivation for model selection

The four models evaluated in this review are chosen because they represent the conceptually distinct routes dedicated to to explain cosmic acceleration. ΛCDM serves as the essential baseline, providing the simplest yet valid explanation of a rigid cosmological constant. The Chevallier–Polarski–Linder (CPL) parametrization introduces a certain amount of complexity by allowing the dark energy equation of state to vary with time, thereby testing whether the data provide evidence for a departure from the strict $w=-1$. Quintessence offers a physically well-motivated, canonical scalar-field realization of dynamical dark energy, rooted in particle-physics considerations. Finally, the Hu–Sawicki $f(R)$ model shifts the paradigm entirely: acceleration is not driven by an exotic fluid, but rather by a modification of the gravitational sector at cosmological scales. Together, these four frameworks cover the three principal theoretical avenues—cosmological constant, dynamical dark energy, and modified gravity—making it possible to compare how different physical mechanisms and theoretical frameworks account for the accelerated expansion.

2.2.2. Basic definition and key features of each model

• *ΛCDM*: The baseline is the flat ΛCDM model, a current standard model characterized by six standard parameters: the physical baryon density $\omega_b=\Omega_b h^2$, the physical cold dark matter density $\omega_c=\Omega_c h^2$, the approximate angular size of the sound horizon $100\theta_{MC}$, the reionization optical depth τ, the scalar spectral index n_s, and the amplitude of primordial fluctuations A_s. The Hubble constant H_0 is a derived quantity, and the dark energy equation of state is fixed to $w=-1$ [1].

•w_0w_aCDM (*CPL dark energy*): The Chevallier–Polarski–Linder (CPL) parametrization extends ΛCDM by allowing dynamical dark energy through the function $w(a)=w_0+w_a(1-a)$, where a is the scale factor $\frac{1}{1+z}$. This introduces two additional parameters, w_0 and w_a, relative to ΛCDM; it reduces to ΛCDM when $w_0=-1$ and $w_a=0$. w_0w_a CDM is the most widely used parametric model

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for describing dynamic dark energy. It was proposed to theoretically solve the "fine-tuning" problem faced by the cosmological constant (Λ), and to explain in observations that the parameter w of the dark energy state equation implied by the latest survey data such as DESI may not be constant at -1, but rather a sign of evolution with redshift [2].

• βφCDM (*Quintessence*): Quintessence models attribute the acceleration to a canonical scalar field ϕ slowly rolling down a potential, but βφCDM also allows for a non-minimal coupling to cold dark matter (CDM) through a coupling constant β [5]. The equation of state is

$$w(z)=\frac{p_\phi}{\rho_\phi}=\frac{\frac{1}{2}\dot{\phi}^2-V(\phi)}{\frac{1}{2}\dot{\phi}^2+V(\phi)} \tag{1}$$

where ρ_ϕ and p_ϕ represents the field's energy density and pressure respectively. The evolution of the field is governed by the modified Klein–Gordon equation with a source term:

$$\ddot{\phi}+3H\dot{\phi}+\frac{dV}{d\phi}=\beta\rho_c \tag{2}$$

where ρ_c is the CDM energy density and β quantifies the interaction strength. This coupling induces an energy transfer between the scalar field and dark matter, capable of altering both the background expansion and the growth of cosmic structures. The extra free parameters are the potential index α, the coupling β, and the initial field value ϕ_i ; the normalization V_0 is fixed by the flatness condition. Quintessence model offers a concrete physical mechanism rooted in scalar-field dynamics, with specific potentials (e.g., power-law or exponential) that predict distinct evolutionary histories. Relative to ΛCDM, the quintessence model introduces at least three extra parameters, and replaces the static Λ with an active field. βφCDM thus provides a physical field theory that makes concrete predictions for the equation-of-state evolution and its coupling to perturbations, and it can be tested against both background and growth-rate data [6].

• *Hu–Sawicki* $f(R)$ (*Modified gravity*): The Hu–Sawicki $f(R)$ model modifies the Einstein–Hilbert action with a non-linear function of the Ricci scalar:

$$f(R)=R-m^2\frac{c_1(R/m^2)^n}{c_2(R/m^2)^n+1} \tag{3}$$

This scalar added the so-called "fifth force" on the basis of general relativity, as a new degree of freedom. In contrast to dark energy models, modified gravity approaches alter the field equations of general relativity through additional parameters, without invoking a new energy component [4, 7].

2.2.3. Similarities and differences among the models

All four models share the same six baseline cosmological parameters, but each extension introduces additional model-specific parameters. They share the common purpose to reproduce the standard thermal history of the universe, so that the physics of recombination and the sound horizon r_d are virtually identical in the early universe. The differences arise in the late-time expansion history and the growth of structure. ΛCDM and the Hu–Sawicki model have very similar background evolutions, but the latter modifies the Poisson equation, which alters the integrated Sachs–Wolfe effect in the CMB and the matter power spectrum. w_0w_a CDM and βφCDM both allow a time-varying equation of state, yet they do so through entirely different physical mechanisms: in w_0w_a CDM the evolution is purely phenomenological, while in βφCDM it follows from a Lagrangian and

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is subject to the Klein–Gordon dynamics. These structural differences propagate into the way CMB and baryon acoustic oscillation (BAO) data constrain H_0. A model with an extended parameter space can shift the preferred H_0 value, but it will also be penalized by the complementary metric Bayesian evidence (see Bayesian evidence and model complexity) unless the improvement in fit is substantial.

2.3. Data extraction and curation

A consistent comparison requires that all H_0 values are tied to the same underlying datasets and calibration standards. The data extraction procedure therefore rests on three main pillars: the observational datasets, the construction of the inverse distance ladder reference, and the retrieval of model-dependent predictions.

2.3.1. Key observational datasets

• *Cosmic microwave background (CMB)*: The Planck 2018 satellite exploration, providing the most precise measurement of the anisotropy of the cosmic microwave background radiation (CMB) temperature [8, 9]. The CMB provides the primary constraint on the sound horizon.

• *Baryon acoustic oscillations (BAO)*: DESI, the distance-ratio measurements from the 6dF Galaxy Survey and other galaxy samples, through the large-scale correlation function of the galaxy [10, 11].

• *Type Ia supernovae*: The Pantheon+ compilation, comprising 1701 light curves of 1550 distinctive, spectroscopically confirmed Type Ia supernovae with full statistical and systematic covariance matrices [12].

2.3.2. Inverse distance ladder reference H_0

The independent reference value of the Hubble constant is obtained through the inverse distance ladder technique [13]. Because this methodology bypasses the calibration of local distance steps (Cepheid variables) and not relying on any assumptions about the energy composition of the late universe, it is referred to as a "late-time dark-energy model-independent" approach. The method starts from the comoving sound horizon at baryon drag, which is a fixed calibration for all subsequent steps,

$$r_d = \int_{z_d}^{\infty} \frac{c_s(z)}{H(z)}\, dz \tag{4}$$

where $c_s(z)$ is the sound speed of the photon–baryon fluid and z_d is the redshift of baryon decoupling. The physical processes that determine r_d are well understood and virtually identical in all models considered, provided their early-time dynamics reduce to standard ΛCDM. A Gaussian prior on is therefore derived from the Planck 2018 TT, TE, EE+lowE likelihood assuming the standard six-parameter cosmology, yielding

$$r_d = 147.09 \pm 0.26 \text{Mpc} \quad .$$

This absolute length is then used to anchor the BAO distance ratios $D_M(z)/r_d$, $D_H(z)/r_d$ and $D_V(z)/r_d$. A fit to the BAO ratios and the Pantheon+ apparent magnitudes, assuming a flat ΛCDM background purely as a kinematic expansion history, yields the posterior distribution of H_0 [14]. The

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free parameters in this fit are benchmark H_0, the matter density parameter Ω_m, and the supernova absolute magnitude M_B ; r_d is kept fixed.

2.3.3. Extraction of model-dependent H_0 and ΔlnZ

For each of the four models, the Hubble constant and logarithmic Bayes factor is collected from recent independent research and articles. In light of the dataset-dependent variation in results across studies, the guiding principle for selecting literature is that the utilization of Planck 2018 high-ℓ TTTEEE + lowE likelihood and DESI datasets is foundational, with minimal inclusion of additional datasets, so that the resulting H_0 reflects the intrinsic prediction of the model. The H_0 and the ΔlnZ for each framework are retrieved from such studies; the values and errors are taken directly from the published tables.

2.4. Evaluation criteria

To compare competing cosmological models while accounting for differences in their parametric complexity, the performance of each model is judged by two complementary statistical metrics—tension metric and Bayesian evidence, respectively—that address the alleviation of the Hubble tension and the precision of the model.

2.4.1. Tension metric

The level of agreement between a model's H_0 value prediction and the inverse-ladder reference H_0 value is expressed as a Gaussian-equivalent tension,

$$T=\frac{H_0^{model}-H_0}{\sqrt{\sigma_{model}^2+\sigma_{inv}^2}} \tag{5}$$

A value $|T|\lesssim 2$ is interpreted as a substantial reduction of the tension between model-dependent prediction of H_0 and referenc H_0, while larger values indicate that the model does not significantly reconcile the early- and late-universe distance scales. This single value provides a transparent, approachable and comparable analysis.

2.4.2. Bayesian evidence and model complexity

Another primary model-selection criterion that integrates the likelihood over the entire prior parameter space in contrast to χ^2. Because a model with more parameters can always fit the data at least as well as a simpler one, an increase in likelihood does not necessarily imply a better description of nature. Consequently, the Bayesian evidence provides a robust, unbiased framework; this approach ensures that model preferences are less likely to be driven by differences in parametrization, but by genuine predictive performance. The Bayesian evidence Z naturally incorporates an Occam's-razor penalty for unnecessary complexity. It is defined as the integral of the likelihood over the prior,

$$Z=\int L(\theta)\,\pi(\theta)\,d\theta \tag{6}$$

For each model, the evidence of the baseline CMB+BAO fit is collected from the same data sources used for the extraction. Models are compared through the logarithmic Bayes factor,

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$$\Delta \ln Z = \ln Z_{\text{model}} - \ln Z_{\Lambda CDM} \quad (7)$$

All models are compared to baseline model ΛCDM. The Bayes factor is interpreted on two complementary aspects, its sign and its absolute magnitude, utilizing the Jeffreys' scale [15]. The sign provides an directional verdict of preference, positive $\Delta \ln Z$ favors the alternative model over ΛCDM, whereas a negative value signals that the standard model remains preferred. The absolute value of $\Delta \ln Z$ quantifies the decisiveness of this preference: $0<|\Delta \ln Z|<1$ indicates an inconclusive or negligible preference, $1\leq|\Delta \ln Z|<2.5$ corresponds to weak evidence, $2.5\leq|\Delta \ln Z|<5$ represents medium preference, and $|\Delta \ln Z|\geq 5$ indicates strong or decisive evidence.

2.4.3. Combined ranking

The final assessment is presented by the combined evaluation of both Bayesian evidence and the σ value obtained from this equation to rank the models' overall performance in calculating the rate of cosmic expansion, verifying the efficiency of each model in solving tension. A model is considered genuinely competitive if it reduces the Hubble tension below the 1σ threshold and is not decisively disfavored by the Bayesian evidence relative to ΛCDM. Through using these methods to quantify the difference between the inferred H_0 of each model and the reference H_0, conducting a standardized and rational comparison between the considered cosmic acceleration models.

3. Results & data

3.1. The H_0 results

The H_0 results obtained are presented below as the first evaluation criteria. All data, drawn from completed studies and research, are summarized into Table 1. For each result, a brief analysis and interpretation is appended respectively. According to Table 1, H_0 predictions predictions against the inverse distance ladder baseline cluster tightly. While ΛCDM, w_0w_a CDM, and Hu–Sawicki $f(R)$ utilize five probes, βϕCDM relies solely on DESI and Planck. Clustering within 1.31σ, none temper the Hubble tension decisively.

Table 1. The H_0 values for each model and their tension with the reference H_0

Model	H_0	ΔH_0 ($kms^{-1}Mpc^{-1}$)	Tension (σ)
Inverse distance ladder (Reference)	66.82 ± 0.74	0	-
ΛCDM	67.77 ± 0.63	+0.95 ± 0.97	≈ 0.98
w_0w_a CDM	66.61 ± 0.76	0.21 ± 1.06	≈ 0.20
βϕCDM*	$67.76^{+0.60}_{-0.58}$	$+0.94^{+0.95}_{-0.94}$	≈ 0.99
Hu–Sawicki $f(R)$	68.10 ± 0.64	+1.28 ± 0.98	≈ 1.31

*The reference H_0 is derived via the inverse distance ladder. All tested models share the identical datasets DESI and Planck 2018 CMB; model ΛCDM, w_0w_a CDM and Hu–Sawicki $f(R)$ each use 3 additional datasets: FRBs, PantheonPlus and CC. For Quintessence, the asymmetric $\sigma_{\Delta}$is symmetrized for the tension estimate [16, 17].

*βφCDM, constrained with DESI + Planck CMB only.

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• Inverse distance ladder (Reference): The one-dimensional posterior distribution of H_0 obtained from the late-time model-independent inverse distance ladder analysis, using a cosmographic expansion up to fifth order in redshift, yields:

$$H_0=66.82\pm0.74\,\mathrm{kms^{-1}Mpc^{-1}} \quad .$$

Set as the reference, all model-dependent H_0 are compared against this baseline H_0 using the same data combination (except for βϕCDM, which uses only DESI and Planck CMB; see below). The result is robust to the choice of polynomial order: fitting with a fourth-order expansion yields $H_0=66.54\pm0.70\,\mathrm{kms^{-1}Mpc^{-1}}$, a shift of only $0.28\mathrm{kms^{-1}Mpc^{-1}}$. Relaxing the assumption of spatial flatness ($k\neq0$) also leaves H_0 essentially unchanged, $H_0=66.59\pm0.69\,\mathrm{kms^{-1}Mpc^{-1}}$. The negligible shifts in the central value produced by systematic variations demonstrate the overall preciseness of the inverse distance ladder measurement of H_0 [13].

• ΛCDM: The standard flat ΛCDM model, constrained by 5 datasets: FRBs, PantheonPlus, DESI, CC and CMB, yields:

$$H_0=67.77\pm0.63\,\mathrm{kms^{-1}Mpc^{-1}} \quad .$$

This result is marginally higher than the reference by $+0.95\pm0.97\mathrm{kms^{-1}Mpc^{-1}}$, with a corresponding tension of only 0.98σ. The tension value remains in a low and acceptable margin, indicating the consistency between the two H_0 values and demonstrating a high consistency with the inverse-ladder reference [16].

• w_0w_a CDM: For the dynamical dark energy model with a Chevallier–Polarski–Linder (CPL) parameterization, constrained by the 5 identical datasets, predicts:

$$H_0=66.61\pm0.76\,\mathrm{kms^{-1}Mpc^{-1}} \quad .$$

This value is slightly lower than the reference by $-0.21\pm1.06\mathrm{kms^{-1}Mpc^{-1}}$, a subtle difference that is statistically negligible at the 0.20σ level. The predicted result of this model has the highest consistency with the reference value, which most effectively resolved the tension [16].

• βφCDM: The quintessence model with a canonical scalar field minimally coupled to gravity gives:

$$H_0=67.76^{+0.60}_{-0.58}\mathrm{kms^{-1}Mpc^{-1}} \quad .$$

The corresponding deviation from the reference is $+0.94^{+0.95}_{-0.94}\mathrm{kms^{-1}Mpc^{-1}}$; the asymmetric errors are symmetrized for the tension significance estimate, which is approximately 1.0σ. In contrast to other tested models, which employ all five datasets, βφCDM uses only the DESI and Planck 2018 CMB datasets. Its predicted value shows no significant deviation from the reference and lies well within the 1σ uncertainty [17].

• Hu–Sawicki $f(R)$: The modified gravity model, which introduces a scale-dependent enhancement of gravitational interactions on large scales and uses all 5 datasets, is constrained to:

$$H_0 = 68.10 \pm 0.64\,kms^{-1}Mpc^{-1} \quad .$$

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This model demonstrates the largest deviation among all the models considered, with $\Delta H_0 = +1.28 \pm 0.98 kms^{-1}Mpc^{-1}$, corresponding to a 1.31σ tension relative to the reference. Although the most prominent shift in the table, it still remains well below the conventional 2σ threshold for statistical significance, providing no robust evidence for a deviation from general relativity or an effective alleviation of the tension [16].

From the H_0 perspective, every model yields a value that lies within 2σ of the reference: 0.98σ, 0.20σ, 0.99σ, and 1.31σ respectively. Even βϕCDM, which uses only two datasets (DESI and Planck CMB) rather than the full five-probe combination, recovers a value that is rational and fully consistent within 1σ. The small deviations between model-dependent H_0 estimations are statistically insignificant, indicating that none of the extended frameworks demonstrates significantly superior performance to standard ΛCDM.

3.2. The ΔlnZ results

As the second evaluation criteria, the ΔlnZ results obtained are presented below. As with the ΔlnZ values above, all data drawn from completed studies and research are compiled in Table 2.

Table 2. The ΔlnZ values for each model

Model	ΔlnZ
ΛCDM	0
w_0w_a CDM	-2.44
βϕCDM*	+0.81
Hu–Sawicki $f(R)$	+0.07

*The ΔlnZ represents the lnZdifference between the tested model and benchmark ΛCDM; all the derived ΔlnZ values and dataset combination for each model are identical to Table 1 [16, 17].

*βφCDM, constrained with DESI + Planck CMB only.

According to Table 2, Bayesian evidence against ΛCDM penalizes overfitting, disfavoring CDM, which yields a value of −2.44. Conversely, marginal shifts for βϕCDM and Hu–Sawicki $f(R)$ fail to secure a statistical advantage.

• ΛCDM: As the fiducial baseline, the standard flat ΛCDM model is assigned ΔlnZ=0 by definition [16]. This choice is natural given that ΛCDM is the most widely accepted and observationally successful framework present, fitting a wide range of independent probes without requiring fine-tuned or ad hoc assumptions; its simplicity also guards against overfitting, making it an ideal benchmark for Bayesian model comparison. Thus, all other models are evaluated against this reference using the same data combination (except for βϕCDM, which uses only DESI and Planck CMB; see below).

• w_0w_a CDM: The dynamical dark energy model with the CPL parameterization yields ΔlnZ=2.44 relative to ΛCDM. This negative value indicates that the data actually favor the simpler ΛCDM over its w_0w_a extension. According to the Jeffreys scale, |ΔlnZ|=2.44 falls in the upper end of the weak evidence range against w_0w_a CDM, demonstrating that the introduction of two additional free parameters (w_0 and w_a) does not provide a sufficiently improved fit to compensate

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for the Occam penalty. The joint dataset does not strongly support a temporal evolution of the dark energy equation of state, and the standard cosmological constant remains relatively preferred [16].

• βφCDM: The βϕCDM model yields $\Delta\ln Z=+0.81$, the highest positive value among 3 test models that indicates a slight preference for this model over ΛCDM. However, a value below 1 is classified as negligible or insignificant evidence. Therefore, while the data show a marginal inclination toward βϕCDM, the improvement in fit is insufficient to conclusively claim that this model is superior to ΛCDM. Simultaneously, it is also crucial to note that βϕCDM is constrained using only DESI and Planck 2018 CMB datasets, whereas the other models employ all five datasets (including FRBs, PantheonPlus, and CC). Despite its reduced datasets, the model still recovers a $\Delta\ln Z$ value that is practically identical to the ΛCDM baseline [17].

• Hu–Sawicki $f(R)$: The modified gravity model gives $\Delta\ln Z=+0.07$. This value indicates that, in terms of Bayesian evidence, Hu–Sawicki $f(R)$ and ΛCDM are statistically equivalent. With the full dataset combination, the model still yields a $\Delta\ln Z$ value practically identical to the ΛCDM baseline. This inconclusive evidence, $0<|\Delta\ln Z|<1$, neither provides any meaningful preference for either model nor suggests that $f(R)$ gravity yields a considerable improvement over ΛCDM [16].

From the Bayesian evidence analysis, w_0w_a CDM model is disfavored relative to ΛCDM; the $f(R)$ model exhibits only weak positive evidence; βϕCDM is virtually indistinguishable from the baseline. Accordingly, none of the models reaches the threshold for a strong and decisive evidence in favor of any extension over the ΛCDM framework.

4. Discussion

The results presented in §3 provide a coherent, evaluative assessment regarding the effectiveness and practicability of the three distinct beyond-ΛCDM models and the standard cosmological model. When interpreted together, the H_0 constraints and Bayesian evidence offer complementary insights into whether the evaluated frameworks are favored by the contemporary multi-probe datasets. Through this analysis, a consistent conclusion emerges: none of the tested models provides a statistically compelling improvement over ΛCDM; the standard model remains as the preference among these candidates.

Among the results, it's noteworthy that the H_0 values estimated across all models remain in a narrow range. Every model yields an expansion rate that falls within 1.3σ of the inverse distance ladder reference. This result not only indicates a high level of confidence, but also achieves consistency with the inverse-ladder reference. The uniformity can be mainly attributed to two aspects: the analogous physical motivations of each model and the constraints from the dataset combination itself. All 4 evaluated models aim to explain the accelerated expansion of the late universe (low redshift). Therefore the H_0 values must match under ideal situations; otherwise, it cannot be consistent with existing data and other independent predictions. Meanwhile, this outcome implies that the datasets employed tight constraints that even substantial modifications to theoretical frameworks cannot shift H_0 away from the baseline.

Each model and its corresponding results are discussed below. Firstly, w_0w_a CDM. It offers the best relief of the tension between model-dependent H_0 value and the reference value, yet it is least favored by the logarithmic Bayes factor. This performance directly indicates the shortcomings of w_0w_a CDM: if a model achieves high-precision fitting on certain observables by introducing extra free parameters without a commensurate improvement in overall fit, the Occam's razor in Bayesian evidence will automatically penalize the model. The tension of 0.20σ does not have a significant advantage over 0.98σ; $\Delta\ln Z=-2.44$ quantifies the insufficient improvement of w_0w_a CDM in

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overall fitting. Although several evidence suggests that w may evolve over time, those evidence is inconclusive and Bayesian evidence significantly reduces the data preference of the model [18]. In summary, the additional complexity of w_0w_a CDM validated that the simplest baseline ΛCDM is still statistically sufficient.

On the H_0 tension criterion alone, βϕCDM's performance is indistinguishable from ΛCDM's. Between these two models, their tension values has only a subtle difference of two decimal places. On the other metric, βϕCDM obtained the highest Bayesian evidence preference among the three beyond-ΛCDM models, with $\Delta\ln Z=+0.81$. Both values lie within the range of inconclusive/insufficient evidence, incapable of determining the model that is more favored by the data and observation. However, this model is unique and differs significantly from other tested models. The H_0 and $\Delta\ln Z$ values were extracted from another independent study that utilized different datasets compared to the other extended models. The βϕCDM, as well as its benchmark ΛCDM, is constrained using only DESI and Planck CMB datasets, whereas all other remaining models employ the full five-probe combination including FRBs, PantheonPlus, and CC [17]. Accordingly, βϕCDM is not on an equal footing with other models in evaluation; the asymmetry in the use of this data makes direct comparison complex and not statistically rigorous. Therefore, this result does not offer a comparable conclusion on βϕCDM's performance.

Lastly, compared to the models above, the overall evaluation result of Hu–Sawicki $f(R)$ is relatively transparent. Hu–Sawicki $f(R)$ gives the largest H_0 value deviation among the cases considered, a tension of 1.31σ. Its Bayes factor, conversely, is only $\Delta\ln Z=+0.07$. These values indicate that the Hu–Sawicki $f(R)$ model is essentially indistinguishable from the ΛCDM baseline; with data providing no meaningful preference for a modified gravity framework over the standard cosmological constant. Furthermore, derived from an independent research, it is worth noting that galaxy morphology provides stringent constraints on the Hu-Sawicki $f(R)$, with no evidence found for a fifth force, thus effectively ruling out astrophysically relevant regimes of the $f(R)$ model [19]. Taking everything into account, it can be concluded that although the observation does not completely rule out the model, Hu-Sawicki $f(R)$ is not statistically favored over the standard ΛCDM framework, and there is no evidence in support of modifications to general relativity.

When interpreting these results as a definitive assessment of the models, two important caveats in procedure must be emphasized. The first restriction is the inconsistent dataset usage of the βϕCDM model previously mentioned. The H_0 value and $\Delta\ln Z$ value of βϕCDM is collected from a distinct study, constrained using only DESI and Planck CMB datasets; whereas all other extended models employ the full five-probe combination including FRBs, PantheonPlus, and CC. Different data combinations have an undeniable impact on model fitting; thus, this discrepancy in data usage is not suitable for direct comparisons. Since the tension and the Bayesian evidence for βϕCDM are not strictly on equal footing with that of the other models, these values serve as a relative reference and cannot be interpreted as a valid final result in this review. If considered exclusively, the outcome demonstrates, at most, weak evidence towards βϕCDM and does not provide a decisive preference for this model. Therefore, the simpler framework of the cosmological constant should still be given priority consideration.

The second caveat concerns the broader context of the Hubble tension. Although in terms of the H_0 value, all models show a high degree of agreement with the inverse distance ladder reference within the 1.3σ range, this reference value itself varies significantly from the local distance-ladder measurements from SH0ES. The local measurement of H_0 yields $H_0=72.9^{+2.4}_{-2.2}\,\mathrm{kms^{-1}Mpc^{-1}}$, considerably higher than our reference $H_0=66.82\pm0.74\,\mathrm{kms^{-1}Mpc^{-1}}$ [20]. This difference arises

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from the distinct methodologies used in these two approaches. The local measurement obtains H_0 directly from Type Ia supernovae observations, whereas the inverse distance ladder adopted in this work uses early-universe probes as an anchor to infer H_0. All models examined remain close to the reference value; none exhibits a shift large enough to alleviate this discrepancy with local measurements. This pattern does not necessarily invalidate the models themselves. Rather, it reflects the nature of the data combinations used here, which rely mainly on probes calibrated against or constrained by early-universe physics. While these models introduce different modifications to ΛCDM, they are all motivated by the same goal of explaining late-time cosmic acceleration. However, these modifications do not substantially alter the sound horizon or the late-time expansion history, and therefore cannot significantly raise the estimated H_0 toward the locally measured value. At present, there is no definitive resolution to this tension. Resolving it may require new physics that couples the early and late universe, or a better understanding of systematic uncertainties in future data measurements [21].

In addition to the two major caveats discussed above, several additional supplements should be acknowledged. Although the priors adopted in the source studies are consistent with those commonly used in the literature, alternative prior choices may lead to variations in the magnitude of ΔlnZ, especially in models involving poorly constrained parameters. The model's performance is also inevitably limited by the dataset combination extracted from current probes. Future observations from surveys such as Euclid may provide more accurate or supplemental information, thereby offer stronger constraints on the differences between the models considered in this review [22]. Moreover, because the H_0 constraint for βϕCDM are asymmetric, it is symmetrized when estimating the tension. This treatment represents a minor approximation and is not expected to affect the main conclusion, but we should still note this point. Lastly, to date, we have not identified a previous study in which exactly the same combination of five datasets was applied across all models considered in this review. A complete five-dataset combination should therefore be applied to βϕCDM in future analyses, ensuring that it is evaluated on an identical observational basis as the other models. Such a consistent comparison can provide a more robust conclusion about the relative performance of these models.

In summary, the joint H_0 and Bayesian evidence analyses point to a consistent conclusion: none of the extended models evaluated here—including w_0w_a CDM, βϕCDM, and Hu–Sawicki $f(R)$ — provides statistically significant evidence for an improvement over the standard ΛCDM framework. In light of the current multi-probe data, ΛCDM remains as the most parsimonious description consistently, achieving a favorable balance between model complexity and observational performance. Related to the Hubble tension, the final results show that all these extensions also fail to produce a meaningful upward shift toward the H_0 value inferred from local measurements. This persistent lack of progress suggests that the Hubble tension is unlikely to be resolved simply by introducing these forms of late-time physics beyond the standard ΛCDM scenario, therefore more comprehensive observation and analysis are required in the future.

5. Conclusion

This review compares four competing models of cosmic acceleration within a common, model-independent calibration of H_0. Rather than performing a new likelihood analysis, we provide a secondary meta-analysis by synthesizing data from completed studies and establishing a unified evaluation framework. Based on the two metrics: Gaussian tension and Bayesian evidence, model

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ΛCDM, w_0w_a CDM dynamic dark energy, βϕCDM quintessence, and Hu–Sawicki $f(R)$ modified gravity were evaluated with their combined results respectively.

All models have shown overall consistency with reference, with all inferred H_0 values lying within about 1.3σ of the reference value. Among models tested, the w_0w_a CDM gives the smallest tension, 0.20σ. This result, however, provides a more significant overall conclusion when interpreted with the Bayesian evidence. The most successful w_0w_a CDM in H_0 fitting has the lowest preference in ΔlnZ value, ΔlnZ=-2.44, suggesting that using extra free parameters suffers a penalty on overfitting. For the other two beyond-ΛCDM models, both of their corresponding results fall within the negligible evidence range, thus failing to provide strong evidence that outperforms ΛCDM.

Ultimately, the outcome can be therefore concluded: none of the considered extensions provides compelling evidence for an improvement over ΛCDM; the simplest standard framework is still regarded as the best-supported description for the accelerated expansion. Simultaneously, the Hubble tension remains unsolved through late-time modifications alone, and model comparison using identical datasets is still missing. Making progress on all issues mentioned will require more thorough observations and a joint analysis of multiple probes in future.

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